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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

MISSISSIPPI RIVER BRIDGE AT DUBUQUE, IOWA

BY R. N. BERGENDOFF[1] AND JOSEF SORKIN,[2] MEMBERS, ASCE

SYNOPSIS

A number of unique and interesting features have been incorporated in the Julien Dubuque Bridge, opened on August 31, 1943, for highway traffic across the Mississippi River between Dubuque, Iowa, and East Dubuque, Ill. Not only is the 845-ft main channel span the second longest clear span across the Mississippi River, but the 1,537-ft three-span continuous truss of which it forms the central span is the longest of its type in the world. There are only two similar structures in North America—the Saint Lawrence River Bridge west of Montreal, Que., Canada, and the Meramec River Bridge south of St. Louis, Mo., both of which have much shorter arch spans (400 ft and 264 ft, respectively).

The 1,537-ft three-span continuous structure is supported on eight shoes, all expansion type. End bearings are 8-ft-long rocker columns; intermediate bearings have shoes riding on nested rollers. At the east end of the central arch span a large pin embedded on the center line of a supporting pier engages a floor beam of the superstructure to transfer longitudinal and transverse forces from the three-span superstructure unit to the substructure. Torque in piers due to transverse forces is thus largely eliminated. The dead load weight of pier 18, at the west end of the central arch span, was materially reduced by using sand fill in the central section of the pier base, thus minimizing the size of the pier and the number of supporting piles required.

The three-span continuous truss structure comprising the 845-ft tied arch center span, flanked by two 346-ft end spans, presents a pleasing appearance, affords to traffic a view unobstructed by diagonal truss members across the 845-ft main channel span, and has inherent advantages for erection. In economy it compared favorably with cantilever, tied arch, and suspension span layouts studied for the crossing.

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **November 1, 1947.**

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[2] Engr. of Design, Howard, Needles, Tammen & Bergendoff, Kansas City, Mo.

Introduction

The bridge serves local traffic between Dubuque, Iowa and East Dubuque, Ill., as well as heavy through traffic on United States Highway No. 20 and the north-south traffic of United States highways Nos. 61 and 51. The new structure replaced a privately-owned toll bridge erected in 1887 some distance upstream from the new location.

Various sites were studied. Navigation requirements at some of the alternate sites permitted shorter channel spans, but none provided as satisfactory alinement or as satisfactory connections to principal routes of highway traffic. Viaduct approaches to the river spans pass above all railroad tracks at both ends of the structure. Both the length of the channel span and the locations of the piers supporting it were dictated by the War Department to satisfy navigation interests and to permit maneuverability of boats approaching the barge terminal on the west bank of the river directly north of the bridge. A vertical clearance of 64 ft above normal pool elevation over a horizontal distance of 800 ft is provided by the main channel span.

Description of Structure

At Dubuque the Mississippi River is about 1,700 ft wide at pool level, at which stage it has a maximum depth of 35 ft. High water at times is more than 15 ft above pool level, and inundates the entire valley from abutment to abutment of the bridge—a distance of approximately 6,000 ft.

Outcrops of limestone are in evidence at the bluffs on both sides of the valley. Within the limits of the bridge site, however, bedrock is 400 ft below pool level, and is overlain with a deep deposit of coarse sand and gravel.

The 7,082-ft total length of the bridge project is divided into 1,320 ft of paved embankment approaches and 5,762 ft of open structure. The 1,537-ft main river channel span unit is a three-span continuous through truss structure, consisting of an 845-ft tied arch center span flanked by 346-ft truss side spans as shown in Fig. 1. Approach viaducts are steel continuous deck girder units,

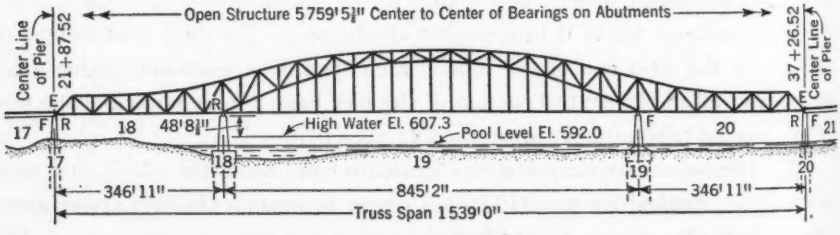

Fig. 1.—Elevation of Three-Span Main River Channel Unit of Mississippi River Bridge at Dubuque, Iowa

with spans up to 187 ft long. Long girder spans adjacent to the river structure have curved bottom flanges; all other spans are of parallel flange girder type. The Iowa approach, made up of seventeen spans from 44 ft to 187 ft long, has a total length of 1,448 ft; the Illinois approach contains twenty-three spans from 85 ft to 184 ft long and has a total length of 2,757 ft. The bridge, which crosses

above twenty-nine tracks of four different railroads, provides one 5-ft sidewalk and a two-lane roadway 24 ft wide. The roadway is designed for H-20 loading as given by the 1935 specifications of the American Association of State Highway Officials (AASHO), modified to assume 12-ft lanes in the design of main trusses and girders, but using 9-ft lanes in the design of the floor system. Fig. 2 shows the roadway cross section of the truss spans.

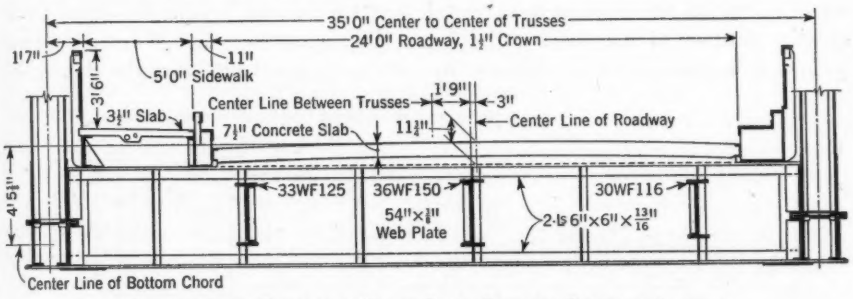

FIG. 2.—CROSS SECTION OF TRUSS SPANS SHOWING ROADWAY LAYOUT

The entire substructure is supported on timber piles. Untreated timber piles were used wherever fully embedded below permanent ground-water elevation; elsewhere creosoted timber piles were used. Except for pier 18, the entire substructure and the viaduct approaches are of conventional design.

The principal purpose of this paper is to present the development of the design and the expedient erection of the main three-span continuous unit.

THREE-SPAN THROUGH TRUSS CONTINUOUS UNIT

With the length of river channel span determined by navigation requirements, the west span of the continuous unit was made sufficiently long to clear the railroad tracks on that bank of the river and to provide a nominal positive dead load reaction at the end bearing. Symmetry was obtained without sacrificing economy by making both end spans of the continuous unit of equal length. By this arrangement a maximum negative moment was introduced at the intermediate supports through utilization of the full dead weight of the end span.

After studies had been made of appearance and section requirements for various alternative heights, the rise of the 845-ft 2-in. central arch span was selected as 106 ft from center of tie to center of bottom chord of the truss. For an 845-ft tied arch span of conventional design and a 1:6 rise ratio, a corresponding height of approximately 130 ft would have been required. Truss depths were selected as 40 ft from center to center of chords at the end portal, 70 ft at the intermediate supports, and 24 ft at the center of the arch. Transverse spacing of trusses is 35 ft on centers. The center line of roadway is offset 2 ft to balance the sidewalk and roadway loads between the two trusses. With this arrangement the maximum stresses in the two trusses are approximately the same, so the trusses were made identical. A saving of floor beam metal would have resulted from a closer spacing of trusses, with sidewalk outside; but truss loadings would have so differed that the trusses would not have been identical and duplication of fabrication would have been sacrificed. Also, closer truss

spacing would have reduced what was regarded as a desirable width-length ratio for lateral stability.

Study of various truss systems resulted in adoption of the Warren system. Compared to the Pratt type, the Warren truss has the advantage of single-panel loading on vertical members and constant chord stresses in double-panel lengths. Subdivided truss systems with shorter floor panels would have secured some saving in the floor system metal, but this saving would be offset by a greater number of members and joints. In appearance the Warren system seemed to harmonize most satisfactorily with the general outline of the structure.

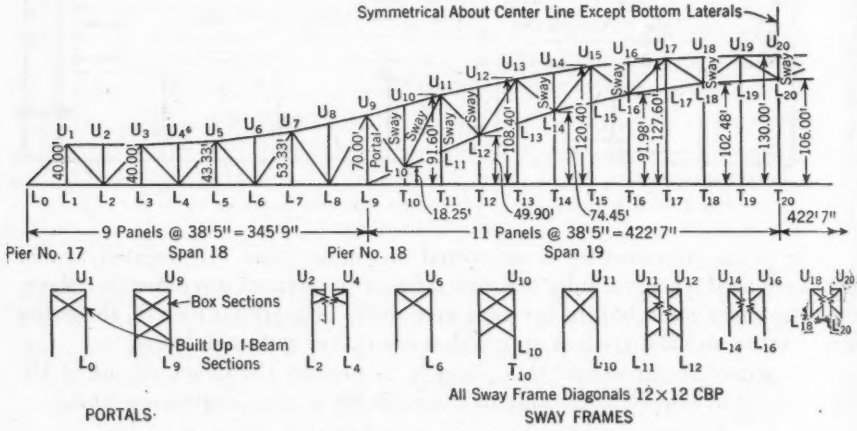

FIG. 3.—GENERAL DIMENSIONS OF TRUSS SPANS, LATERAL BRACING, SWAY FRAMES, AND PORTALS

The general arrangement of lateral and sway bracing is shown in Fig. 3. The bottom lateral system in the plane of the roadway deck is a conventional rigid single-plane system of double-angle members. The top lateral system in the planes of the top and bottom chords of the arch span is a rigid double-plane system of H-members. Sway frames are provided in the plane of all vertical posts except U8–L8. They are also provided in the planes of hangers L11–U11 and in the planes of diagonals L10–U11. The end posts and the posts at the intermediate piers are braced with massive portals. Horizontal members were omitted from sway frames and portals in end spans to improve appearance.

All panels are 38 ft 5 in. long, there being nine panels in the end spans and twenty-two panels in the center span. Floor beams rigidly framed to the trusses and hangers support three lines of roadway stringers on 10-ft, 3-in. centers, together with a sidewalk fascia beam. Transverse crossbeams, 4 ft $9\frac{5}{8}$ in. on centers, rest on the top flanges of the stringers. The roadway deck is a $7\frac{1}{2}$-in. concrete slab, reinforced transversely with rail steel bars. A steel safety curb separates the 24-ft roadway, from the 5-ft sidewalk on the north. Along the south side of the roadway a similar curb is provided, also steel. The south handrail is 24 in. outside the edge of the roadway, and the intervening space is floored with steel plate to provide an auxiliary sidewalk. Handrails 3 ft 6 in. high above the sidewalks are steel, with steel posts. Roadway curbs

are 10 in. high. Above streets and railroad tracks the curbs are closed; elsewhere they are left open to provide full drainage and to facilitate cleaning the roadway and removing snow. To eliminate undesirable chord stress participation and to relieve possible temperature stresses due to unequal thermal movement of the floor system, floor contraction joints and stringer expansion bearings were provided at L4, L7, T11, T15, and T18.

Design Procedure

Because of the magnitude and statical indeterminacy of the structure, erection methods had to be predetermined and sections provided for stresses encountered during the various stages of erection. The design finally adopted contemplated a three-stage erection program comprising: (1) Cantilever erection of the trusses and bracing of the central span; (2) erection of hangers, floor beams, and arch span ties after the trusses had been connected to act as a three-span continuous unit; and (3) installation of the floor system and deck after making the tie connection of the arch span. Although a 50% increase of normal allowable stresses was permitted for erection purposes, a number of members nevertheless were governed by erection stresses.

The form of the trusses of the central span lends itself to an economical arrangement for cantilever erection with maximum depth over intermediate supports tapering to a minimum depth at the center of the span. To minimize the added material required for erection purposes, erection of the arch by cantilever methods was limited to placement of trusses and bracing. After closure of the trusses at the center of the arch span, added carrying capacity of the trusses became available through conversion to a three-span continuous system by vertical adjustment of the end reactions. Utilizing this increased capacity, the hangers, floor beams, and ties of the center span were added. By adjustment of the horizontal distance between L9 and L20, the tie closure could then be made. Stringers, crossbeams, curbs, railings, and concrete floor slabs were then added to complete the span. The design drawings were complete in every detail, including the make-up of the members and details of splices and joints. It may be noted that the superstructure contractor elected to carry out erection generally in full accordance with the original design scheme.

Floor System.—For design of the concrete roadway slab the so-called Westergaard theory[3,4] of wheel load distribution was used. Welded bar trusses were planned originally for the slab reinforcement; subsequently, emergencies of World War II made it necessary to use bent bars of equivalent area. Crossbeams, designed as continuous beams over the stringer supports, are 12-in. I at 31.8 lb in intermediate panels and 12-in. I at 40.8 lb in end panels. Stringers were designed as simple beams spanning between floor beams with maximum load reactions transmitted by the crossbeams, and are rolled beams. Generally, all floor beams are plate girders 4 ft 6 in. deep. At intermediate piers, special double-web floor beams, 6 ft 2 in. deep, are provided as an integral part of the portal frames.

[3] "Distribution of Wheel Loads and Design of Reinforced Concrete Bridge Floor Slabs," by H. R. Erps, A. L. Googins, and J. L. Parker, *Public Roads*, October, 1937, p. 149.

[4] "Computation of Stresses in Bridge Slabs Due to Wheel Loads," by H. M. Westergaard, *ibid.*, March, 1930, p. 1.

Lateral and Sway Bracing.—Three lateral systems are provided as follows: (*a*) a lower lateral system in the plane of the bottom chords of the end spans and the arch tie of the center span; (*b*) a lateral system in the plane of the truss bottom chord of the arch span; and (*c*) a top lateral system in the plane of the top chord of the three spans.

The bottom lateral system was designed as a three-span continuous truss, with the bottom chords of the end spans and the arch ties of the center span acting as chords for the lateral truss. This system comprises a rigid single plane with diagonals of double-angle section proportioned to take either tension or compression. Except for the six central panels, where carbon steel was used, all diagonals of this lateral system are silicon steel. In panels L8′–L9′ and L9′–L10′, adjacent to pier 19, special diagonals were provided to transmit all longitudinal forces of the structure to the vertical pin anchor.

The lateral system in the plane of the bottom chord of the arch truss was provided primarily for the purpose of bracing the chord. This system extends only from L11 to L11′, because of clearance requirements. Chord bracing in other panels is provided by the sway frames in the planes of L10–U10, L11–U11, and L10–U11.

The top lateral system was designed as a three-span continuous truss with the spans equivalent to the developed lengths of the top chord plane. A rigid, double-plane, double-web system was used with H-section diagonals proportioned to resist either tension or compression.

Nominal sway bracing is provided in the plane of all vertical posts and at hanger U11–L11. The H-sections were used for all sway bracing diagonals. Portal frames in the plane of the end posts and in the plane of the vertical posts at intermediate piers, L9–U9, were designed to transmit computed lateral loads to the foundation. To obtain the desired rigidity, and, because of esthetic considerations, the struts and diagonals of these frames have fairly massive sections. Deep plate girder sections were used for end portal framing. Box girder sections were used for the intermediate portals.

Main Trusses.—Truss chords, arch ties, and some of the heavier diagonals are made up of box girder sections consisting of two I-beams, top and bottom cover plates, and additional side plates as required. Handholes, 9 in. by 18 in. spaced at about 3-ft 6-in. centers, were provided to permit riveting and painting. All other diagonals are also box sections consisting of channels and plates, with handholes 6 in. by 12 in., at 3-ft centers. Box sections were used throughout, to the complete exclusion of laced members. Nearly all truss members are silicon steel. Carbon steel was used for such details, as diaphragms, fills, connection angles, etc., and for members of minimum section, where size of material rather than stress governed. In proportioning the members, the material in line of the handholes was considered ineffective in carrying the primary stresses. All posts and hangers are made up of single-web girders, consisting of four bulb angles and a web plate.

Bearings.—Substantially to eliminate pier torsion, which would have resulted with fixed shoes, because of lateral deformation of the lower lateral system under transverse wind loads, expansion-type bearings are provided at

all supports. Longitudinal forces of the entire three-span structure are transmitted to pier 19 through a large pintle pin anchored in an encased structural frame embedded in the pier. The forged steel pin has a 5-in.-diameter hollow center, is 4 ft 4½ in. long, and varies in outside diameter from 12 in. at the top to 8 in. at the bottom. The upper end, 13 in. in diameter, extends 9 in. into the double floor beam of the superstructure to engage a 5-in.-thick bearing plate on the floor beam.

Expansion shoes at both ends of the central arch span are each supported on a nest of five 12-in.-diameter alloy steel rollers 5 ft 8 in. long. Rotation resulting from span deflection is provided through a line bearing detail connecting upper and lower sections of the shoe rather than by the conventional pin type arrangement. The shoes are 6 ft 8 in. square and 5 ft 10 in. high.

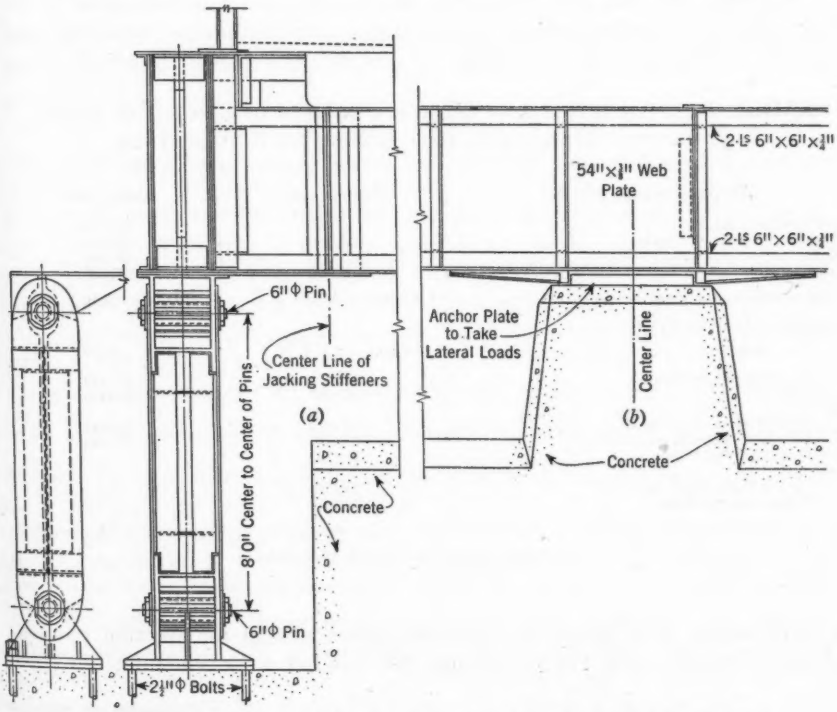

FIG. 4.—EXPANSION ROCKER COLUMNS AND SPECIAL GUIDES FOR TRANSMISSION OF TRANSVERSE WIND SHEARS FROM END FLOOR BEAMS TO PIERS 17 AND 20

A number of factors governed the design of bearings for ends of the three-span unit. Provision had to be made for: (1) The lowering of panel point L0, during erection operations, 3.56 ft below its final position; (2) a tie-down anchorage to resist a 125.5-kip uplift during cantilevering operations; (3) a 403-kip positive reaction for final dead plus live load; (4) an expansion of 1,190 ft of structure; and (5) the transmission of a transverse horizontal wind shear of 159 kips from the superstructure to the pier.

During erection the ends of the trusses were supported and tied down by two 4-in.-diameter anchor bolts embedded in the pier. The raising and lowering of the end of the span was controlled by a 250-ton hydraulic jack placed between pairs of beams engaging these bolts. After erection was completed, rocker columns 8 ft high (see Fig. 4(*a*)) were inserted to act as bearings for the ends of the span. These care for expansion movements of the span with only a nominal change in roadway elevation, and were designed to take both positive and negative reactions. Transverse wind shears are transmitted to a built-up section of the pier through special guides in the plane of the lower flanges of the end floor beams.

Deflection and Camber.—The maximum dead load deflection of 15⅛ in. occurs at the center line of the arch span.

Materials and Stresses.—Properties of the principal materials used in the superstructure, together with the design stresses used for normal dead, live, and impact loadings, are given in Table 1. For stresses caused by gravity loads

TABLE 1.—PROPERTIES AND DESIGN STRESSES OF PRINCIPAL SUPERSTRUCTURE MATERIALS, IN POUNDS PER SQUARE INCH

Property or design stress	Carbon steel	Silicon Steel
ASTM[a] designation	A7–39	A94–39
Minimum tensile strength	60,000	80,000
Minimum yield point	30,000	45,000
Axial tension, net section	18,000	24,000
Axial compression, gross section	$15{,}000 - 0.25\left(\frac{L}{r}\right)^2$	$20{,}000 - 0.46\left(\frac{L}{r}\right)^2$
Bending on Extreme Fiber:		
Compression	$18{,}000 - 5.0\left(\frac{L}{b}\right)^2$	$24{,}000 - 6.67\left(\frac{L}{b}\right)^2$
Tension, net section	18,000	24,000
Pins	27,000	36,000
Shear:		
Girder webs, gross section	11,000	14,000
Pins and shop-driven rivets	13,500	18,000
Power-driven rivets	11,000	....
Bearing:		
Pins and shop-driven rivets	27,000	....
Power-driven rivets	22,500	....

[a] American Society for Testing Materials.

in combination with lateral or longitudinal loads, and for erection stresses, overstress was permitted in accordance with the following schedule:

Loads	Permissible overstress, % of normal
Dead load + 30-lb wind	0
Dead load + live load + impact + 30-lb wind	25
Dead load + 50-lb wind	50
Erection stresses	50

Compression Members.—In compression members consisting of segments connected by cover plates or of segments connected by webs, the thickness of the segment webs was limited to a minimum of 1/28 of the unsupported distance of plate for silicon steel, and 1/32 of that for carbon steel. Cover plates con-

necting the segments were limited in thickness to a minimum of 1/36 of the unsupported distance of the plate for silicon steel, and 1/40 of that for carbon steel.

Design Loads

Dead Load.—The dead load per linear foot of bridge is 8,545 lb for the arch span and 6,745 lb for the end spans, and is made up as follows:

Load	Arch span (lb)	End spans (lb)
Concrete roadway and sidewalk	2,540	2,540
Curbs and railing	210	210
Floor system	845	845
Truss steelwork (average)	4,950	3,150
Total	8,545	6,745

For final stress calculations, the actual estimated weights of truss members, lateral system, and other component parts were properly distributed among the various panel points.

Live Load.—The floor system was designed for an H-20 loading on 9-ft lanes in accordance with the 1935 AASHO specifications. This loading consists of a 20-ton truck followed and preceded by 15-ton trucks spaced 30 ft apart. The main trusses were designed for the same loading, except that the lanes were assumed to be 12 ft wide. Such a modification of the design loading was deemed justifiable since a concentration of a great number of exceptionally heavy trucks, required to produce maximum stresses in the various truss members of the long spans, is highly improbable. Equivalent uniform loads were used for the truss design.

Impact.—For dynamic vibratory and impact effects, stresses produced by live load were increased in accordance with the formula: $I = \frac{50}{L + 125}$, in which I is the impact fraction, and L is the length, in feet, of the section of the span which is loaded to produce the maximum stress in the member.

Sidewalk Load.—The sidewalk slab and the immediate supports were designed to carry 100 lb per sq ft. For design of the trusses, 50 lb per sq ft was used.

Wind Load.—Wind forces on the structure were assumed as a moving horizontal load equal to 30 lb per sq ft on one and one half times the area of the structure as seen in elevation. The lateral force due to the moving live load and wind force against this load were considered as 200 lb per lin ft of bridge, acting 6 ft above the roadway. Stability of the structure and of the stresses in the members was also checked for a 50-lb wind pressure.

Longitudinal Forces.—Provision was made for the effect of a longitudinal force equivalent to 10% of the live load on the structure, acting 4 ft above the floor. Appropriate provisions were also made for longitudinal wind loads.

Calculation of Stresses

The continuous tied arch unit is statically indeterminate in the third degree, requiring for its solution three elastic equations. For the analysis of the structure the theory of redundant members was used.

As shown in Fig. 5(*b*), the end reactions and the arch tie were assumed to be the redundant forces. With these forces removed, the truss is a simple span with two end cantilevers, in which the stresses can be calculated readily for any given loading. Referring to Fig. 5(*b*), and with the redundants removed as

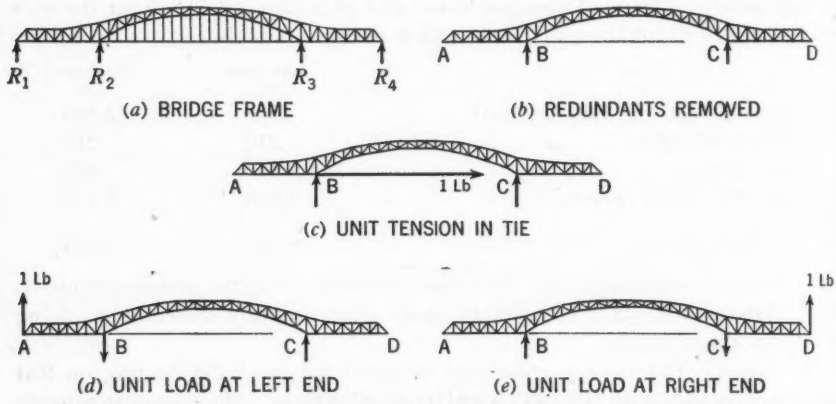

FIG. 5.—METHODS OF LOADING TO DETERMINE UNIT DEFLECTIONS AT VARIOUS POINTS

shown, let S' be the stress in each member due to a given loading; u be the stress due to a horizontal unit load applied as shown in Fig. 5(*c*); v be the stress due to a vertical unit load applied as shown in Fig. 5(*d*); w be the stress due to a vertical unit load applied as shown in Fig. 5(*e*); L be the length of each member, in inches; E be the modulus of elasticity of the material, in pounds per square inch; and A be the average gross area of each member, in square inches. The deflections and horizontal movements due to unit loads can be expressed as in Table 2, in which the first line, for example, denotes vertical deflection at point A due to a horizontal unit load applied as shown in Fig. (5*c*), and is represented by $\sigma\, u\, a = \Sigma u\, v\, L/(E\, A)$.

TABLE 2.—DEFLECTIONS AND MOVEMENTS DUE TO THE UNIT LOADINGS SHOWN IN FIG. 5

Direction of movement (1)	Point (2)	Method of loading (3)	Designation (4)	Eq. No. (5)
Vertical...	A	(See Fig. 5(*c*))	$\sigma\, u\, a = \Sigma u\, v\, L/(E\, A)$	1
Horizontal.	B	(See Fig. 5(*c*))	$\sigma\, u\, b = \Sigma u^2\, L/(E\, A)$	2
Vertical...	D	(See Fig. 5(*c*))	$\sigma\, u\, d = \Sigma u\, w\, L/(E\, A)$	3
Vertical...	A	(See Fig. 5(*d*))	$\sigma\, v\, a = \Sigma v^2\, L/(E\, A)$	4
Horizontal.	B	(See Fig. 5(*d*))	$\sigma\, v\, b = \Sigma u\, v\, L/(E\, A)$	5
Vertical...	D	(See Fig. 5(*d*))	$\sigma\, v\, d = \Sigma v\, w\, L/(E\, A)$	6
Vertical...	A	(See Fig. 5(*e*))	$\sigma\, w\, a = \Sigma v\, w\, L/(E\, A)$	7
Horizontal.	B	(See Fig. 5(*e*))	$\sigma\, w\, b = \Sigma u\, w\, L/(E\, A)$	8
Vertical...	D	(See Fig. 5(*e*))	$\sigma\, w\, d = \Sigma w^2\, L/(E\, A)$	9
Vertical...	A	Any given load	$\sigma\, a = \Sigma S'\, v\, L/(E\, A)$	10
Horizontal.	B	Any given load	$\sigma\, b = \Sigma S'\, u\, L/(E\, A)$	11
Vertical...	D	Any given load	$\sigma\, d = \Sigma S'\, w\, L/(E\, A)$	12

In accordance with the law of reciprocal deflections, however,

$$\sigma\, u\, a = \sigma\, v\, b \quad \text{(13a)}$$

$$\sigma\, u\, d = \sigma\, w\, b \quad \text{(13b)}$$

$$\sigma\, v\, d = \sigma\, w\, a \quad \text{(13c)}$$

Furthermore, because of symmetry of the structure,

$$\sigma u a = \sigma u d \quad \ldots\ldots (14a)$$

$$\sigma v a = \sigma w d \quad \ldots\ldots (14b)$$

To satisfy the condition of zero deflection at point A and point D and no movement at point B, the following expressions are developed:

$$R_1 \sigma v a + H \sigma u a + R_4 \sigma w a + \sigma a = 0 \quad \ldots\ldots (15a)$$

$$R_1 \sigma v b + H \sigma u b + R_4 \sigma w b + \sigma b = 0 \quad \ldots\ldots (15b)$$

$$R_1 \sigma v d + H \sigma u d + R_4 \sigma w d + \sigma d = 0 \quad \ldots\ldots (15c)$$

in which R_1 and R_4 are the final end reactions and H is the final stress in the arch tie, for any given loading.

Substituting in these equations the values of Eqs. 13 and 14, Eqs. 15 can be rewritten as:

$$R_1 \sigma v a + H \sigma u a + R_4 \sigma v d + \sigma a = 0 \quad \ldots\ldots (16a)$$

$$R_1 \sigma u a + H \sigma u b + R_4 \sigma u a + \sigma b = 0 \quad \ldots\ldots (16b)$$

$$R_1 \sigma v d + H \sigma u a + R_4 \sigma v a + \sigma d = 0 \quad \ldots\ldots (16c)$$

in which there are three unknowns—R_1, the final left end reaction for a given loading; R_4, the final right end reaction; and H, the final stress in the arch tie. The other components of the equations are constants. Thus, $\sigma v a$, $\sigma u a$, $\sigma v d$, and $\sigma u b$ are constants based on the geometry of the statically determinate structure (Fig. 5(*b*)) and the areas of members, and σa, σb, and σd are constants based on the position and magnitude of a given loading on the statically determinate structure.

Direct general expressions for the redundants can be developed from Eqs. 16. Such expressions, however, would be quite involved, and would serve no practical purpose. It is simpler and more expedient first to determine the numerical values of $\sigma v a$, $\sigma u b$, $\sigma u a$, and $\sigma v d$, and then to solve the simultaneous equations, thus obtaining general expressions for the redundants in terms of σa, σb, and σd.

Computations leading to determination of values of $\sigma v a$, $\sigma u b$, $\sigma u a$, and $\sigma v d$ are shown in Table 3. Dividing the totals of Cols 5, 7, 8, and 10 by E (equal to 30,000,000 lb per sq in.) gives the following:

$$\sigma v a = 0.0002538867 \text{ in.} \qquad \sigma u a = -0.000106734 \text{ in.}$$
$$\sigma u b = 0.0000730703 \text{ in.} \qquad \sigma v d = 0.000143508 \text{ in.}$$

Using these numerical values, Eqs. 16 become:

$$0.0002538867 R_1 - 0.000106734 H + 0.000143508 R_4 + \sigma a = 0 \quad \ldots (17a)$$

$$-0.000106734 R_1 + 0.0000730703 H - 0.000106734 R_4 + \sigma b = 0 \quad \ldots (17b)$$

$$0.000143508 R_1 - 0.000106734 H + 0.0002538867 R_4 + \sigma d = 0 \quad \ldots (17c)$$

from which:

$$R_1 = -10{,}372.1\,\sigma a - 17{,}067.78\,\sigma b - 1{,}312.5\,\sigma d \quad \ldots\ldots (18)$$

$$H = -17{,}067.7\,\sigma a - 63{,}545.5\,\sigma b - 17{,}067.7\,\sigma d \quad \ldots\ldots (19)$$

$$R_4 = -1{,}312.5\,\sigma a - 17{,}067.7\,\sigma b - 10{,}372.1\,\sigma d \quad \ldots\ldots (20)$$

For symmetrical loading, such as for dead load, $\sigma a = \sigma d$, and, consequently:

$$R_1 = R_4 = -11{,}684.6\,\sigma a - 17{,}067.7\,\sigma b \quad \ldots\ldots (21)$$

and

$$H = -34{,}035.4\,\sigma a - 63{,}545\,\sigma b \quad \ldots\ldots (22)$$

With these reactions determined, R_2 and R_3 were obtained by statics, and influence lines were developed for the reactions, as well as for the stresses in individual truss members. From Eqs. 18, 19, and 20, other pertinent proper-

TABLE 3.—COMPUTATIONS FOR VALUES OF $\sigma v a$, $\sigma u b$, $\sigma u a$, AND $\sigma v d$

Member	Length, L (in.)	Area, A (sq in.)	$\sigma v a E$		$\sigma u b E$		$\sigma u a E$	$\sigma v d E$	
			v	$\frac{v^2 L}{A}$	u	$\frac{u^2 L}{A}$	$\frac{u v L}{A}$	w	$\frac{w v L}{A}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
L0–U1	665	86.5	−1.39	14.85	....	....	....	....	....
U1–U3	922	67.0	−1.92	50.70	....	....	....	....	....
U3–U5	923	79.5	−3.69	158.00	....	....	....	....	....
U5–U7	927	89.0	−4.82	242.0	....	....	....	....	....
U7–U9	944	108.0	−5.08	225.0	....	....	....	....	....
L0–L2	922	67.0	+0.96	12.67	....	....	....	....	....
L2–L4	922	71.0	+2.88	107.50	....	....	....	....	....
L4–L6	922	80.7	+4.44	225.00	....	....	....	....	....
L6–L8	922	93.5	+5.05	251.00	....	....	....	....	....
L8–L9	461	111.5	+4.94	101.00	....	....	....	....	....
U1–L2	665	42.4	+1.39	30.30	....	....	....	....	....
L2–U3	665	42.4	−1.39	30.30	....	....	....	....	....
U3–L4	665	42.4	+1.16	21.10	....	....	....	....	....
L4–U5	695	46.5	−1.13	19.10	....	....	....	....	....
U5–L6	695	52.0	+0.506	3.42	....	....	....	....	....
L6–U7	788	47.2	−0.466	3.62	....	....	....	....	....
U7–L8	788	57.3	−0.098	0.13	....	....	....	....	....
L8–U9	958	50.7	+0.086	0.16	....	....	....	....	....
U9–L9	840	160	−0.93	4.54	+0.48	1.21	− 2.34	−0.41	+ 2.00
U9–U11	960	100.0	−5.47	287.00	+0.302	0.876	− 15.85	−0.288	+ 15.10
U11–U13	944	78.0	−6.10	450.00	+1.02	12.60	− 75.20	−0.963	+ 71.15
U13–U15	934	78.0	−6.76	547.00	+1.88	42.30	−152.00	−1.99	+161.00
U15–U17	925	93.2	−7.36	537.0	+2.86	81.30	−208.70	−3.44	+251.00
U17–U19	922	144.5	−7.75	383.0	+3.90	97.20	−192.50	−5.37	+265.00
U19–U'19	922	169.6	−7.20	282.5	+4.42	106.20	−173.40	−7.20	+282.50
U'19–U'17	922	144.5	−5.37	184.0	+3.90	97.20	−133.50	−7.75	+265.00
U'17–U'15	925	93.2	−3.44	117.6	+2.86	81.30	− 97.80	−7.36	+252.00
U'15–U'13	934	78.0	−1.99	47.4	+1.88	42.30	− 44.80	−6.76	+161.00
U'13–U'11	944	78.0	−0.963	11.2	+1.02	12.60	− 11.88	−6.10	+ 71.15
U'11–U'9	960	100.0	−0.288	0.8	+0.302	0.876	− 0.83	−5.47	+ 15.10
U'9–L'9	840	160.0	−0.41	0.89	+0.48	1.21	− 1.03	−0.93	+ 2.00
L9–L10	511	254.0	+5.46	60.0	−1.11	2.485	− 12.20	0	0
L10–L12	997	199.0	+5.89	173.7	−1.71	14.63	− 50.40	+0.59	+ 17.40
L12–L14	970	150.0	+6.42	266.5	−2.46	39.20	−102.00	+1.42	+ 58.85
L14–L16	946	92.0	+6.90	489.0	−3.32	113.2	−235.50	+2.62	+186.00
L16–L18	931	77.2	+7.30	643.0	−4.24	217.0	−373.50	+4.15	+365.50
L18–L20	924	71.2	+7.30	694.0	−5.05	331.0	−478.00	+6.11	+580.00
L20–L'18	924	71.2	+6.11	484.0	−5.05	331.0	−400.00	+7.30	+580.00
L'18–L'16	931	77.2	+4.15	208.0	−4.24	217.0	−212.50	+7.30	+365.50
L'16–L'14	946	92.0	+2.62	70.6	−3.32	113.2	− 89.50	+6.90	+186.00

TABLE 3—(*Continued*)

Member	Length, L (in.)	Area, A (sq in.)	$\sigma v a E$		$\sigma u b E$		$\sigma u a E$	$\sigma v d E$	
			v	$\frac{v^2 L}{A}$	u	$\frac{u^2 L}{A}$	$\frac{u v L}{A}$	w	$\frac{w v L}{A}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
L'14–L'12...	970	150.0	+1.42	13.0	−2.46	39.2	− 22.50	+6.42	+ 58.85
L'12–L'10...	997	199.0	+0.59	1.7	−1.71	14.63	− 5.04	+5.89	+ 17.40
L'10–L'9....	511	254.0	0	0	−1.11	2.48	0	+5.46	0
U9–L10.....	773	46.5	+0.561	5.23	−0.487	3.94	− 4.54	+0.40	+ 3.73
L10–U11....	995	91.6	−0.396	1.71	+0.643	4.60	− 2.81	−0.629	+ 2.71
U11–L12....	681	42.4	+0.707	8.03	−0.597	5.73	− 6.78	+0.582	+ 6.61
L12–L13....	840	79.3	−0.275	0.80	+0.646	4.43	− 1.88	−0.760	+ 2.21
U13–L14....	615	42.4	+0.755	8.28	−0.698	7.08	− 7.65	+0.806	+ 8.84
L14–U15....	719	70.1	−0.104	0.11	+0.586	3.52	− 0.62	−0.895	+ 0.95
U15–L16....	574	42.4	+0.726	7.14	−0.791	8.48	− 7.78	+1.113	+ 10.96
L16–U17....	630	61.2	+0.164	0.28	+0.451	2.09	+ 0.76	−0.952	− 1.61
U17–L18....	551	42.4	+0.637	5.28	−0.842	9.21	− 6.97	+1.468	+ 12.17
L18–U19....	567	45.2	+0.550	3.77	+0.187	0.44	+ 1.28	−0.894	− 6.15
U19–L20....	544	39.7	−0.141	0.27	−0.437	2.62	+ 0.86	+1.30	− 2.55
L20–U'19....	544	39.7	+1.30	23.15	−0.437	2.62	− 8.15	−0.141	− 2.55
U'19–L'18...	567	45.2	−0.894	10.00	+0.187	0.44	− 2.31	+0.550	− 6.15
L'18–U'17...	551	42.4	+1.468	28.00	−0.842	9.21	− 16.10	+0.637	+ 12.17
U'17–L'16...	630	61.2	−0.952	9.32	+0.451	2.09	− 4.42	+0.164	− 1.61
L'16–U'15...	574	42.4	+1.113	16.80	−0.791	8.48	− 11.95	+0.726	+ 10.96
U'15–L'14...	719	70.1	−0.895	8.20	+0.586	3.52	− 5.35	−0.104	+ 0.95
L'14–U'13...	615	42.4	+0.806	9.44	−0.698	7.08	− 8.17	+0.755	+ 8.84
U'13–L'12...	840	79.3	−0.760	6.12	+0.646	4.43	− 5.20	−0.275	+ 2.21
L'12–U'11...	681	42.4	+0.582	5.43	−0.597	5.73	− 5.58	+0.707	+ 6.61
U'11–L'10...	995	91.6	−0.629	4.30	+0.643	4.60	− 4.45	−0.396	+ 2.71
L'10–U'9....	773	46.5	+0.400	2.66	−0.487	3.94	− 3.24	+0.561	+ 3.73
L9–T10.....	461	142.5	....	....	1.0	3.24	....	....	....
T10–T11....	461	137.5	....	....	1.0	3.35	....	....	....
T11–T16....	2,305	127.5	....	....	1.0	18.10	....	....	....
T16–T18....	922	137.5	....	....	1.0	6.70	....	....	....
T18–T'18....	1,844	142.5	....	....	1.0	12.84	....	....	....
T'18–T'16...	922	137.5	....	....	1.0	6.70	....	....	....
T'16–T'11...	2,305	127.5	....	....	1.0	18.10	....	....	....
T'11–T'10...	461	137.5	....	....	1.0	3.35	....	....	....
T'10–L'9....	461	142.5	....	....	1.0	3.24	....	....	....
Total.....	....	....	....	7,616.60	....	2,192.10	−3,202.02	....	4,305.24

ties of the structure are readily obtainable. Thus, for a 1-in. settlement at point A, with $\sigma b = 0$ and $\sigma d = 0$, $R_1 = -10{,}372.1$ lb; $H = -17{,}067.7$ lb; and $R_4 = -1{,}312.5$ lb. Similarly, for a 1-in. horizontal displacement at point B, with $\sigma a = 0$ and $\sigma d = 0$, $R_1 = -17{,}067.7$ lb; $H = -63{,}545.5$ lb; and $R_4 = -17{,}067.7$ lb. Also, with $\sigma a = 0$ and $\sigma d = 0$, for $H = 1$ lb, $R_1 = R_4 = \frac{17{,}067.7}{63{,}545.5} = 0.269$ lb. Other reactions, as well as stresses, can be obtained by statics. These relations were important for studying the effect of settlement of supports, as well as for determination of erection stresses through the various stages and operations.

As is necessary for statically indeterminate structures, the areas of truss members had to be assumed or estimated before a stress analysis could be made. Furthermore, any effect the method of erection might have on the size of truss members had to be predetermined, since any subsequent change of member area for erection conditions would have unbalanced the system. Since a cantilever erection method was indicated, it was apparent that some of the members

TABLE 4.—STRESSES,[a] IN KIPS, AND GENERAL

Member	Static Loads				Erection	Erection wind	Col. 6 plus Col. 7	Design for		Effective Area		$\frac{L}{r}$
	Dead	Live	Impact	Total				Col.	Stress	Gross	Net	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
U1–U3...	242 *C*	200 *C*	19 *C*	461 *C*	500 *T*	390 *T*	890 *T*	5	461 *C*	64.13	52.7	45
U3–U5...	44 *C*	250 *C*	28 *C*	322 *C*	1,260 *T*	444 *T*	1,704 *T*	8	1,704 *T*	77.04	63.79	...
U11–U13.	428 *T*	191 *T*	18 *T*	637 *T*	1,730 *T*	422 *T*	2,152 *T*	8	2,152 *T*	74.79	61.54	...
U13–U15.	430 *C*	218 *C*	22 *C*	670 *C*	1,240 *T*	385 *T*	1,625 *T*	8	1,625 *T*	74.79	61.54	...
L0–U1...	256 *C*	172 *C*	17 *C*	445 *C*	....	....	....	5	445 *C*	74.79	60.29	37
L2–L4...	197 *T*	240 *T*	27 *T*	464 *T*	945 *C*	208 *C*	1,153 *C*	8	1,153 *C*	68.97	55.4	45
L18–L20.	465 *T*	237 *T*	23 *T*	725 *T*	....	....	....	5	725 *T*	68.97	55.40	...
L4–L6...	203 *C*	205 *C*	20 *C*	428 *C*	1,613 *C*	287 *C*	1,900 *C*	8	1,900 *C*	78.10	....	45
L6–L8...	820 *C*	245 *C*	42 *C*	1,107 *C*	2,100 *C*	487 *C*	2,587 *C*	8	2,587 *C*	89.85	....	45
L16–L18.	350 *C*	240 *C*	23 *C*	613 *C*	393 *C*	....	....	5	613 *C*	74.91	....	45
U5–U7...	496 *T*	221 *T*	25 *T*	742 *T*	1,870 *T*	637 *T*	2,507 *T*	8	2,507 *T*	85.96	71.6	...
U15–U17.	1,335 *C*	359 *C*	36 *C*	1,730 *C*	692 *T*	....	....	5	1,730 *C*	90.00	81.00	44
L14–L16.	1,238 *C*	376 *C*	36 *C*	1,650 *C*	951 *C*	....	....	5	1,650 *C*	91.21	....	46
U7–U9...	1,137 *T*	298 *T*	37 *T*	1,472 *T*	2,260 *T*	776 *T*	3,036 *T*	8	3,036 *T*	104.59	87.09	...
U9–U11..	1,126 *T*	281 *T*	27 *T*	1,434 *T*	2,210 *T*	910 *T*	3,120 *T*	8	3,120 *T*	97.72	80.97	...
L8–L9...	1,380 *C*	353 *C*	34 *C*	1,767 *C*	2,360 *C*	440 *C*	2,800 *C*	8	2,800 *C*	107.51	....	45
U17–U19.	2,160 *C*	468 *C*	47 *C*	2,675 *C*	129 *T*	....	....	5	2,675 *C*	140.26	....	43
L9–T10..	2,005 *T*	439 *T*	42 *T*	2,486 *T*	928 *T*[e]	2,933 *T*[f]	....	7	2,933 *T*	....	119.37	...
L12–L14.	2,105 *C*	520 *C*	50 *C*	2,675 *C*	1,490 *C*	....	....	5	2,675 *C*	145.26	....	48
T10–T11.	2,005 *T*	439 *T*	42 *T*	2,486 *T*	720 *T*[e]	2,725 *T*[f]	....	7	2,725 *T*	....	115.14	...
T11–T14.	2,005 *T*	439 *T*	42 *T*	2,486 *T*	544 *T*[e]	2,549 *T*[f]	....	7	2,549 *T*	....	106.64	...
T14–T16.	2,005 *T*	439 *T*	42 *T*	2,486 *T*	466 *T*[e]	2,471 *T*[f]	....	7	2,471 *T*	....	106.64	...
T16–T18.	2,005 *T*	439 *T*	42 *T*	2,486 *T*	617 *T*[e]	2,622 *T*[f]	....	7	2,622 *T*	....	115.14	...
T18–T20.	2,005 *T*	439 *T*	42 *T*	2,486 *T*	683 *T*[e]	2,688 *T*[f]	....	7	2,688 *T*	....	119.39	...
L0–L2...	177 *T*	119 *T*	13 *T*	309 *T*	....	....	....	5	340 *T*[g]	64.13	52.7	...
U19–U'19	2,565 *C*	539 *C*	52 *C*	3,156 *C*	....	....	....	5	3,156 *C*	165.26	....	43
L9–L10..	3,788 *C*	762 *C*	73 *C*	4,623 *C*	2,630 *C*	....	...	5	4,643 *C*	248.72	....	46
L10–L12.	2,920 *C*	628 *C*	60 *C*	3,608 *C*	2,000 *C*	....	....	5	3,608 *C*	194.91	....	46
U1–L2...	94 *T*	131 *T*	15 *T*	240 *T*	365 *C*	85 *C*	450 *C*	8	450 *C*	38.81	....	94
L2–U3...	64 *T*	94 *T*	23 *T*	181 *T*	450 *T*	85 *T*	535 *T*	8	535 *T*	....	32.81	...
U3–L4...	223 *C*	113 *C*	23 *C*	359 *C*	462 *C*	82 *C*	544 *C*	8	544 *C*	38.81	....	94
L6–U7...	572 *T*	171 *T*	23 *T*	766 *T*	....	....	....	5	766 *T*	....	36.06	...
U9–L10..	521 *T*	141 *T*	14 *T*	682 *T*	....	....	....	5	682 *T*	....	36.06	...
U11–L12.	437 *T*	150 *T*	17 *T*	604 *T*	....	....	....	5	604 *T*	....	32.81	...
U13–L14.	494 *T*	144 *T*	20 *T*	658 *T*	....	....	....	5	658 *T*	....	32.81	...
U15–L16.	520 *T*	156 *T*	21 *T*	697 *T*	....	....	....	5	697 *T*	....	32.81	...
U17–L18.	478 *T*	172 *T*	17 *T*	667 *T*	....	....	....	5	667 *T*	....	32.81	...
L18–U19.	301 *C*	190 *C*	18 *C*	509 *C*	....	....	....	5	509 *C*	38.81	....	81
U19–L20.	154 *T*	174 *T*	17 *T*	345 *T*	....	....	....	5	366 *T*[i]	....	30.56	...
L4–U5...	374 *T*	145 *T*	25 *T*	544 *T*	....	....	....	5	544 *T*	....	36.00	...
U5–L6...	443 *C*	145 *C*	22 *C*	610 *C*	....	....	....	5	610 *C*	47.76	....	91
U7–L8...	472 *C*	153 *C*	17 *C*	642 *C*	....	....	....	5	642 *C*	53.26	....	94
L8–U9...	590 *T*	179 *T*	20 *T*	789 *T*	....	....	....	5	789 *T*	....	39.26	...
L14–U15.	684 *C*	198 *C*	27 *C*	909 *C*	....	....	....	5	909 *C*	64.56	....	86
L16–U17.	559 *C*	193 *C*	19 *C*	771 *C*	...	....	....	5	771 *C*	55.56	....	82
L12–U13.	734 *C*	200 *C*	20 *C*	954 *C*	....	....	....	5	954 *C*	73.65	....	97
L10–U11.	813 *C*	216 *C*	21 *C*	1,050 *C*	....	....	....	5	1,050 *C*	85.90	....	106
U9–L9...	963 *C*	263 *C*	29 *C*	1,255 *C*	....	....	....	5	1,228 *C*	132.38	....	...
U1–L1, U3–L3, U5–L5, U7–L7, U15–L15, U17–L17, U19–L19									167 *T*	20.34	17.02	224
U2–L2, U4–L4, U6–L6, U14–L14, U16–L16, U18–L18, U20–L20									167 *T*	20.34	....	203
U8–L8, U12–L12									167 *T*	23.94	....	210
U11–L11, U13–L13									167 *T*	23.94	....	196
L11–T11, L12–T12, L13–T13, L14–T14, L15–T15									167 *T*	23.94	....	284
U10–L10									167 *T*	27.06	....	178
L16–T16, L17–T17, L18–T18, L19–T19, L20–T20									167 *T*	27.06	....	302
L10–T10									....[m]	69.39	....	52

[a] *T* = tension, *C* = compression. Unit stresses as shown for members governed by static loads were re- carbon steel, S = silicon steel. [d] Add two plates 20 × ¾ from L0 to 12 ft 6 in. up, and at lower portal connection [e] L0–L2 section determined by Col. 5 + 15-lb wind at 1.25 normal stress. Bending at L0 = 466,000 ft-lb. [h] For these members see Col. 1. [m] Designed for direct stress of 167 kips plus a moment of 1,250 ft-kips.

MAKE-UP OF CONTINUOUS-SPAN MEMBERS

UNIT STRESS[b]		Material[c]	MAKE-UP OF SECTION				Shape	Member
f_c	f_t		Top plate	Bottom plate	Side plates	Is, [s, or ∠s		
(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
14,500	27,000	C	30 × $\frac{3}{8}$	30 × $\frac{3}{8}$	....			U1–U3
....	27,000	C	32 × $\frac{1}{2}$	32 × $\frac{3}{8}$	....			U3–U5
....	36,000	S	30 × $\frac{1}{2}$	30 × $\frac{3}{8}$	....			U11–U13
....	27,000	C	30 × $\frac{1}{2}$	30 × $\frac{3}{8}$	....	2-24 I 79.9		U13–U15
13,775	....	C	30 × $\frac{1}{2}$	30 × $\frac{3}{8}$	....[d]			L0–U1
21,750	....	C	30 × $\frac{7}{16}$	30 × $\frac{7}{16}$	....			L2–L4
....	24,000	S	30 × $\frac{7}{16}$	30 × $\frac{7}{16}$	....			L18–L20
28,600	....	S	30 × $\frac{1}{2}$	30 × $\frac{1}{2}$	....			L4–L6
28,600	....	S	32 × $\frac{1}{2}$	32 × $\frac{1}{2}$	....	2-24 I 90		L6–L8
14,500	....	C	30 × $\frac{7}{16}$	30 × $\frac{7}{16}$	....			L16–L18
						2-24 I 95		
....	36,000	S	32 × $\frac{1}{2}$	32 × $\frac{1}{2}$	....			U5–U7
19,110	....	S	33 × $\frac{1}{2}$	33 × $\frac{1}{2}$	....	2-24 I 100		U15–U17
19,000	....	S	32 × $\frac{1}{2}$	32 × $\frac{1}{2}$	....			L14–L16
....	36,000	S	32 × $\frac{1}{2}$	32 × $\frac{1}{2}$	....	2-24 I 115		U7–U9
....	36,000	S	32 × $\frac{1}{2}$	32 × $\frac{1}{2}$	....			U9–U11
28,600	....	S	32 × $\frac{1}{2}$	32 × $\frac{1}{2}$	....			L8–L9
19,100	....	S	32 × $\frac{1}{2}$	33 × $\frac{1}{2}$	2-20 × $\frac{5}{8}$			U17–U19
....	24,000	S	32 × $\frac{1}{2}$	32 × $\frac{1}{2}$	4-20 × $\frac{7}{16}$			L9–T10
18,900	....	S	32 × $\frac{3}{4}$	33 × $\frac{7}{8}$	2-20 × $\frac{3}{4}$	2-24 I 120		L12–L14
....	24,000	S	32 × $\frac{1}{2}$	32 × $\frac{1}{2}$	2-20 × $\frac{1}{2}$			T10–T11
....	24,000	S	32 × $\frac{1}{2}$	32 × $\frac{1}{2}$	2-20 × $\frac{1}{2}$			T11–T14
....	24,000	S	32 × $\frac{1}{2}$	32 × $\frac{1}{2}$	2-20 × $\frac{1}{2}$			T14–T16
....	24,000	S	32 × $\frac{1}{2}$	32 × $\frac{1}{2}$	2-20 × $\frac{3}{4}$			T16–T18
....	24,000	S	32 × $\frac{1}{2}$	32 × $\frac{1}{2}$	4-20 × $\frac{7}{16}$			T18–T20
....	18,000	C	30 × $\frac{1}{2}$	30 × $\frac{1}{2}$	2-20 × $\frac{3}{8}$[h]	2-24 I 79.9		L0–L2
19,100	....	S	32 × $\frac{1}{2}$	33 × $\frac{1}{2}$	4-20 × $\frac{5}{8}$	2-24 I 120		U19–U19
18,330	....	S	34 × $\frac{1}{2}$	34 × $\frac{7}{8}$	{2-32 × $\frac{1}{2}$, 2-32 × $\frac{5}{8}$}	2-36 I 194		L9–L10
19,000	....	S	33 × $\frac{1}{2}$	33 × $\frac{7}{8}$	{2-26 × $\frac{1}{2}$, 2-26 × $\frac{3}{8}$}	2-30 I 132		L10–L12
19,200	....	C	....	....	2-22 × $\frac{7}{16}$			U1–L2
....	27,000	C	....	....	2-22 × $\frac{7}{16}$			L2–U3
19,200	....	C	....	....	2-22 × $\frac{7}{16}$			U3–L4
....	21,200	S	....	....	2-24 × $\frac{1}{2}$			L6–U7
....	21,100	S	....	....	2-24 × $\frac{1}{2}$			U9–L10
....	21,200	S	....	....	2-22 × $\frac{7}{16}$	2-18 [42.7		U11–L12
....	21,200	S	....	....	2-22 × $\frac{7}{16}$			U13–L14
....	21,700	S	....	....	2-22 × $\frac{7}{16}$			U15–L16
....	21,700	S	....	....	2-22 × $\frac{7}{16}$			U17–L18
14,600	....	S	....	....	2-22 × $\frac{1}{2}$			L18–U19
....	10,800	C	....	....	2-22 × $\frac{1}{2}$			U19–L20
....	18,000	C	....	....	2-22 × $\frac{1}{2}$			L4–U5
13,400	....	S	....	....	2-24 × $\frac{9}{16}$	2-18 [45.8		U5–L6
13,040	....	S	....	....	2-26 × $\frac{5}{8}$			U7–L8
....	20,500	S	....	....	2-26 × $\frac{1}{2}$			L8–U9
14,000	....	S	....	....	2-26 × $\frac{1}{2}$	2-18 [58		L14–U15
14,100	....	S	....	....	2-24 × $\frac{1}{2}$	2-18 [58		L16–U17
13,260	....	S	....	....	2-30 × $\frac{11}{16}$	2-18 I 70		L12–U13
12,400	....	S	....	....	{2-30 × $\frac{5}{8}$, 2-16 × $\frac{1}{2}$}	2-18 I 70		L10–U11
19,480	....	S	42 × $\frac{3}{4}$	42 × $\frac{3}{4}$	2-17$\frac{1}{2}$ × $\frac{1}{2}$[f]	8-5 × 5 × $\frac{1}{2}$ ∠		L10–U11 U9–L9
....	18,000	C	19 × $\frac{5}{16}$[f]	....	....	4-6 × 3$\frac{1}{2}$ ∠ 12.3[k]		U1–L1[i]
....	....	C	19 × $\frac{5}{16}$[f]	....	....	4-6 × 3$\frac{1}{2}$ ∠ 12.3[k]		U2–L2[i]
....	....	C	19 × $\frac{5}{16}$[f]	....	....	4-7 × 3$\frac{1}{2}$ ∠ 15.3[k]		U8–L8[i]
....	18,000	C	19 × $\frac{5}{16}$[f]	....	....	4-7 × 3$\frac{1}{2}$ ∠ 15.3[k]		U11–L11[i]
....	18,000	C	19 × $\frac{5}{16}$[f]	....	....	4-7 × 3$\frac{1}{2}$ ∠ 15.3[k]		L11–T11[i]
....	....	C	19 × $\frac{5}{16}$[f]	....	....	4-8 × 3$\frac{1}{2}$ ∠ 18[k]		U10–L10
....	18,000	C	19 × $\frac{5}{16}$[f]	....	....	4-8 × 3$\frac{1}{2}$ ∠ 18[k]		L16–T16[i]
....	24,000	S	17$\frac{1}{2}$ × $\frac{1}{2}$[f]	....	2-18 × $\frac{1}{2}$	4-8 × 6 × $\frac{1}{2}$ ∠		L10–T10

duced to compensate for bending stresses due to weight of member itself. [b] In pounds per square inch. [c] C = 8 ft below and 5 ft above portal brace. [d] 30-lb quartering wind. [f] Dead load plus 30-lb quartering wind. 10 ft from L0 only. [i] Determined by live load reversal. [f] Web plates. [k] Bulb angles. [i] For complete listing of

would be governed by erection stresses. The following design procedure was, therefore, adopted:

1. Areas of members were approximated for the final structure, based on specified loading.
2. The areas were checked for erection stresses as determined by the conditions shown in Fig. 6, and increased as required.
3. The structure was analyzed for final loading stresses.

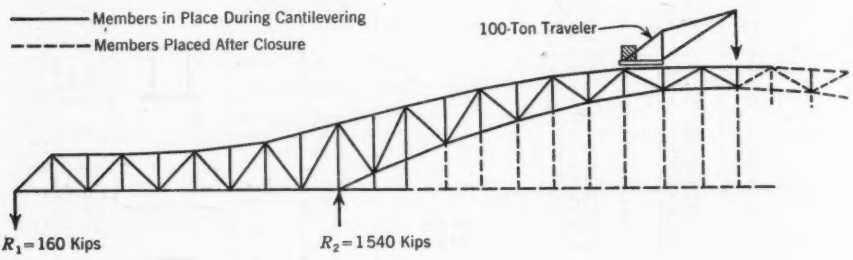

FIG. 6.—MAXIMUM LOADING ASSUMED IN DESIGNING FOR ERECTION STREESSS

A summary of the final design stresses and of the general make-up of members is given in Table 4, and a summary of dead load panel concentrations in Table 5.

TABLE 5.—DEAD LOAD

Item	Weight, kips per lin ft	Panel Points								
		0	1	2	3	4	5	6	7	8
Roadway concrete	1.316	28.00	50.60	50.60	50.60	50.60	50.60	50.60	50.60	50.60
Handrail and curbs	0.107	2.25	4.10	4.10	4.10	4.10	4.10	4.10	4.10	4.10
Crossbeams	0.092	2.30	3.35	3.35	3.35	4.00	3.35	3.35	4.00	3.35
Stringers	0.223	4.85	8.70	8.70	8.70	9.00	8.70	8.70	9.00	12.70
Floor beams[a]	0.112	6.20	4.32	4.32	4.32	4.50	4.32	4.32	4.50	4.32
Bottom laterals[a]	0.090	1.25	2.25	2.00	2.00	2.50	3.00	3.15	3.25	6.35
Top laterals[a]	0.112	....	1.82	3.64	3.64	3.64	3.64	3.64	3.65	3.65
Sway frames[a]	0.032	....	....	4.50	0.93	4.70	0.93	5.00	0.93	0.93
Portals and arch chord laterals	0.058	2.50	8.70	....	....	....	....	....	....	....
Truss	....	15.75	32.75	32.80	34.20	36.10	39.00	42.40	46.90	53.10
Miscellaneous	....	1.20	....	....	....	0.26	....	....	....	....
Total	....	64.30	116.59	114.01	111.84	119.40	117.64	125.26	127.20	139.10

[a] One

ERECTION PROCEDURE

Completion of the entire project in fifteen months was provided in the contract. Construction operations began in June, 1941. Following the attack on Pearl Harbor, Hawaii, on December 7, 1941, with the resulting emergency, construction progress was considerably slowed down, primarily because of shortages of materials and delayed deliveries. As a result, the project was

finally completed in April, 1944, although the bridge was opened to traffic in August, 1943.

General Procedure.—Erection of the viaduct girders presented no particular problem. A locomotive crane, a crawler crane, and a traveling derrick were used to place steel. Detailed erection plans and a stress analysis covering all principal steps for the three-span unit were developed by the contractor and reviewed and compared with the engineers' analysis. The diagrams in Fig. 7 illustrate the general procedure.

Three-Span Continuous Truss Unit.—Erection of the trusses of the three-span unit was started at the west end span. Commencing at pier 17, the four west panels were erected on falsework bents and the balance of the truss of span 18 was then placed by cantilevering to pier 18. On completion of this work simultaneous erection operations were carried out from pier 18 eastward and from pier 19 westward and eastward.

The east half of the three-span unit was erected by balanced cantilever methods, with pier 19 acting as the sole supporting substructure unit. In addition to resting on the shoes placed on the pier, the span was temporarily supported during erection at the first panel point east of the pier on inclined steel columns framed into the pier base. Fig. 8 shows the first stage of the balanced construction operations. Working eastward from pier 19 a guy derrick mounted at floor level erected the steel of span 20 simultaneously with placement of the steel of the east half of span 19 by a 55-ton traveling derrick operating on the top chord. Prior to landing at pier 20, 699 ft of the structure with

PANEL CONCENTRATIONS

POINTS											
9	10	11	12	13	14	15	16	17	18	19	20
52.00	50.60	50.60	50.60	50.60	50.60	50.60	50.60	50.60	50.60	50.60	50.60
4.10	4.10	4.10	4.10	4.10	4.10	4.10	4.10	4.10	4.10	4.10	4.10
3.35	3.35	4.00	3.35	3.35	3.35	4.00	3.35	3.35	4.00	3.35	3.35
16.70	12.70	9.00	8.70	8.70	8.70	9.00	8.70	8.70	9.00	8.70	8.70
26.00	4.32	4.50	4.32	4.32	4.32	4.50	4.32	4.32	4.50	4.32	4.32
12.40	7.00	4.15	3.50	3.25	3.15	2.75	2.50	2.50	2.50	2.25	2.00
3.70	3.70	3.65	3.65	3.65	3.64	3.64	3.64	3.64	3.64	3.64	3.64
....	6.60	6.60	8.15	1.86	5.56	1.86	5.23	1.86	5.00	1.86	5.00
33.50	.....	18.20	3.64	3.64	3.64	3.64	3.64	3.64	3.64	3.64	3.64
111.90	110.90	94.10	84.00	79.70	72.90	69.80	68.80	71.50	73.10	74.30	75.90
....	....	0.28	....	....	....	0.26	....	....	0.26	....	....
263.65	203.27	182.80	174.01	163.17	159.96	154.15	154.88	154.21	160.34	156.76	161.25

half.

a weight of 1,520 tons, exclusive of 110 tons of erecting equipment, were balanced on pier 19.

The west half of span 19 was placed by cantilevering out from pier 18 with a 55-ton traveling derrick mounted on the top chord. Fig. 7(*a*) shows the geometric position of the trusses at this stage. On completion of the erection of the trusses and bracing, closure was made at the lower chord panel point L20

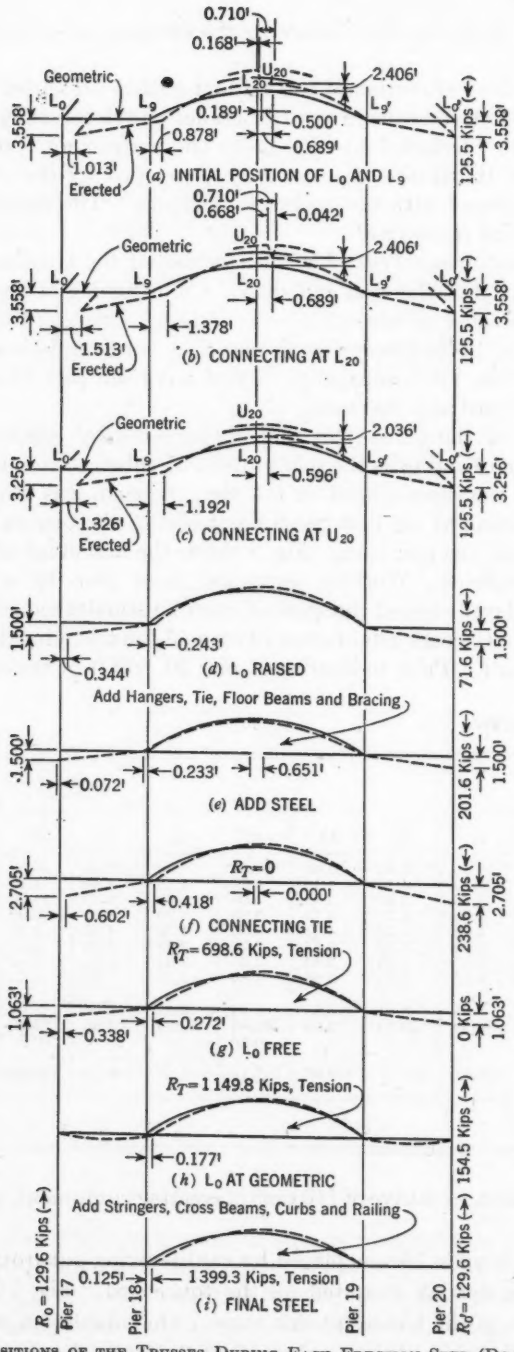

FIG. 7.—POSITIONS OF THE TRUSSES DURING EACH ERECTION STEP (DOTTED LINES) COMPARED WITH UNSTRAINED POSITION (SOLID LINES)

followed by closure of the top chord at panel points U19 and U'19. Vertical adjustments at these points were controlled by regulating the jacks and tie downs at L0 and L'0, as shown in Figs. 7(*b*) and 7(*c*), respectively. Horizontal control was provided by jacking against the ends of the trusses at pier 17 with the shoes at pier 18 free to roll. Illustrative of the accuracy of the fabrication and erection is the fact that, upon completion of the truss erection, prior to closing, only $\frac{1}{4}$-in. discrepancy was found in the horizontal length of the trusses.

At this stage of erection the unit became a three-span continuous truss. To limit erection stresses to a predetermined economic maximum, the end reactions were raised as shown in Fig. 7(*d*), resulting in a decrease in the negative end reactions. A redistribution of stresses set up in step (*a*) was thus effected. After closure of the trusses, the hangers, floor beams, and ties were placed as the traveling derricks were backed down off the span.

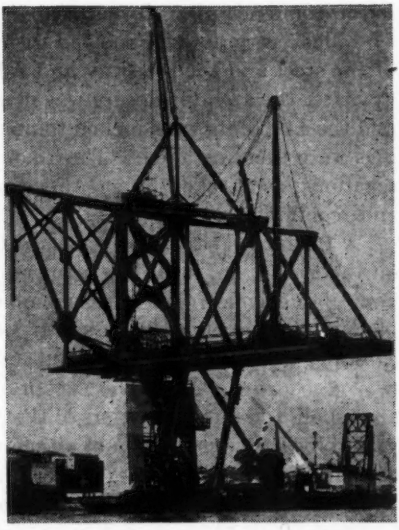

FIG. 8.—EARLY STAGE OF BALANCED ERECTION ON PIER 19, WITH STABILITY PROVIDED WHOLLY BY DIAGONAL STRUTS TO PANEL POINT L'8

During these operations all arch tie splices were riveted, except at the west end of the span at panel point T12, where final closure of the tie was to be effected. The geometric position of the trusses, and the end reaction at the end of this step are shown in Fig. 7(*e*). Fig. 9 shows the truss spans with all the hangers and ties erected just prior to closing the tie.

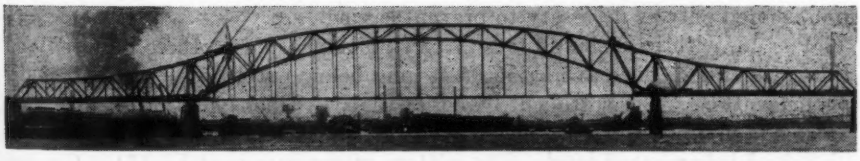

FIG. 9.—TRUSS SPAN FUNCTIONING AS A CONVENTIONAL THREE-SPAN CONTINUOUS STRUCTURE FOLLOWING CANTILEVER CLOSURE AND PRIOR TO FINAL CLOSURE OF ARCH TIE

The design contemplated lowering the ends of trusses, as shown in Fig. 7(*f*), a distance sufficient to shorten the arch span and to effect a closure of the tie in unstressed condition. To avoid the increased erection stresses in the top chord over the intermediate support, the contractor elected to effect a closure by a direct horizontal pull on the ties.

Closure of arch ties at this stage of erection required a 10-in. horizontal shortening of the arch span. Special jacking devices were used, incorporating two 350-ton hydraulic jacks per tie. Each pair of jacks was connected to a

single pump, thus assuring a uniform pull on the component parts of each tie. The jacking device consisted of a temporary diaphragm bolted near the end across each length of unconnected tie, with connecting links extending from each diaphragm to the jacking beams. To control simultaneous closure of both trusses, the pump pressures throughout the operation were limited to a maximum difference of 50 lb per sq in. After thus shortening the arch span, the tie splices were riveted, and the structure was then ready to receive the balance of the steel and the roadway and sidewalk slab.

After the arch tie closure was made, the end truss anchorages were released, and the L0 points were jacked to final elevation. The rocker column end bearings were then installed. The amount of vertical movement and the resulting reactions, as checked by gage readings, corresponded closely to the computed end reactions.

A 16-ton floor traveler was used for the erection of the balance of steelwork on the arch span, including stringers, crossbeams, curbs, and railing. To prevent undue overstress of truss members due to unbalanced loading, symmetrical placing of concrete slab, in so far as practical, was required, conforming to a carefully devised pouring schedule.

Substructure.—With the exception of piers 18 and 19, at the ends of the arch span, the forty-two piers of the structure are of conventional type. Excavation for the construction of all piers was done in the open. Cofferdams were used in constructing bases of certain piers—timber crib cofferdams for piers 18 and 19, and steel sheet pile cofferdams for pier 17 and piers 20 to 25, inclusive.

Piers 26 to 31, inclusive, were constructed in open excavations kept essentially free of ground water by a system of well points sunk around the perimeter of the excavated area. Excavation was opened for a single trench enclosing all piers on which work was under way. After the bases of the first group of piers were completed, the excavation was extended to include a second group. Well points were advanced and the area occupied by the first group of piers was backfilled. The bases of piers 1 to 16 and piers 32 to 41 and of the two abutments are above ground-water level and were constructed in open excavation, shored only as necessary to prevent caving.

Pier 18, with base size 43 ft by 75.5 ft, is supported on three hundred and thirty-six timber piles averaging 95 ft long below cutoff level—23.5 ft below pool elevation. The bottom of base is at El. 531. Pier 19, with base size 49 ft by 75.5 ft, is supported on three hundred and eighty-four timber piles averaging 95 ft long below cutoff level. The bottom of the base is at El. 551. Greater depth of water and stronger current encountered at pier 18 required a deeper pier than at pier 19.

To provide the penetration necessary at pier 18 to withstand scour and yet to reduce the weight of the pier to a minimum, the design incorporated in the pier base a sand core 27 ft by 57.5 ft by 25 ft high. This arrangement effected a saving both in the number of supporting piles and in the volume of concrete. The 8-ft and 9-ft concrete walls surrounding the sand core, together with a cover 7 ft thick, were poured under water by a tremie. The sand core was placed simultaneously through tremie pipes so as to maintain the top of sand to conform with the rate of placing concrete walls. After pumping out the

temporary cofferdam, the piles were cut off and the pier base was finished with a 12-ft thickness of concrete poured in the dry. A concrete shaft 70 ft high was then built to complete the pier.

A saving of 1,438 cu yd of concrete resulted from using the sand core instead of a solid concrete base and effected a saving of 5.1 tons per pile in dead load weight. To have constructed a solid concrete base providing the same penetration and the same maximum load per pile would have required a base 52 ft by 81 ft founded on four hundred and forty piles. On the basis of contract unit prices, such a design would have increased the cost of the pier $133,500.

Quantities and Cost

For the truss spans, 3,850,100 lb of structural silicon steel and 4,583,975 lb of carbon steel were required. In addition, 6,410,492 lb of structural steel were required in the girder spans. Also required were 2,463,500 lb of reinforcing steel, 34,087 cu yd of concrete, and 2,909 timber piles. The total cost of the project was $3,175,341.63.

Conclusions

A significant advantage of the continuous tied arch structure as compared to the conventional truss types is the economy of metal, due to a more advantageous distribution of weight for cantilever erection.

Although the subject of esthetics does not lend itself to argument based on reason only, since it is rather a matter of individual taste, an opinion is ventured that this type of structure does possess elements of beauty not found in the conventional type trusses. Similarly, the almost unobstructed river view from the main span is also a favorable feature.

The tied arch type permits erection of a self-supporting part of the structure (the trusses) spanning between piers to be utilized subsequently in completing erection of the balance of the structure. The full weight of the stress carrying framed structure must be supported on the conventional cantilever and continuous designs during erection before any closure can be made.

Acknowledgment

The Julien Dubuque Bridge is operated by the City of Dubuque Bridge Commission. The project was financed by a revenue bond issue and contributions of the state highway departments of Illinois and Iowa. The Bridge Commission was created by an Act of Congress and consists of Charles T. Landon, chairman, and Charles T. Kretschmer, William M. Clemens, Thomas M. Stampfer, and Russel E. Werner.

The cooperation and competent counsel of state and government officials were in large measure responsible for the successful completion of the project under restrictive wartime conditions. Acknowledgment is made of the able assistance of F. R. White, M. ASCE, chief engineer of the Iowa State Highway Commission; Walter A. Rosenfield, director, and W. W. Polk, M. ASCE, chief highway engineer of the Illinois Highway Commission; and Clifford Shoemaker and J. T. Voshell, Members, ASCE, district engineers of the Public Roads Administration at Kansas City, Mo., and Chicago, Ill., respectively.

The ingenious methods used by the contractors in placing foundations and in the erection of superstructure successfully accomplished the conceptions of the design. Careful and detailed analysis of all erection steps and problems and accurate fabrication of steel by the Bethlehem Steel Company were reflected in the manner in which erection proceeded. This company was represented by Jonathan Jones, M. ASCE, chief engineer; S. C. Campbell, general superintendent; and W. W. Oskin, Assoc. M. ASCE.

Contractors for the substructure, the F. J. Robers Construction Company and the La Crosse Dredging Corporation, were represented on the project by P. C. Peterson, engineer in charge. Gund-Graham Company, Melberg and Taylor, and Ulrich Willy Company were contractors for other miscellaneous work. Construction was begun on June 18, 1941, and the bridge was opened to traffic on August 31, 1943.

The bridge was planned, designed, and the construction supervised by Howard, Needles, Tammen and Bergendoff. Ned L. Ashton, Josef Sorkin, and Jacob Karol, Members, ASCE, participated in the design. Detailed plans were prepared under the supervision of C. S. Harper, M. ASCE. Ivan P. Hanson, Assoc. M. ASCE, served as resident engineer for the consultants.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

CLASSIFICATION AND IDENTIFICATION OF SOILS

BY ARTHUR CASAGRANDE,[1] M. ASCE

SYNOPSIS

The purpose of this paper is threefold: (1) To review and compare existing soil classifications used in civil engineering, with emphasis on their limitations; (2) to present a new soil grouping tentatively adopted by the United States Engineer Department in 1942 for use on airfield projects; and (3) to outline field identification procedures, requiring no apparatus. In abbreviated form, this paper was presented at the Annual Meeting of the Society in New York, N. Y., in January, 1943.

1. INTRODUCTION

The two most controversial chapters in the study of soils for civil engineering purposes are: (*a*) Soil classification, the most confused chapter; and (*b*) the shear strength of soils, the most difficult chapter. If the art of soil classification is far from satisfactory, the confusion is often made worse in that users are unaware of its limitations and apply it for purposes other than that originally intended.

Thorough knowledge of not only one but all classification systems important in civil engineering is the best protection against pitfalls and is the best assurance of an intelligent application of an individual classification. In addition, it provides tools with which the engineer can fashion, if necessary, a new classification to fit his needs in applying soil mechanics to a particular problem. It also frees him from the danger of becoming narrow-minded in this field. Those who really understand soils can, and often do, apply soil mechanics without any formally accepted soil classification.

Limitations of Soil Classification Systems.—It is not possible to classify all soils into a relatively small number of groups such that the relation of each soil to the many divergent problems of applied soil mechanics will be adequately presented. Some systems require detailed tests to permit proper grouping of

NOTE—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **November 1, 1947.**

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soils; others are specifically designed so that classification is possible on the basis of field identification without quantitative test results. Some classifications were created principally to facilitate making soil surveys, whereas others are chiefly to aid in digesting and correlating empirical knowledge in a specific engineering problem.

Development of Systems.—To some extent, soil classifications used in civil engineering reflect in their chronological order the development of soil mechanics. In its beginnings this leaned heavily on the soil testing methods

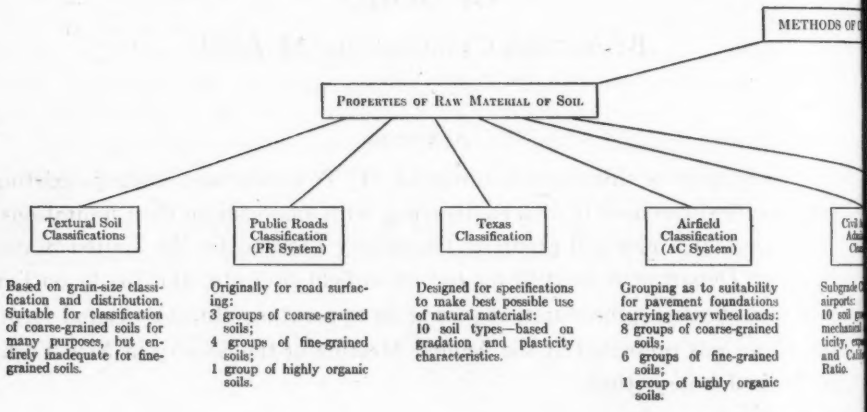

METHODS OF
PROPERTIES OF RAW MATERIAL OF SOIL
Textural Soil Classifications
Public Roads Classification (PR System)
Texas Classification
Airfield Classification (AC System)
Based on grain-size classification and distribution. Suitable for classification of coarse-grained soils for many purposes, but entirely inadequate for fine-grained soils.
Originally for road surfacing:
3 groups of coarse-grained soils;
4 groups of fine-grained soils;
1 group of highly organic soils.
Designed for specifications to make best possible use of natural materials:
10 soil types—based on gradation and plasticity characteristics.
Grouping as to suitability for pavement foundations carrying heavy wheel loads:
8 groups of coarse-grained soils;
6 groups of fine-grained soils;
1 group of highly organic soils.

FIG. 1.—METHODS

developed by agricultural soil science. Since this science originally stressed classification by grain size, the first soil classifications used by civil engineers were textural classifications. Most of these were based on the relative contents of three grain-size fractions called sand, silt, and clay. Later classifications used the entire grain-size curve.

Probably the first departure from the grain-size analysis as the exclusive basis for classification was proposed in 1908 by A. Atterberg in Sweden for agricultural purposes. Later, the Swedish Geotechnical Commission devised and used a similar classification in its extensive research. Still later, the United States Bureau of Public Roads classification was created, with identification and classification of fine-grained soils based almost exclusively on tests other than the grain-size analysis. Finally, new soil classifications have been developed to fill the needs in design and construction of airfields for heavy airplanes.

The essential elements of all classifications described in this paper are summarized in Fig. 1.

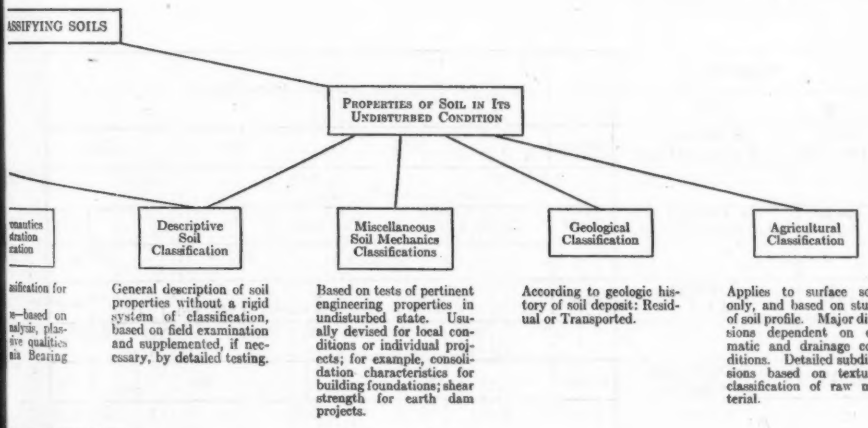
2. GRAIN-SIZE SCALES

The division of grain sizes into fractions with arbitrarily standardized boundaries is usually called grain-size classification. It is not really a soil classification but rather a grain-size scale, and will be so termed in this paper,

to prevent confusion. Its purpose is to form the basis of grain-size classifications of soils, usually termed textural soil classifications.

The most important grain-size scales, summarized in Fig. 2, will be discussed.

United States Bureau of Soils Grain-Size Scale.—This scale is the one most commonly used in the United States. For practical purposes, and particularly for the textural soil classification as developed by the Bureau, the fraction between 1 mm and 2 mm has always been grouped together with the sand fraction (1).[2]


ASSIFYING SOILS
PROPERTIES OF SOIL IN ITS UNDISTURBED CONDITION
Descriptive Soil Classification
Miscellaneous Soil Mechanics Classifications
Geological Classification
Agricultural Classification
General description of soil properties without a rigid system of classification, based on field examination and supplemented, if necessary, by detailed testing.
Based on tests of pertinent engineering properties in undisturbed state. Usually devised for local conditions or individual projects; for example, consolidation characteristics for building foundations; shear strength for earth dam projects.
According to geologic history of soil deposit: Residual or Transported.
Applies to surface soils only, and based on study of soil profile. Major divisions dependent on climatic and drainage conditions. Detailed subdivisions based on textural classification of raw material.

CLASSIFYING SOILS

International Grain-Size Scale.—A. Atterberg's (2) scale was proposed as international standard in 1913 when the International Society of Soil Science (3) stated:

> "It was decided to accept the group division of soil grains recommended by Dr. Atterberg. For the time being the following nomenclature shall be used in the German language [English terms added by the author]:
>
> | | | |
> |---|---|---|
> | Grains larger than 20 mm..... | Stein und Geröll | [stones] |
> | Grains from 20 mm to 2 mm..... | Kies | [gravel] |
> | Grains from 2 mm to 0.2 mm..... | Grobsand | [coarse sand] |
> | Grains from 0.2 mm to 0.02 mm..... | Feinsand | [fine sand] |
> | Grains from 0.02 mm to 0.002 mm... | Schluff | [silt] |
> | Grains finer than 0.002 mm..... | Kolloidale Teilchen oder Rohton | [colloidal particles or raw clay]" |

A noteworthy exception to this proposal was taken in 1914 by J. Kopecky (4), who recommended a modification based on the following divisions: 0.2, 0.06, 0.02, 0.006, and 0.002 mm. It is of interest that the scale adopted by the Massachusetts Institute of Technology (M.I.T.), Cambridge, Mass. (Fig. 2), has essentially the same numerical values.

M.I.T. Grain-Size Scale.—The advantages of the M.I.T. scale are (*a*) that the 0.002-mm limit for the clay fraction has been adopted to correspond to the

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (Appendix).

international grain-size scale; and (*b*) that it, like that of the Bureau of Soils, possesses the simplicity of division into three major fractions, and permits the plotting of results on a triangular chart, as discussed in Section 3, textural soil classifications.

United States Department of Agriculture (U.S.D.A.) Grain-Size Scale.—The U.S.D.A. modified the original Bureau of Soils scale by (1) lowering the upper boundary for the clay fraction from 0.005 mm to 0.002 mm, to bring this scale in line with the international standard and (2) designating the entire range from 2.0 mm to 0.05 mm as sand.

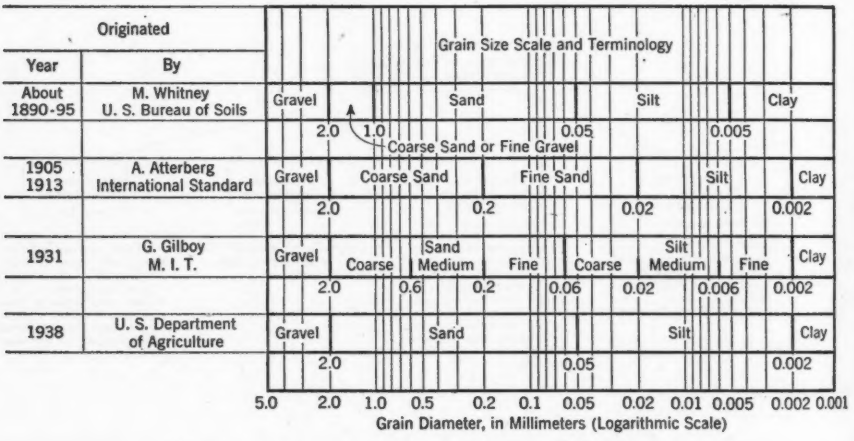


FIG. 2.—COMPARISON OF PRINCIPAL GRAIN-SIZE SCALES

Comparison of the M.I.T. and the new U.S.D.A. scales (Fig. 2) shows that their major divisions are practically identical. The various subdivisions are of no consequence since, in fact, they are never used in practical applications.

Limitations of Grain-Size Scales.—Below the range of the sieve analysis, all preceding grain-size scales represent merely two points of the grain-size curve. This is a very incomplete presentation as compared with the complete curve, which can be obtained readily by modern methods of wet-mechanical analysis.

The use of the terms "silt" and "clay" for arbitrary grain-size fractions is misleading. Such terms should be reserved exclusively for the description of soils having the corresponding physical characteristics. To designate the size fractions, one should either use the terms "sand sizes," "silt sizes," "clay sizes," together with the name of the classification used, or state the size without a name. For example, "30% clay sizes according to the Bureau of Soils Classification" would mean "30% < 0.005 mm." Obviously, designation of the grain size itself is much more brief and leaves less doubt as to its meaning. If a publication merely refers to 12% clay, the reader must usually guess what particular grain-size fraction is meant, or whether perhaps 12% of the total material consists of a plastic soil.

Because of the limited usefulness of the grain-size scales, the lack of a uniform standard is less serious than would be expected. Since the new U.S.D.A. scale is almost identical with the M.I.T. scale, either would probably cause less confusion if adopted as a United States standard. In the absence of such a uniform standard, it is desirable to adhere to the designation of the numerical limits of the grain-size fraction, omitting any names for the individual fractions.

In 1945, R. Glossop and A. W. Skempton (5) proposed the M.I.T. grain-size scale as a standard in their valuable contribution on this subject.

3. Textural Soil Classifications

Basis and Types.—The principle of classifying soils purely on the basis of their grain-size distribution dates back to the time when it was not yet realized that the physical properties of fine-grained soils, having the same grain-size distribution, can be widely different. The simplicity of textural soil classifications, and the fact that they can be applied with little experience, are the main reasons why they are still widely used.

In the United States the original textural classifications are based on the relative percentages of the sand, silt, and clay size fractions. They are facilitated by plotting these three fractions on a triangular chart. Therefore, they will be referred to as triangle textural classifications.

For individual engineering projects, textural classifications have been devised which are based on a comparison of the entire grain-size curves of soils with a set of master curves. These classifications will be called master curve textural classifications.

Triangle Textural Classifications.—The most widely used triangle textural classification is the U. S. Bureau of Soils classification (6) shown in Fig. 3. Soils are divided into the ten groups whose names appear inside the triangle. This classification is applied only to that part of a soil smaller than 2 mm in size, with the sand fraction extending from 2.0 mm to 0.05 mm.

The principle of the triangular plot is that any point represents a number on each of the three scales, and these three numbers always add up to 100. Therefore, to classify a soil, it is only necessary to determine the sand and silt fractions in per cent, and to find the corresponding point in the triangle. For example, if the fraction of a soil smaller than 2 mm in size contains 26% of sand sizes, and 32% of silt sizes (automatically making the contents of clay sizes 42%), then the position of this soil on the chart is found by the intersection of the 26% line on the sand scale with the 32% line on the silt scale. It can be seen that this point P (Fig. 3(*a*)) is to be classified as a "clay." If particles larger than 2 mm in size were present in quantities sufficient to be especially noticeable, this would be indicated by adding the term "gravelly" to the name of the soil.

Engineers who have used this classification have often noticed serious discrepancies between the names as derived from the triangle and the actual physical characteristics of the soils. Therefore, they have introduced other subdivisions and, in some cases, other names. For example, Fig. 3(*b*), used by the Lower Mississippi Valley Division of the U. S. Engineer Department,

represents an attempt to adjust this method of classification to the soils of the lower Mississippi Valley. It should be noted that "loam," which is really an agricultural term, is not used in Fig. 3(*b*).

The difficulty of all triangle classifications lies in the fact that those physical characteristics of fine-grained soils which are of interest to the civil engineer often are not reflected by the three grain-size fractions. Point P in Fig. 3(*a*) may represent a highly plastic soil in one part of the United States and a slightly plastic silt in another locality.

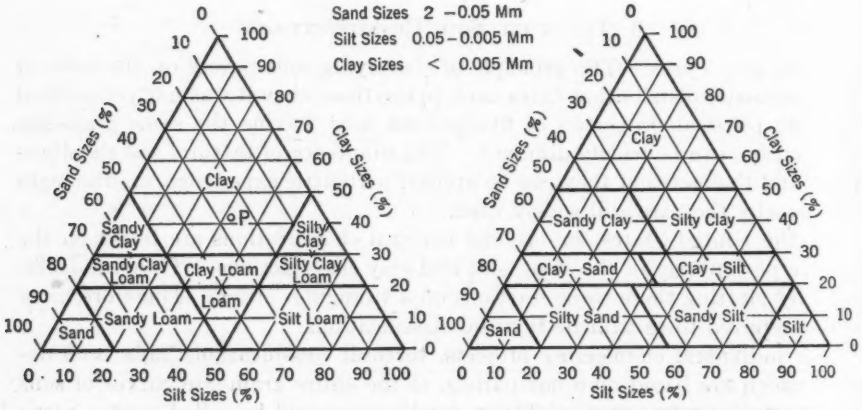


FIG. 3.—COMPARISON OF SOILS CLASSIFICATION TRIANGLES (SIZES IN MILLIMETERS: SAND FRACTION, 2 TO 0.05; SILT FRACTION, 0.05 TO 0.005; AND CLAY FRACTION, LESS THAN 0.005)

The more successful use of the classification in Fig. 3(*a*) in agricultural soil studies may be because in well-developed surface soils certain fine-grained materials such as uniform, nonplastic, rock flour do not exist. It would seem as if the weathering due to the mechanical, chemical, and biological forces, which are responsible for the development of the surface soils, tends to reduce the range of physical properties which a soil with a given grain-size distribution may possess; or, in other words, the grain-size distribution is more likely to reflect the physical properties of a weathered surface soil than of the underlying unweathered materials.

Master Curve Textural Classifications.—The development of modern methods of wet-mechanical analysis which yield the entire grain-size curve as readily as the three fractions used in the triangle classification has led to utilizing the entire grain-size curve for soil classification. This can be accomplished by plotting a set of typical grain-size curves, called master curves, covering the desired range of soils.

Examples of master curve textural classifications are the classifications developed by the Boston Metropolitan District Water Supply Commission (7), by the U. S. Engineer Office in Providence, R. I. (8), and by Donald M. Burmister, Assoc. M. ASCE (8*a*). In the Providence classification, the ne-

cessity for further differentiation of fine-grained soils into nonplastic and plastic soils is also recognized.

Textural soil classifications by means of master curves have been reasonably successful for noncohesive soils of the same general geologic origin. However, for most fine-grained soils the same objections apply as for the triangle classification.

TABLE 1.—COMPARISON OF GRAIN-SIZE DISTRIBUTION AND PHYSICAL CHARACTERISTICS OF SOME FINE-GRAINED SOILS

No.	Location (1)	Material (2)
1	Winnipeg, Canada	Very tough, highly plastic clay
2	Massena, N. Y.	Soft, medium plastic, Laurentian clay
3	Massena, N. Y.	Soft, somewhat organic, silty Laurentian clay of low plasticity
4	Vera, Wash.	Kaolin-type clay, with typical lack of toughness
5	California	Diatomaceous earth with characteristics of clayey rock flour
6	Wayland, Mass.	Diatomaceous earth with characteristics of nonplastic rock flour
7	Missouri	Lean, silty clay
8	Hartwick, N. Y.	Typical rock flour

TABLE 1.—(*Continued*)

No.	PERCENTAGE BY WEIGHT SMALLER THAN: (DIAMETERS IN MILLIMETERS)				Liquid limit	Plastic limit	Plasticity index	Dry strength	AC group symbols[a]
	0.05 (3)	0.005 (4)	0.002 (5)	0.001 (6)	(7)	8)	(9)	(10)	(11)
1	90	75	54	35	96	29	67	Very high	CH
2	96	77	62	50	55	26	29	Medium	CH
3	92	70	50	37	37	24	13	Medium	CL–OL
4	96	76	50	34	55	31	24	Low	MH–CH[b]
5	96	70	47	34	122	80	42	Very low	MH
6	92	52	27	15		Nonplastic		Almost none	MH
7	90	26	16	10	35	22	13	Medium	CL
8	88	20	10	6	24	21	3	Very low	ML

[a] Airfield Classification System symbols; see Table 4. [b] The new KH-group.

Limitations of Textural Soil Classifications.—Table 1 shows the large range of soils covered by almost identical grain-size curves. Soils Nos. 1 to 5 range from a highly plastic, very tough clay having high dry strength, through kaolin-type clays lacking toughness and having only low dry strength, to diatomaceous soils with very low plasticity and only slight dry strength.

It is also instructive to compare the grain-size distribution of soils Nos. 6 to 8 (Table 1) with their other characteristics. The highest percentage of grains smaller than 0.005 mm is shown by the diatomaceous earth which is practically noncohesive and nonplastic. The clay (No. 7) with only 26% material smaller than 0.005 mm actually shows more plasticity and greater dry strength than soil No. 5 containing three times as much material smaller than 0.005 mm.

Many similar discrepancies between grain-size distribution and physical characteristics of fine-grained soils could be cited. Nevertheless, the grain-

size curves of fine-grained soils are still extensively used as the exclusive basis for soil classification. In contrast to the misleading impressions which these curves may convey, the pertinent physical properties of such soils can easily be distinguished on the basis of simple field identification procedures.

4. Agricultural Soil Classification

A classification of surface soils for agricultural purposes, called pedological classification, is a complicated system in which the classification of the physical properties of an individual soil layer represents merely one element. Since surface soils have "grown" under the influence of climate, vegetation, type of underlying parent material, topographic features, and age, all these factors, and particularly their influence on the chemistry of the surface soils, play a major role in a pedological soil classification.

Of main interest to the civil engineer is that part of such a classification which describes the physical properties and composition of the surface soils and of the underlying parent materials. In the United States, such description is confined to a textural classification.

Because in a textural classification soils with different physical characteristics would be classified the same, A. Atterberg in Sweden suggested the classification (3) of the physical properties of soils for agricultural purposes shown in Table 2. Further subdivisions are made to distinguish between soils rich in

TABLE 2.—Atterberg Soil Classification

Major divisions	Secondary divisions	Description
A.—Clays (Plastic Soils)		
	I.—Sticky Clays (Highly Plastic)	This group contains only the heaviest clays
	II.—Loamy Clays (Not Sticky)	Subdivided into medium heavy and fairly heavy clays
B.—Loams (Nonplastic, More or Less Cohesive Soils)		
	I.—Fairly Heavy Loams	Clayey loams
	II.—Light Loams	Sandy loams and loess soils
C.—Sand, *Mo*, and Silt Soils (Noncohesive Soils)		
	I.—Capillarity Greater Than 34 Cm	Fine-grained sandy soils; dust loess; subdivided by mechanical analysis
	II.—Capillarity Less Than 34 Cm	Coarse, dry sandy soils, useful only for forestry

humus or lime, and soils containing gravel, stones, and diatoms. To assist in identification, Atterberg proposed the use of his plasticity tests as well as a dry strength number based on the dry strength of the soil. Thus, his classification emphasizes pertinent physical characteristics whose differentiation is not possible by using grain-size analysis alone.

Use of Agricultural Soil Maps in Civil Engineering.—The surface soils of large areas of the United States and other countries have been surveyed and

mapped from an agricultural standpoint. Often it is possible to correlate, by means of field observations and tests, the classifications used on such maps with the pertinent engineering properties of these soils so as to save much time in soil surveys for roads and airfields, and in finding sources of satisfactory material for fill construction. Also, with sufficient experience, or advice from agricultural soil experts, the engineer can derive information on the subdrainage characteristics of a given area.

A systematic correlation between agricultural soil classifications, as found on the soil maps, and the engineering properties, has been published for the State of Indiana (9).

5. Geologic Soil Classifications

Every comprehensive soil investigation should also include a geologic classification. In most cases differentiation between the two major subdivisions, that is, residual and transported soils, will be possible on the basis of a general field examination. Further subdivisions of the transported soils, such as glacial, fluvial, lacustrine, marine, aeolian, and volcanic, are a useful and sometimes necessary part of a comprehensive soil classification which, however, may require the assistance of a geologist. When geologic maps are available, they should be utilized in connection with foundation investigations for important civil engineering projects.

6. United States Bureau of Public Roads Classification System

Development and Limitations.—The Public Roads (PR) classification (10) was developed in the years 1927 to 1929 from extensive research that the U. S. Bureau of Public Roads had directed particularly toward the use of soils in secondary road construction. The original definition of its groups is based on the stability characteristics of soils when directly acted upon by wheel loads. This classification, therefore, is essentially a grouping applicable to the use of soil in a road surface or as a base beneath a thin bituminous wearing surface. Unfortunately, it has been termed "subgrade classification." Attempts to use it for judging the characteristics of subgrades beneath a substantial thickness of pavement and base have frequently led to disqualification of excellent subgrade and subbase materials. The susceptibility of soils to frost action is also not properly recognized in this classification because at the time it was developed, knowledge of frost action was still limited.

A simplified presentation of the PR classification system is shown in Table 3. It differs from the usual presentation (10) to place more emphasis on those characteristics which aid in field identification. To understand the use of the letter A in the group designations, it is noted that the original PR classification also provided for three additional groups designated B–1, B–2, and B–3. These soon fell into disuse and need not be considered here.

Classification Procedure.—The original publication (10) contains recommendations for classifying soils based on grain-size distribution, plasticity, and shrinkage characteristics, as well as on other tests which are not so commonly used. The numerical limits of the test results, which determine the group classification of a particular soil, have undergone many revisions, the latest (11) being dated February, 1942.

Modifications of PR System.—Some of the shortcomings of the PR system were recognized by soils engineers in various highway departments, who tried either to adapt it to their local soil conditions and problems, or to develop new classifications. In New Hampshire, where frost action is the most destructive agent, engineers have eliminated the A–1 group entirely, since soils in this group (such as some well-graded glacial tills) were found to be just as subject to frost heaving as soils in the A–2 group. Therefore, all sandy and gravelly soils of the A–1 and A–2 groups were classified as A–2 materials, and the new A–2 group was subdivided into three subgroups, depending on their frost-heaving characteristics. The definition of the A–3 group was not changed; all soils in this group are considered better subgrades than the soils in the combined A–1 and A–2 groups (12).

TABLE 3.—SIMPLIFIED CHART OF PUBLIC ROADS CLASSIFICATION SYSTEM

Major divisions		Group	General description	Examples
Coarse-Grained Soils		A–1	Coarse with fines—well graded; considerable dry strength and practically no swelling and shrinkage.	Sand-clays in Southern States; some types of glacial till.
		A–2	Coarse with fines—compared with A–1 either deficient in dry strength or subject to volume change.	Many types of glacial deposits and of residual soil.
		A–3	Coarse-grained soils containing less than 10% passing No. 200 mesh and having no dry strength.	Clean, well graded glacial gravel; beach and dune sands.
Fine-Grained Soils	Silts	A–4	Silts and clays of low plasticity	Rock flour; many sandy and silty clays; loess.
		A–5	Elastic silts	Highly micaceous silt; diatomaceous earth.
	Clays	A–6	Highly colloidal types of inorganic clays; medium to high plasticity.	Some inorganic clays from southern and tropical regions.
		A–7	The great majority of inorganic clays and some types of organic clays; medium to high plasticity.	Glacial clays; some types of gumbo; kaolin-type clays; some organic clays.
Peat and Muck		A–8	Materials having very high compressibility.	Peat and organic clays.

Realizing that the PR system is seriously deficient for classifying subgrades, highway engineers have given consideration to a radical revision of this system without changing the designations A–1, A–2, etc. Under a tentative proposal, all gravelly and coarse sandy soils of the A–3 group, which are excellent subgrades, would be classified as A–1 materials, together with those originally defined as A–1. Equally radical changes are proposed for the fine-grained soils. If this is adopted, it would not be possible to gain a clear idea of the type of soil from the PR designation, except by noting the "edition" of the classification. Such confusion could easily be avoided by giving this radical modification an entirely new designation not mistakable for the original, as by using a prefix letter other than A.

7. TEXAS HIGHWAY DEPARTMENT SOIL CLASSIFICATION SYSTEM

About 1939, the Texas Highway Department developed a classification of Texas soils for writing specifications such that natural materials would be used

to best advantage in road construction (13). The ten soil types are based essentially on gradation and plasticity characteristics. Type 1, which is typical, is defined as follows:

> Type 1 soil is a material which will retain high stability under the direct impact of traffic loads. Soils of this type are the only materials permitted in the upper six inches of the finished roadway. This type of material can also be used for the roadbed below six inches, if economically available. Type 1 soil includes caliche, rock, shell, iron ore, or gravel with good grading, and excellent soil binder. The detailed requirements vary depending upon whether the aggregate is angular or rounded by nature.

The other soil types are classified in the order of decreasing value in road construction. Types 7 to 9 represent clays of increasing plasticity and type 10 represents a highly compressible material, usually of organic character, having a liquid limit greater than 90.

8. Civil Aeronautics Administration (CAA) Soil Classification System

The CAA system, established in 1944 for classifying soils in the order of their value as subgrade of airport pavements, is based on the mechanical analysis, plasticity characteristics, expansive qualities, and California Bearing Ratio (CBR).

Soils are divided into groups designated by the symbols E–1 to E–10. Their characteristics are briefly described by A. H. Hadfield (14):

> "* * * there are two groups: the E–1 to E–4 'granular soils,' containing 50% or more of sand and the E–5 to E–10 'non-granular soils,' containing less than 55% sand (the E–5 has 65% or less sand).
>
> "The granular soils are further divided into non-frostheave soils, groups E–1 and E–2, and soils subject to frost heave, groups E–3 and E–4. The E–1 soil is a free draining, non-plastic sand corresponding to the PRA classification A–3. The E–2 soil is a sand containing slightly more silt and clay than the E–1. The E–3 corresponds to a non-plastic and moderately plastic PRA A–2 type, and the E–4 is equivalent to the PRA A–2 plastic type.
>
> "In the non-granular group the E–5 soil corresponds to the non-plastic or moderately plastic A–4 silt, and the E–6 to the more plastic A–4 silts, the A–4 and A–6 silty clays, and the A–6 or A–7 clays of low plasticity. The E–7, E–8 and E–9 groups include clay soils of average plasticity, high plasticity and very high plasticity, respectively, covered by PRA's A–6 and A–7 soils groups. The E–10 is the highly elastic soil classed as A–5 by PR."

9. Airfield Classification System for U. S. Engineer Department

Development.—As a result of difficulties in the use of other classifications for the design and construction of military airfields, the Office of the Chief of Engineers tentatively adopted in 1942 a new soil classification which will be referred to as the Airfield Classification (AC) System. It was originally developed by the writer for instruction in Army courses on "Control of Soils in Military Construction" offered at the Graduate School of Engineering, Harvard University, Cambridge, Mass., from 1942 to 1944.

TABLE 4.—AIRFIELD CLASSIFICATION SYSTEM—PHYSICAL CHARACTERISTICS OF SOIL GROUPS AS AFFECTING CONSTRUCTION OF FOUNDATIONS FOR AIRFIELD PAVEMENTS[a]

1		2	3	4		5	6
Major divisions		Soil groups and typical names	Group symbol	General Identification (On Disturbed Samples)		Observations and tests relating to material in place	Principal classification tests (on disturbed samples)
				Dry strength[b]	Other pertinent examinations		
Coarse-grained soils	Gravel and gravelly soils	Well-graded gravel and gravel-sand mixtures, little or no fines	GW	None	Gradation; Grain shape; Examination of binder wet and dry; Durability of grains	Dry unit weight or void ratio; Degree of compaction; Cementation; Stratification and drainage characteristics; Ground water conditions; Traffic tests; Large-scale load tests; California bearing ratio tests	Mechanical analysis
		Well-graded gravel-sand mixtures with excellent clay binder	GC	Medium			Mechanical analysis, liquid and plastic limits on binder
		Poorly graded gravel and gravel-sand mixtures, little or no fines	GP	None			Mechanical analysis
		Gravel with fines, silty gravel, clayey gravel, poorly graded gravel-sand-clay mixtures	GF	Very slight to high			Mechanical analysis, liquid and plastic limits on binder if applicable
	Sands and sandy soils	Well-graded sands and gravelly sands, little or no fines	SW	None			Mechanical analysis
		Well-graded sand with excellent clay binder	SC	Medium to high			Mechanical analysis, liquid and plastic limits on binder
		Poorly graded sands, little or no fines	SP	None			Mechanical analysis
		Sand with fines, silty sands, clayey sands, poorly graded sand-clay mixtures	SF	Very slight to high			Mechanical analysis, liquid and plastic limits on binder if applicable

TABLE 4—(*Continued*)

1		2	3	4		5	6
Major divisions		Soil groups and typical names	Group symbol	General Identification (On Disturbed Samples)		Observations and tests relating to material in place	Principal classification tests (on disturbed samples)
				Dry strength[b]	Other pertinent examinations		
Fine-grained soils containing little or no coarse grained material	Fine-grained soils having low to medium compressibility; liquid limit < 50	Silts (inorganic) and very fine sands, *mo*, rock flour, silty or clayey fine sands with slight plasticity	ML	Very slight to medium	Shaking test and plasticity	Dry unit weight, water content and void ratio	Mechanical analysis, liquid and plastic limits if applicable
		Clays (inorganic) of low to medium plasticity sandy clays, silty clays, lean clays	CL	Medium to high	Examination in plastic range	Consistency, undisturbed and remolded	Liquid and plastic limits
		Organic silts and organic silt-clays of low plasticity	OL	Slight to medium	Examination in plastic range, odor, color	Stratification, root holes and fissures Drainage and groundwater conditions	Liquid and plastic limits from natural condition and after oven drying
	Fine-grained soils having high compressibility; liquid limit > 50	Micaceous or diatomaceous fine sandy and silty soils, elastic silts	MH	Very slight to medium	Shaking test and plasticity	Traffic tests Large-scale load tests	Mechanical analysis, liquid and plastic limits if applicable
		Clays (inorganic) of high plasticity, fat clays	CH	High to very high	Examination in plastic range	California bearing ratio tests	Liquid and plastic limits
		Organic clays of medium to high plasticity	OH	Medium to high	Examination in plastic range, odor, color	Compression tests	Liquid and plastic limits from natural condition and after oven drying
Fibrous organic soils with very high compressibility		Peat and other highly organic swamp soils	Pt	Readily identified		Consistency, texture and natural water content	

TABLE 4—(*Continued*)

3	7	8		9	10	11	12	13[d]	14	15
Group symbols	Value as foundation when not subject to frost action	Value as Wearing Surface for Stage or Emergency Construction		Potential frost action	Compressibility and expansion	Drainage characteristics[c]	Field compaction characteristics and equipment	Solids at optimum compaction (lb per cu ft) and void ratio, e	California bearing ratio for compacted and soaked specimen	Comparable groups in PR classification
		With dust palliative	With bituminous surface treatment							
GW	Excellent	Fair to poor	Excellent	None to very slight	Almost none	Excellent	Excellent; crawler tractor, rubber tired equipment	>125 e <0.35	>50	A-3
GC	Excellent	Excellent	Excellent	Medium	Very slight	Practically impervious	Excellent; tamping roller, rubber tired equipment	>130 e <0.30	>40	A-1
GP	Excellent	Poor	Poor to fair	None to very slight	Almost none	Excellent	Good to excellent; crawler tractor, rubber tired equipment	>115 e <0.45	25–60	A-3
GF	Good to excellent	Poor to good	Fair to good	Slight to medium	Almost none to slight	Fair to practically impervious	Good to excellent; crawler tractor, rubber tired equipment, tamping roller	>120 e <0.40	>20	A-2
SW	Excellent	Poor	Good	None to very slight	Almost none	Excellent	Excellent; crawler tractor, rubber tired equipment	>120 e <0.40	20–60	A-3
SC	Excellent	Excellent	Excellent	Medium	Very slight	Practically impervious	Excellent; tamping roller, rubber tired equipment	>125 e <0.35	20–60	A-1
SP	Good	Poor	Poor	None to very slight	Almost none	Excellent	Good to excellent; crawler tractor, rubber tired equipment	>100 e <0.70	10–30	A-3
SF	Fair to good	Poor to good	Poor to good	Slight to high	Almost none to medium	Fair to practically impervious	Good to excellent; crawler tractor, rubber tired equipment, tamping roller	>105 e <0.60	8–30	A-2

TABLE 4—(*Continued*)

3	7	8		9	10	11	12	13[d]	14	15
Group symbols	Value as foundation when not subject to frost action	Value as Wearing Surface for Stage or Emergency Construction		Potential frost action	Compressibility and expansion	Drainage characteristics[c]	Field compaction characteristics and equipment	Solids at optimum compaction (lb per cu ft) and void ratio, e	California bearing ratio for compacted and soaked specimen	Comparable groups in PR classification
		With dust palliative	With bituminous surface treatment							
ML	Fair to poor	Poor		Medium to very high	Slight to medium	Fair to poor	Good to poor; close control essential; rubber tired roller	>100 e <0.70	6-25	A-4
CL	Fair to poor	Poor		Medium to high	Medium	Practically impervious	Fair to good; tamping roller	>100 e <0.70	4-15	A-4 A-6 A-7
OL	Poor	Very poor		Medium to high	Medium to high	Poor	Fair to poor; tamping roller	>90 e <0.90	3-8	A-4 A-7
MH	Poor to very poor	Very poor		Medium to very high	High	Fair to poor	Poor to very poor	>100 e <0.70	<7	A-5
CH	Poor to very poor	Very poor		Medium	High	Practically impervious	Fair to poor; tamping roller	>90 e <0.90	<6	A-6 A-7
OH	Very poor	Useless		Medium	High	Practically impervious	Poor to very poor	<100 e >0.70	<4	A-7 A-8
Pt	Extremely poor	Useless		Slight	Very high	Fair to poor	Compaction not practical			A-8

[a] Table 4 represents the third revision since it was first prepared by the author in 1942. [b] For binder = fraction passing No. 40 U. S. standard mesh. [c] These characteristics do not apply to undisturbed materials having fissures and root holes, such as most surface soils. [d] These weights apply only to soils having specific gravities from 2.65 to 2.75.

In the AC system, it is recognized that a clean gravelly soil, which would be classified A–3 in the PR system, is a better foundation material for pavements than a gravelly soil of the A–1 or A–2 groups, because it is not subject to frost action and is free draining. The AC system also recognizes that a gravelly soil is generally a better foundation material for heavy wheel loads than a sand.

The fifteen groups of the AC system represent a considerably greater number than are used in the other systems. However, the grouping is specifically so devised that soils can readily be classified by an experienced soils engineer on the basis of visual and manual examination.

10. Soil Groups and Group Symbols of AC System

Table 4, representing the third revision by the writer, summarizes the essential information on the AC system including the group symbols. Each symbol consists of two letters which may be considered as initials of the name of the most typical soil of that group. Their meaning will be understood from the following outline:

Coarse-grained soils are subdivided into:

(1) Gravel and gravelly soils; symbol G.
(2) Sands and sandy soils; symbol S.

The gravels and the sands are each divided into four groups:

(*a*) Well-graded, fairly clean material; symbol W, in combinations GW and SW.
(*b*) Well-graded material with excellent clay binder (corresponding to PR A–1 group); symbol C, in combinations GC and SC.
(*c*) Poorly graded, fairly clean material; symbol P, in combinations GP and SP.
(*d*) Coarse materials containing fines, not covered by preceding groups; symbol F, in combinations GF and SF.

Fine-grained soils are subdivided into three types:

(1) The inorganic silty and very fine sandy soils; symbol M, which stands for the Swedish terms *mo* and *mjala* (flour), used for fine-grained nonplastic, or slightly plastic, soils and, in the international grain-size scale, designating the fractions from 0.2 mm to 0.02 mm and from 0.02 mm to 0.002 mm, respectively.
(2) The inorganic clays; symbol C.
(3) The organic silts and clays; symbol O.

Each of these three types of fine-grained soils is grouped according to its liquid limit into:

(*a*) Fine-grained soils having liquid limits less than 50; that is, of low to medium compressibility; symbol L, in combinations ML, CL, and OL.
(*b*) Fine-grained soils having liquid limits greater than 50; that is, of high compressibility; symbol H, in combinations MH, CH, and OH.

Important Note.—In classifying fine-grained soils the term compressibility refers to the slope of the virgin pressure-void ratio curve and not to the actual

condition of the undisturbed soil which may be partly dried or otherwise preconsolidated. Experience has proved that the liquid limit is a good measure of relative compressibility.

Highly organic soils, usually fibrous, such as peat and swamp soils having very high compressibility, are not subdivided and are placed in one group; symbol Pt.

Use of Group Symbols.—The sequence of the group symbols need not be memorized, but the meaning of the symbols and the sequence of the major divisions, that is, G–S–L–H should be learned.

When a material does not clearly fall into one group, boundary classifications such as GW–SW or ML–CL should be used.

A study of the soil names in Col. 2, Table 4, will show that these represent no new soil terminology, but are those that experienced soils engineers use when describing a soil on the basis of visual and manual examination.

Emphatically the group symbols are not intended to be substituted for detailed descriptions. However, the use of these symbols is proper when comparing physical characteristics of different soil groups, as in Table 4, references (15)(16).

11. Coarse-Grained Soil Groups of AC System

GW–SW Groups.—These soils are well graded. They comprise gravelly and sandy soils having little or no fines. The presence of the fines must not noticeably change the strength characteristics of the coarse-grained fraction and must not interfere with its free-draining characteristics. In areas subject to frost action, the material should not contain more than about 3% of soil grains smaller than 0.02 mm in diameter. Where frost action is not a factor, contents of the order of 10% passing No. 200 mesh may not be objectionable. The GW and SW soils would be classified in the PR system as A–3 materials.

GC–SC Groups.—These soils also are well graded. They comprise gravel-sand mixtures and sands with an excellent clay binder, or other natural cementing agent, so proportioned that the mixture shows negligible swelling and shrinkage. Thus, dry strength of GC and SC materials is usually provided by a small amount of highly colloidal clay, and in some cases by the cementation of calcareous material (as in coral), or iron oxides (as in iron ore gravels). The letter C in these group symbols conveys the meaning of both "clay" and "cementation."

Materials in the GC and SC groups correspond to the A–1 group in the PR system. They may be used as road surfacing material because they are stable under wheel loads both in wet and dry condition. However, they are usually subject to frost action.

GP–SP Groups.—These soils are poorly graded. They comprise gravels and sands, containing little or no fines, as stated for the GW and SW groups. Materials in the GP and SP groups can be subdivided into:

(*a*) Uniform gravels, such as the clean, round-grained gravel deposited in gravel banks along the middle courses of rivers;

(*b*) Uniform sands, such as beach sand and dune sand; and

(*c*) Nonuniform mixtures of very coarse material with fine sand, with intermediate sizes lacking, as often results during excavation from mixing of gravel and sand layers.

These groups would be classified A–3 in the PR system.

GF–SF Groups.—Any coarse-grained material which contains detrimental fines (either silty or clayey) and which does not qualify for the GC–SC groups is classified in the GF–SF groups. Silty gravels, silty sands, poorly graded gravel-sand-clay mixtures, and sand-clay mixtures are typical examples. In the PR system they would be classified as A–2 materials.

Identification of Coarse-Grained Soils.—In field identification of coarse-grained materials, the gradation, grain size, shape, and the mineral composition are examined by spreading a dry sample on a flat surface.

The division between gravelly sands and sandy gravels has not been standardized. Many highway engineers classify gravelly sands containing only 20% to 30% of gravel sizes (retained on No. 10 mesh) as gravelly soils. An exact point of division is unimportant, except for specifications, and boundary cases may be classified as GW–SW, and so forth.

Considerable experience is required to differentiate, on the basis of a visual examination, between some well-graded soils and poorly graded soils. When in doubt, a mechanical analysis should be made and the grain-size curve plotted, from which the gradation characteristics can be judged.

Examination of fines is confined to the fraction passing No. 40 mesh (United States Standard), which fraction is usually called the "binder." In the absence of a sieve, this separation can be performed tolerably well by hand. The binder is mixed with water, and its plasticity characteristics and dry strength are examined. The dry strengths are listed in Col. 4, Table 4. For more accurate identification, quantitative determination of the liquid and plastic limit of the binder, and the grain-size curve of the entire material are required.

Identification of active cementing agents other than clay is usually not possible by visual and manual examination, since such agents require a curing period of days, and even weeks. It is on the basis of the actual use of such soils that they are classified into the GC–SC groups. In the absence of such experience, they may be classified tentatively into other groups, neglecting any possible development of strength due to cementation.

Depending on the use to which a coarse-grained soil is put, the durability of its grains may require careful examination. Pebbles and sand grains consisting of sound, igneous rock are easily identified. Weathered material is recognized from its discolorations and the relative ease with which the grains can be crushed. Pebbles consisting of weathered granitic rocks and quartzite are not necessarily objectionable when used for the foundations of pavements. In the process of compaction, some round-grained quartzite gravels, found in southeastern United States, will break up into much smaller, but angular and quite durable, quartz fragments which will form a well-graded stable base. Disintegrated granite in California is extensively used in highway construction. On the other hand, coarse-grained soils containing fragments of shaley rock

may be dangerous because alternate wetting and drying may result in partial or complete disintegration. This tendency can be identified by a slaking test. The particles are first thoroughly oven-dried, or sun-dried, then submerged in water for at least 24 hours; finally, their strength is examined and compared with their original strength. Some types of shales will completely disintegrate when subjected to such a slaking test.

The properties of coarse-grained soils in the undisturbed state (natural deposite or man-made fill) require examination if they are to be used in that condition. Depending on the problem, several of the observations and tests listed in Col. 5, Table 4, will be required.

12. Fine-Grained Soil Groups of AC System

In the transition between coarse-grained and fine-grained soils, usually those soils containing more than 50% of material smaller than 0.1 mm in size (passing No. 150 mesh) are classified as fine-grained soils (ML or CL); and soils containing less than 50% smaller than 0.1 mm in size are classified as coarse grained (SF or SC). A non-plastic, silty, fine sand with 40% to 60% passing a No. 150 mesh would be classified as an SF–ML soil.

Of basic importance in the classification of fine-grained soils is the plasticity as measured by the Atterberg liquid and plastic limit tests, or estimated by manual examination (17)(18)(19)(20)(21).

The Plasticity Chart.—The writer's method of using the liquid and plastic limit values for soil identification is illustrated in the plasticity chart, Fig. 4.

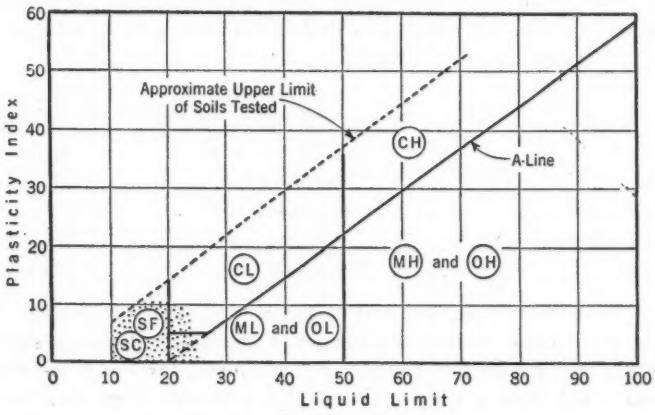


Fig. 4.—Plasticity Chart for AC System

The inclined A-line represents an important empirical boundary. It goes through the point on the base line having a liquid limit of 20, and through the point with a liquid limit of 50 and a plasticity index of 22.0. Therefore, its equation is: Plasticity index $= 0.73 \times$ (liquid limit $- 20$).

The A-line represents the empirical boundary between typical inorganic clays (CL–CH groups) which are generally above the A-line, and plastic soils containing organic colloids (OL–OH groups) which are below it. Also located

below the A-line are typical inorganic silts and silty clays (ML and MH groups), except for liquid limits less than 30, for which values inorganic silts may range slightly above the A-line.

For soils having liquid limits less than 25, there is considerable overlapping of several soil groups as shown by the dotted area in Fig. 4. For example, a soil with a liquid limit of 20 and a plasticity index of 8 could be classified according to its grain-size composition and physical characteristics either as a sandy clay (CL), or a very clayey sand (SF), or it may be such a well-graded mixture of sand and clay as to fall into the SC group. Frequently, such soils will be classified as boundary cases CL–SC or CL–SF; or SF–SC. A soil with a liquid limit of 20, and a plasticity index of 0 to 2, may be either a typical rock flour (ML), or a very silty sand with a trace of plasticity (SF), or a boundary case designated SF–ML. In spite of this overlapping, an experienced soils engineer will be able to differentiate between these soil groups purely on the basis of field identification.

Relation Between Position on Plasticity Chart and the Physical Properties.—Experience has shown that the compressibility, when compared at equal preconsolidation load, is approximately proportional to the liquid limit, and that materials having the same liquid limit possess approximately equal compressibility. In comparing the physical characteristics of soils having the same liquid limit, it will be found that, with increasing plasticity index, the cohesive characteristics (toughness and dry strength) increase, whereas permeability decreases.

The behavior of soils at equal liquid limit, with plasticity index increasing, as compared with their behavior at equal plasticity indexes, with liquid limit increasing, is summarized as follows:

Characteristic	Comparing soils at equal liquid limit with plasticity index increasing	Comparing soils at equal plasticity index with liquid limit increasing
Compressibility	About the same	Increases
Permeability	Decreases	Increases
Rate of volume change	Decreases	....
Toughness near plastic limit	Increases	Decreases
Dry strength	Increases	Decreases

After plotting the results of limit tests on a number of samples from the same fine-grained deposit (Fig. 5), generally the points lie on a straight line approximately parallel to the A-line. For example, for the thick stratum of the so-called "Soft Boston Blue Clay," which underlies large areas of Greater Boston, the liquid limit ranges from 30 to more than 70 along a line corresponding approximately to the lower boundary of the area marked "Glacial Clays." Of hundreds of limit determinations on this clay, the average liquid limit is 47 and the average plastic limit is 23.

Position of Typical Inorganic Clays.—The belt of glacial clays in Fig. 5 seems to cover most inorganic clays from Boston, Detroit, Chicago, and northeastern Canada. Much tougher clays are represented by the line showing test results on clays from one locality in Venezuela and by the belt which covers gumbo clays from several southern states.

Kaolin-Type Clays.—Interesting types of fine-grained soils are the kaolin-type clays, usually derived from feldspar in granitic rocks. On the plasticity chart, they plot below the A-line. Although they are generally referred to as clays, they possess some of the characteristics of inorganic silts. They have relatively low dry strength; and in the wet state show some reaction to the shaking test, which is characteristic of the ML–MH groups. However, they are extremely fine grained and smooth, and often contain as great a clay-size fraction as many tough clays located well above the A-line, as shown by soil No. 4 in Table 1. In Fig. 5 the kaolin-type clays in a belt slightly below the A-line are from widely separate deposits. They may be classified as boundary cases, ML–CL, and MH–CH. Test results on a different type of kaolin (from Mica, Wash.) are located along a line which lies in the MH area well below the A-line.

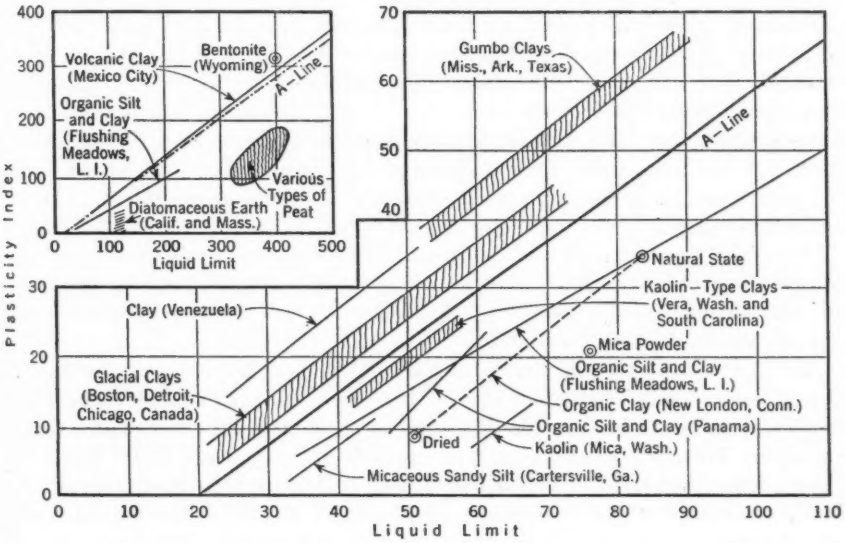

Plasticity Index
Liquid Limit
Volcanic Clay (Mexico City)
Bentonite (Wyoming)
A-Line
Organic Silt and Clay (Flushing Meadows, L. I.)
Various Types of Peat
Diatomaceous Earth (Calif. and Mass.)
Gumbo Clays (Miss., Ark., Texas)
Natural State
Kaolin-Type Clays (Vera, Wash. and South Carolina)
Clay (Venezuela)
Mica Powder
Organic Silt and Clay (Flushing Meadows, L. I.)
Glacial Clays (Boston, Detroit, Chicago, Canada)
Organic Clay (New London, Conn.)
Organic Silt and Clay (Panama)
Dried
Kaolin (Mica, Wash.)
Micaceous Sandy Silt (Cartersville, Ga.)

FIG. 5.—RELATION BETWEEN LIQUID LIMIT AND PLASTICITY INDEX FOR TYPICAL SOILS

Inorganic Silts, Rock Flour, Micaceous, and Diatomaceous Soils.—The common types of inorganic silts and rock flour have liquid limits less than 30 and are located in the ML area, Fig. 4. A typical rock flour, such as is found in glaciated areas, is usually nonplastic.

Eolian deposits of the loess-type have liquid limits ranging usually between 25 and 35 and vary from nonplastic (that is, the base line) to the A-line, and in exceptional cases also into the CL group slightly above the A-line.

Micaceous soils have a substantially greater liquid limit than a similar soil without the admixture of mica, as illustrated in Fig. 5 by the micaceous, sandy silt from Cartersville, Ga.

Fairly pure diatomaceous earth is generally nonplastic, with liquid limits above 100. With other fine-grained admixtures, diatomaceous soils were

found to range throughout the ML and MH groups. The locations on the plasticity chart of two types of fairly pure diatomaceous earth are shown by the small shaded area in the inset of Fig. 5.

Organic Soils.—Even a very small admixture of organic colloidal matter will cause an inorganic clay to increase its liquid limit, without an appreciable change in the plasticity index, thus causing a relative shift to a position below the A-line. In Fig. 5 are shown the positions of several organic silts and clays. Most of the organic silts, silt clays, and clays deposited by the rivers along their lower reaches near the Atlantic seaboard (for example, Hudson River silt, and Potomac River silt) range in liquid limit between 40 and more than 100 and are located on the plasticity chart from 5 units to 15 units below the A-line.

An important property of all fine-grained, organic soils is the radical drop in plasticity due to oven-drying. This is due to the fact that organic colloids are very sensitive and undergo irreversible changes upon drying. In Fig. 5 this drying effect is illustrated by the two points, connected by a dashed line, for an organic clay from New London, Conn. The liquid limit dropped from 84 to 51, and the plastic limit from about 50 to 42, with the corresponding drop in plasticity index from 34 to 9. (Note: Oven-drying also affects the limits of inorganic soils, but to a much more limited extent. Furthermore, the limits may be either raised or lowered by drying, depending on the soil. For this reason, it is not advisable to dry soil samples before subjecting them to the limit tests.)

The limit tests can also be performed on most peaty soils after they are thoroughly remolded. The liquid limit is generally several hundred per cent, and the position of such materials on the plasticity chart is well below the A-line, as illustrated in the inset of Fig. 5.

Volcanic Clays.—Liquid limits much greater than 100 are uncommon for typical, sedimentary, inorganic clays. However, clays which are formed by chemical breakdown of volcanic ash, such as bentonite and the volcanic clays underlying Mexico City, Mexico, have been found to possess liquid limits as high as 500. The inset in Fig. 5 shows the position for a commercially available bentonite, as well as the tremendous range over which the Mexico City clays extend.

13. Field Identification of Fine-Grained Soils

Basis of Field Identification.—It has been shown how the limit tests, supplemented if necessary by other tests, are used to classify fine-grained soils. However, the AC system is so devised that it is possible, with experience, to classify most soils correctly on the basis of field identification procedures alone. The easiest way of learning field identification is under the guidance of an experienced man. To learn without such assistance, one should compare systematically the numerical test results for typical soils in each group with the "feel" of the material while performing the field identification procedures.

The distinguishing characteristics between coarse-grained soils and fine-grained soils have already been described. The principal procedures for field identification of fine-grained soils are (*a*) the shaking test, (*b*) the examination of plasticity characteristics, and (*c*) the examination of dry strength. In

addition, observation of color and odor is of value, particularly for organic soils.

Shaking Test.—In this method a wet pat of soil is alternately shaken in the palm of the hand and then squeezed between the fingers. A nonplastic, fine-grained soil will become "livery" and show free water on the surface while being shaken. Squeezing will cause the water to disappear from the surface and the sample to stiffen and finally crumble under increasing finger pressure, like a brittle material. If the water content is just right, shaking the broken pieces will cause them to liquefy again and flow together. The speed with which the pat changes its consistency and the water on the surface appears or disappears, is distinguished as (1) rapid, (2) sluggish or slow, and (3) no reaction. Rapid reaction to the shaking test is typical for nonplastic, uniform, fine sand (SP, SF) and inorganic silts (ML), particularly of the rock flour type; also for diatomaceous earths (MH). With decreasing uniformity the reaction becomes more sluggish. Even a slight content of colloidal clay will impart some plasticity and slow up materially the reaction to the shaking test; this is true, for example, of slightly plastic, inorganic, and organic silts (ML, OL); very silty clays (CL–ML); and many kaolin-type clays (ML, ML–CL, MH, and MH–CH). Extremely slow or no reaction to the shaking test is typical for all clays above the A-line (CL, CH), as well as for highly plastic organic clays (OH).

Plasticity Characteristics.—Examining of the plasticity characteristics should start with a small sample having a soft, but not too sticky, consistency. This sample is alternately rolled into a thread and folded into a lump, and the increase in stiffness of the thread is observed as the plastic limit is approached. After the water content has dropped below the plastic limit, the pieces of the threads should be lumped together, and a slight kneading action on the lump continued until it crumbles.

The higher the position of a soil above the A-line on the plasticity chart (CL, CH), the stiffer are the threads near the plastic limit, and the tougher are the lumps below the plastic limit. Examples for such tough clays are the gumbo clays (CH) shown in Fig. 5. For soils slightly above the A-line (as the glacial clays (CL, CH)), the threads at the plastic limit are medium stiff and the lumps soon start crumbling when the water content drops. Soils below the A-line (ML, MH, OL, and OH) show, with very few exceptions, weak threads near the plastic limit. For organic soils and micaceous inorganic soils, located considerably below the A-line, the threads are very weak and spongy. All soils below the A-line, except OH soils near the A-line, lack cohesiveness when the water content drops below the plastic limit, and the lumps will readily crumble.

When working in a locality where the air humidity is fairly constant, the length of time to reach the plastic limit furnishes a rough indication of the magnitude of the plasticity index. For example, for a gumbo clay with a liquid limit of 70 and a plasticity index of 50 (CH), or a highly plastic organic clay with a liquid limit of 100 and a plasticity index of 50 (OH), a very much longer time will elapse before the plastic limit is reached, as compared with a glacial clay of the CL group. For a slightly plastic silt (ML) the plastic limit

will be reached very quickly. To prevent a misleading conclusion it is important that the examination of the plasticity characteristics be started for all soils at about the same consistency.

Dry Strength.—The resistance of a piece of dried soil to crushing by finger pressure is an indication of the character of the colloidal fraction of a soil.

Almost no dry strength characterizes nonplastic ML and MH soils. The dried lumps will fall apart under the slightest finger pressure. Typical rock flour and fairly pure diatomaceous earth are examples.

Low dry strength is representative of all soils of low plasticity, located below the A-line, and of very silty, inorganic clays in the CL group, slightly above the A-line.

Medium dry strength is typical of most clays of the CL group, and of those clays of the CH, MH (kaolin-type clays), and OH groups which are located near the A-line.

High dry strength is indicative of most CH clays, also of those CL clays located high above the A-line, as well as some organic clays of the OH group having very high liquid limits and located near the A-line.

Very high dry strength is developed only by those clays of the CH group located high above the A-line, as the gumbo clays shown in Fig. 5.

Color.—In field soil surveys color is often helpful in distinguishing between various soil strata, and, with sufficient preliminary experience with the local soils, also in identifying the individual soils. To the experienced eye certain dark or drab shades of gray and brown, including almost black colors, are indicative of fine-grained soils containing organic-colloidal matter (OL, OH). In contrast, clean and bright-looking colors, including medium and light gray, olive green, brown, red, yellow, and white, are generally associated with inorganic soils.

Odor.—Organic soils of the OL and OH groups usually have a distinctive odor which, with experience, can be used as an aid in identification. This odor is especially apparent in fresh samples. It is gradually reduced by exposure to air, but can again be brought out by heating a wet sample.

Observations and Tests Relating to Material in Place.—For foundations, the properties of soils in the undisturbed state require special examination. Most important are the denseness of coarse-grained soils and the consistency of fine-grained soils. Also, the consistency of the same soil remolded with its natural water content should be observed. For example, a clay may be described as "stiff and brittle in undisturbed state and very soft and sticky in remolded state." Where the drainage characteristics are of importance, one should observe stratification, root holes, and fissures. Depending on the problem, other tests, such as those listed in Col. 5, Table 4, may be required.

14. Possible Expansion of AC System

General.—Numerous suggestions have been received for the expansion of the airfield classification system. Some have recommended the use of subscripts or letters and numbers added in parentheses, to supply more detailed information. Such additional symbols are useful abbreviations but have the disadvantage that they can be read only by those who have taken the trouble

to acquire a working knowledge of such a "shorthand" system. In general, readers of reports and publications should not be burdened with more than a minimum number of symbols; full description in words is preferred.

For soil specialists it may be desirable to arrive at an agreement on the use of possible symbols for describing soils more in detail or on the addition of soil groups. For this purpose the most practical of the suggestions received are summarized below, including the writer's ideas, based on experience accumulated since the new system was formulated.

Coarse-Grained Soils.—Simple field identification methods permit differentiation within the GP and SP groups between uniform soils (such as beach sand) and those materials covering a wide range of sizes but of poor gradation (as a mixture of gravel and fine sand which is deficient in intermediate sizes). By adding the new groups GU and SU for uniform gravels and sands, respectively, and restricting the use of the GP and SP symbols to those materials which have a wide but poor gradation, it is easy to provide for this differentiation.

To distinguish between those soils in the GF and SF groups which possess a silt binder and those having a plastic (clay) binder, the subscripts m and c, respectively, can be used, for example, GF_c, SF_m. The gradation of the coarse fraction could also be described by subscripts, such as, S_uF_m, S_pF_c, and S_wF_m.

Fine-Grained Soils.—The entire range of plasticity is at present divided into the two main groups designated with the letters L and H. Experience has indicated that the L group covers too large a range and that with practice one can distinguish by field identification between soils having a liquid limit ranging well below 35 and those from 35 to 50. The soils in such an intermediate group could be identified with the letter I. Thus, CL would designate a very lean clay having a liquid limit below 35, and located in the plasticity chart above the A-line; CI would be a clay with a liquid limit between 35 and 50, above the A-line; and CH would be a fat clay with a liquid limit larger than 50 above the A-line.

In localities where kaolin-type clays require attention, it would be desirable to add such clays as separate groups for which the symbols KL, KI, and KH suggest themselves. This illustrates the flexibility of this classification, which permits adding new groups without affecting the main framework. It should be kept in mind, however, that any expansion should not be beyond the possibilities of field identification. From that viewpoint there is no objection to introducing the kaolin-type clays as a separate group, because anyone who has handled such clays and learned their "feel" will be able to recognize them readily.

A comparison of the original groups of fine-grained soils with this possible expansion is illustrated in Fig. 6. It will be noted that the order has also been changed to conform better to the order of usefulness of soils in foundation and earthwork engineering.

15. Descriptive Soil Classification

Describing a soil in detail on the basis of a careful visual and manual inspection, utilizing more or less generally accepted nomenclature, classifies this soil, although no rigid system is used. This approach will be designated as a

"descriptive soil classification." As such, it may be considered the oldest and most widely used classification. Not only field engineers and boring foremen, but also soil mechanics experts use this extensively, often in preference to a more rigid system.

The disadvantage of the descriptive classification lies in a lack of uniformity of soil terminology. The abuse of classifications by giving only the group number of a soil without any further information, has perhaps been the greatest obstacle to the development of a more uniform soil nomenclature. Therefore, the application of any system of classification as an excuse to reduce the description to a minimum or to eliminate it entirely must be strongly condemned. Instead, the main emphasis should always be on a complete verbal description of every pertinent characteristic of a soil, in its undisturbed state as well as after it is reworked.

Original Groups	Suggested Expansion
ML CL OL	CL KL ML OL — Liquid Limit = 30 CI KI MI OI — Liquid Limit = 50
MH CH OH	CH KH MH OH

FIG. 6.—SUGGESTED EXPANSION OF AC GROUPS FOR FINE-GRAINED SOILS

For example, a soil might be described as "olive green, homogeneous, medium-plastic, silty clay, medium stiff and brittle in the undisturbed state, very soft when remolded." This descriptive classification will convey to the reader, familiar with this terminology, a fairly clear picture of this soil. In contrast, if it is designated as an A–4 or A–7 soil, the information is entirely inadequate for any engineering use.

If a soil surveyor is forced to describe each soil in detail, he will develop the habit of examining it with greater thoroughness than if he is asked merely to give the group number or symbol. He can, for convenience, use his own abbreviations for frequently recurring characteristics thereby creating a more detailed classification than is afforded by any existing rigid classification. Such abbreviations may be in the form of numerous subdivisions of certain classes within an existing system.

For some purposes the descriptive classification should emphasize specific soil characteristics. Regarding compaction, it was often found useful to group soils into (*a*) noncohesive, coarse-grained soils (that is, soils having no dry strength); (*b*) typical cohesive soils (soils having considerable dry strength); and (*c*) those fine-grained soils having little or almost no dry strength.

Descriptive Adjectives.—The most common adjectives used in describing soils are:

Grain Size

(1) Very coarse
(2) Coarse
(3) Medium coarse
(4) Medium fine
(5) Fine
(6) Very fine

Grain Shape

(1) Angular
(2) Subangular
(3) Subrounded
(4) Rounded
(5) Well rounded

Gradation

(1) Very uniform
(2) Uniform
(3) Poorly graded
(4) Fairly well graded
(5) Well graded
(6) Very well graded

Compactness

(1) Very loose
(2) Loose
(3) Medium dense
(4) Dense, firm, compact
(5) Very dense, well compacted

Consistency

a. Consistency Grading—

(1) Very soft
(2) Soft
(3) Medium stiff
(4) Stiff
(5) Hard

b. Other Consistency Characteristics—

(1) Brittle
(2) Friable
(3) Elastic
(4) Spongy
(5) Sticky

Plasticity

a. Plasticity Grading—

(1) Nonplastic
(2) Trace of plasticity
(3) Medium plastic
(4) Highly plastic

b. Cohesiveness Near Plastic Limit—

(1) Very weak
(2) Weak
(3) Firm
(4) Medium tough
(5) Tough
(6) Very tough

Dry Strength

(1) Very slight
(2) Slight
(3) Medium
(4) High
(5) Very high

Examples of Descriptive Soil Classification.—Examples of descriptive soil classification will indicate how the adjectives can be combined into a detailed description:

(1) Uniform, fine, clean sand with rounded grains. Equivalent to A–3; SP (or SU in expanded AC system).

(2) Well-graded, gravelly silty sand; angular gravel particles, ½-in. maximum size; rounded and subangular sand grains; a very silty binder of low dry strength with no plasticity; well compacted and moist in undisturbed state. Equivalent to A–2–3; GW–GF.

(3) Organic silt; medium stiff and brittle in undisturbed state and very soft and sticky in remolded state; medium dry strength; weak and spongy at plastic limit; crumbles readily below plastic limit. Equivalent to A–7, OL–OH.

(4) Highly plastic (fat) clay; stiff at the plastic limit; forms a tough lump when drying is continued considerably below the plastic limit; medium stiff in undisturbed state, soft and sticky when remolded. Equivalent to A–6–7; CH.

Acknowledgment.—The writer wishes to express his appreciation to S. D. Wilson for stimulating assistance.

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PAPERS

CONSOLIDATION OF FINE-GRAINED SOILS BY DRAIN WELLS

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SYNOPSIS

Drain wells have been used to accelerate the consolidation of fine-grained compressible soils subjected to new loads. However, mathematical analyses of such wells have become available only recently, and are confined to wells having infinitely pervious well backfill and no peripheral smear. Complete formulas for consolidation by vertical and radial flow to wells, for cases with or without peripheral smear and drain well resistance, are presented in this paper; necessary derivations are given in Appendix II. Because soils are not homogeneous, and because of incomplete knowledge of stress-strain consolidation characteristics of soils, these solutions should be regarded as approximate when applied to practical problems.

INTRODUCTION

When a soil mass is subjected to a new load, compressive stresses cause a reduction of pore space and, if saturated, the excess water is expelled. For slightly pervious soil, time is required to accomplish this, and, initially, before any reduction of pore space has occurred, the new compressive stresses are carried by the water (called hydrostatic excess or excess pore-water pressure). As the excess water drains, the new compressive stresses are transferred to the soil grains increasing the intergranular pressures. This process is known, in soil mechanics terminology, as consolidation.

New loads applied to foundation soils create shearing stresses; to maintain stability the soil must not be overstressed. For slightly pervious compressible soils, the factors affecting the shearing strength are not fully understood at present; however, it is known that the shearing strength increases with a reduction of the soil pore water.

Most compressible soils are alluvial deposits and are more pervious in the direction of the bedding plane than in a perpendicular direction. This is very

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pronounced in varved deposits[2] laid down in lakes during the wasting away of the continental glaciers. When such soils are loaded, horizontal flow accelerates the consolidation of the soil mass as compared with strictly vertical flow. Because of the horizontal migration of water, however, this process imposes a dangerous condition on the stability of the foundation due to increased pore-water pressure[3] reducing the intergranular stresses at the toes of fills.

The limitation that the shearing strength shall not be exceeded may be met by widening the loading area (thus reducing the shearing stresses in the foundation), or by stage construction, thereby permitting a gain in shearing strength from the consolidation process. These methods are costly. Sometimes a more economical method may be to accelerate the consolidation process, and thus hasten a gain in shearing strength, by the use of vertical sand-filled holes known as drain wells. Drain wells are especially efficient in stratified soils because of the greater perviousness parallel to the bedding. Furthermore, drain wells permit control over the water migrating in a horizontal direction, thus reducing the excess pore-water pressures that might be built up in the toe area of an earth fill.

Drain wells have been used in this country mainly by the California State Highway Department,[4,5] and also at an eastern shipyard.[6] They have also been used by the Providence (R. I.) District, United States Engineer Department, as a repair measure to a dike failure. Further use is planned by the latter organization to permit construction of a moderately high dam on a foundation containing a large deposit of soft varved silt.

The theory of consolidation by drain wells was developed by the author during the design of the two Providence District projects previously noted. The theory for ideal wells having no resistance to flow up the well bore and having no smear at the well periphery has been presented elsewhere,[7,8,9,10,11] but consolidation theory which includes the effect of well smear and well resistance is presented here publicly for the first time.

Notation.—The letter symbols in this paper are defined where they first appear, in the text or by diagrams, and are assembled for convenience in Appendix I. In so far as possible the symbols conform with those given in "Soil Mechanics Nomenclature," *Manual of Engineering Practice No. 22.*

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Basic Considerations

The basic theory of consolidation has been developed by Karl Terzaghi, M. ASCE.[12] For the purpose of this paper the following basic assumptions are made:

(1) All vertical loads are initially carried by excess pore-water pressure u.

(2) All compressive strains within the soil mass occur in a vertical direction. Thus, the basic partial differential equation for consolidation by three-directional flow in stratified soils is:

$$\frac{k_h}{\gamma_w}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{k_v}{\gamma_w}\left(\frac{\partial^2 u}{\partial z^2}\right)=\frac{a_v}{1+e}\left(\frac{\partial u}{\partial t}\right)\ldots(1a)$$

in which k_h is the coefficient of permeability of the soil in a horizontal direction; k_v is the coefficient of permeability in a vertical direction, γ_w is the unit weight of water; x, y, and z are rectangular coordinates; e is the void ratio of the soil; t is time; and a_v, the coefficient of compressibility of the soil, equals $-\frac{de}{dp}$. For symmetrical flow to a central drain well Eq. 1a becomes:

$$\frac{k_h}{\gamma_w}\left(\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}\right)+\frac{k_v}{\gamma_w}\left(\frac{\partial^2 u}{\partial z^2}\right)=\frac{a_v}{1+e}\left(\frac{\partial u}{\partial t}\right)\ldots(1b)$$

in which r and z are cylindrical coordinates as defined by Fig. 1.

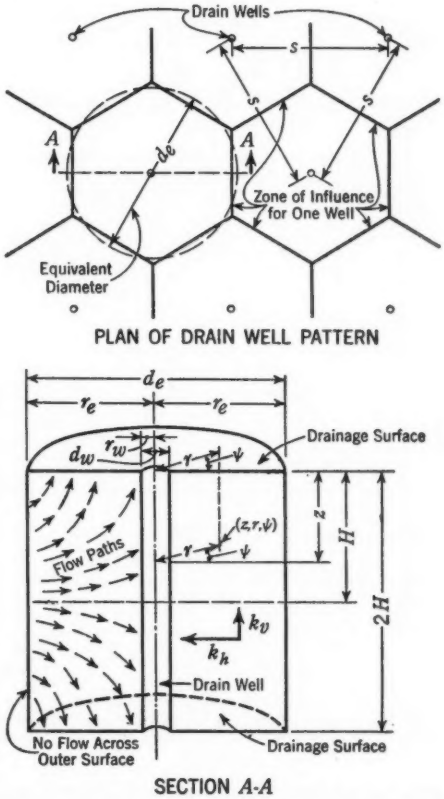

Fig. 1.—Plan of Drain Well Pattern and Fundamental Concepts of Flow Within Zone of Influence of Each Well

(3) The most economical pattern of drain wells is that shown in Fig. 1. An exact analysis should include the load distribution and the effect of each well on the rate of consolidation at any point in the foundation. However, it is believed that such a solution would be extremely cumbersome and rather academic when consideration is given to the heterogeneity of the foundation, and the present incomplete knowledge of the three-dimensional stress-strain consolidation properties of fine-grained soils.

(4) The zone of influence of each well is a circle (see Fig. 1).

(5) Load distribution is uniform over this area.

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Ideal Drain Wells

Wherever drain wells are installed construction operations generally cause some remolding or smear of the fine-grain soil adjacent to the well periphery, especially in varved soils. Also, the backfilling of the well, even with pervious material, results in some resistance to flow of the expelled water up the well bore. For ideal drain wells these effects are ignored, although solutions for these effects are discussed subsequently.

Case of Free Strain.—The initial solution for ideal drain wells is made on the assumptions that the load is uniform over the circular zone of influence for each well and that differential settlements occurring over such a zone, as consolidation progresses, have no effect on redistribution of stresses by arching of the fill. Furthermore, it is assumed that shear strains developed in the

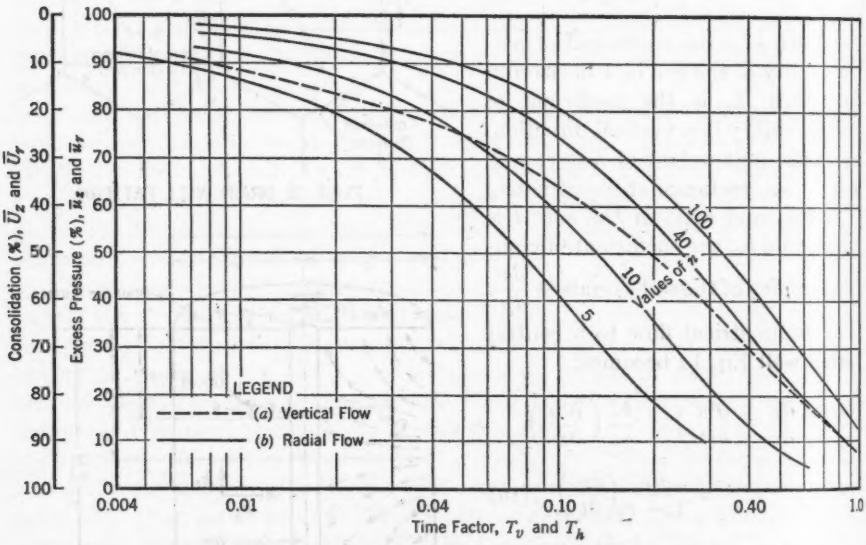

FIG. 2.—AVERAGE CONSOLIDATION RATES
(*a*) For Vertical Flow in a Clay Stratum of Thickness 2 *H* Drained on Both Upper and Lower Surfaces
(*b*) For Radial Flow to Axial Drain Wells in Clay Cylinders Having Various Values of *n*

foundation by differential settlements will have no effect on the consolidation process. The boundary conditions are:

(1) Initial excess pore-water pressure, u_0, is uniform throughout the soil mass when $t = 0$.

(2) Excess pore-water pressure at the drain well surface (r_w) is zero when $t > 0$.

(3) The external radius, r_e, is impervious or, because of symmetry, no flow occurs across this boundary; that is, $\frac{\partial u}{\partial r} = 0$ when $r = r_e$.

(4) Excess pore-water pressure at the upper horizontal boundary of the soil mass ($z = 0$) is zero when $t > 0$.

(5) The lower horizontal boundary of the soil mass ($z = H$) is impervious or, because of symmetry, no flow occurs across this boundary; that is, $\frac{\partial u}{\partial z} = 0$ at $z = H$.

Subject to the foregoing conditions a solution of Eq. 1*b* for excess pore-water pressure, $u_{r,z}$, at any point due to consolidation by both radial and vertical flow is:

$$u_{r,z} = \frac{u_r\, u_z}{u_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which u_r and u_z are, respectively, the excess pore-water pressures due to radial flow only and vertical flow only. Similarly, the average excess pore-water

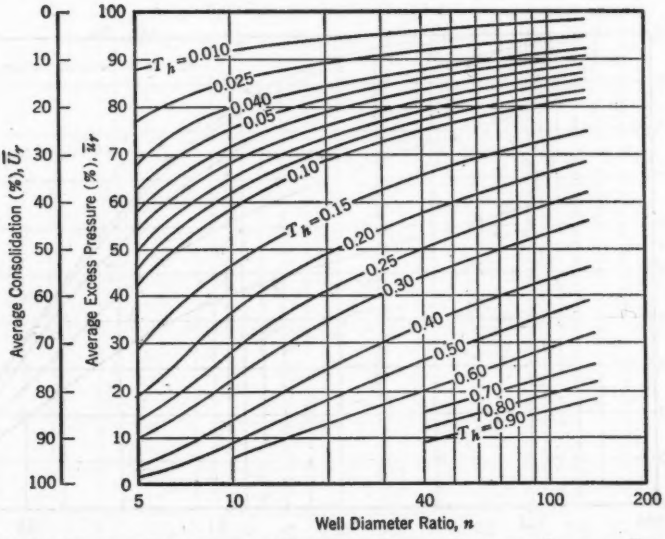

FIG. 3.—AVERAGE CONSOLIDATION RATES FOR RADIAL DRAINAGE ONLY, IN TERMS OF WELL DIAMETER RATIO, n

pressure throughout the entire soil mass is:

$$\bar{u}_{r,z} = \frac{\bar{u}_r\, \bar{u}_z}{u_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which $\bar{u}_r$ and $\bar{u}_z$ are the average excess pore-water pressures throughout the entire soil mass due to radial flow and vertical flow, respectively. Equations for u_r and $\bar{u}_r$ were developed by R. E. Glover;[13] those for u_z and $\bar{u}_z$ were developed by Karl Terzaghi.[11,12] A more general solution may be obtained when the initial excess pore-water pressure is a function of z, provided no expansion of the soil occurs. Equations for u_z and $\bar{u}_z$ for this case were developed by G. Gilboy,[14] Assoc. M. ASCE.

[13] *Technical Memorandum No. 158*, by R. E. Glover, Bureau of Reclamation, Denver, Colo., 1930.
[14] "Soil Mechanics," by G. Gilboy, Mass. Inst. Tech., Cambridge, Mass., 1930 (mimeographed notes).

Curves of average excess pore-water pressures $\bar{u}_r$ due to purely radial flow, versus time factor $T_h = \dfrac{k_h\,(1+e)\,t}{\gamma_0\,a_v\,d_e^2}$ for various values of n are shown in Fig. 2, in which n is the ratio of the zone of well influence to well diameter. Also given in Fig. 2 is a curve of average excess pore-water pressure $\bar{u}_z$ due to vertical flow only, versus time factor $T_v = \dfrac{k_v\,(1+e)\,t}{\gamma_0\,a_v\,H^2}$. Curves of $\bar{u}_r$ versus well diameter n for various time factors T_h are shown in Fig. 3. These curves permit the construction of average pore-water pressures versus time factor curves similar to those shown in Fig. 2 for any value of n between 5 and 100. Variations of excess pore-water pressures u_r at various points, due to radial flow, for a well diameter ratio $n = 40$, are shown in Fig. 4, indicating a much

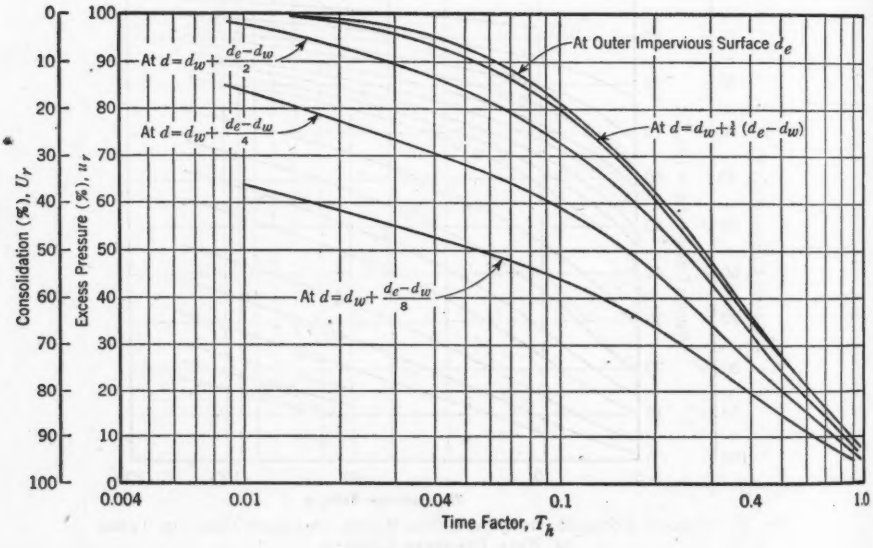

FIG. 4.—RADIAL CONSOLIDATION RATES AT VARIOUS RADIAL SURFACES FOR CLAY CYLINDER HAVING WELL DIAMETER RATIO, $n = 40$

faster rate of consolidation nearer to the well as compared to that nearer the outer zone of well influence. Shown in Fig. 5 is excess pore-water pressure, u_z, due to vertical flow only, versus relative depth, $Z\%$, of clay layer for various time factors T_v. The relative depth, $Z\%$, is defined as the ratio of the depth z to the point to the total thickness H of the clay deposit. Use of the curves shown in Fig. 2 and Fig. 5 is explained in the following example.

Example.—A bed of varved clay, 40 ft thick, resting on impervious rock, and having free drainage at its upper surface is quickly loaded with a very wide fill to 1 ton per sq ft. Tests on undisturbed soil samples furnish the following average data: $a_v = 13 \times 10^{-5}$ cm² per g; $e = 1.500$; $k_v = 100 \times 10^{-8}$ cm per sec; $k_h = 500 \times 10^{-8}$ cm per sec (assumed); and $\Delta e = 0.125$. Settlement

under the 1-ton loading will be:

$$\Delta H = \frac{H\,\Delta e}{1+e} = \frac{(40\text{ ft})\,(0.125)}{1+1.500} = 2.0\text{ ft}$$

Fig. 6(*a*) shows a time-average consolidation curve for vertical flow only. (The effect of horizontal flow has been neglected.) For time in months,

$$t_v = \frac{T_v\,\gamma_w\,a_v\,H^2}{k_v\,(1+e)} = \frac{T_v\,(1)\,(13\times10^{-5})\,(40\times30.5)^2}{(100\times10^{-8})\,(1+1.500)\,(60\times60\times24\times30)} = 30\,T_v$$

Values for T_v at various stages of consolidation are obtained from time factor consolidation curve for vertical flow only, as given in Fig. 2. After the average

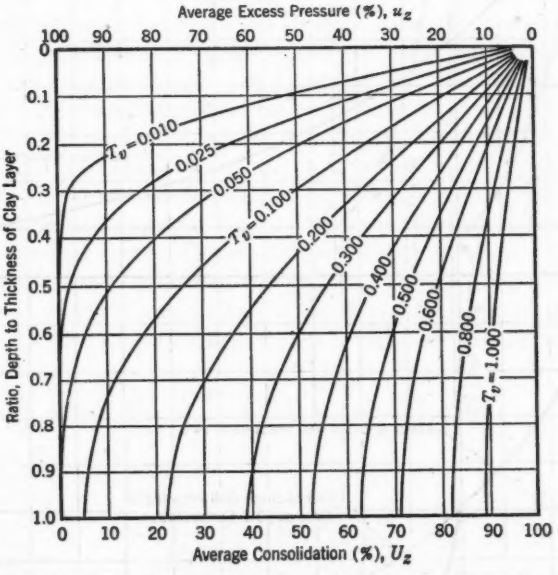

FIG. 5.—VARIATION IN CONSOLIDATION BY VERTICAL FLOW FOR VARIOUS TIME FACTORS AND DEPTH RATIOS

consolidation has reached 50%, sand-filled drain wells 1 ft in diameter are installed 40 ft apart in an equilateral pattern. For time in months,

$$t_h = \frac{T_h\,\gamma_w\,a_v\,d_e^2}{k_h\,(1+e)} = \frac{T_h\,(1)\,(13\times10^{-5})\,(40\times30.5)^2}{(500\times10^{-8})\,(1+1.500)\,(60\times60\times24\times30)} = 6\,T_h$$

Values for T_h are obtained from the time factor consolidation curve for $n = 40$ in Fig. 2. The time for 50% consolidation by vertical flow only is

$$t_v = 30\times0.195 = 5.85\text{ months}$$

which is, therefore, the origin for the radial-average consolidation curve. At any time, t, after installations of the wells, $\bar{u}_{r,z}$ is obtained by use of Eq. 3.

For example, at nine months

$$\bar{u}_{r,z} = \frac{\bar{u}_r \times \bar{u}_z}{u_0} = \frac{24\% \times 39\%}{100\%} = 9.4\%$$

The average consolidation is $100 - 9.4 = 90.6\%$.

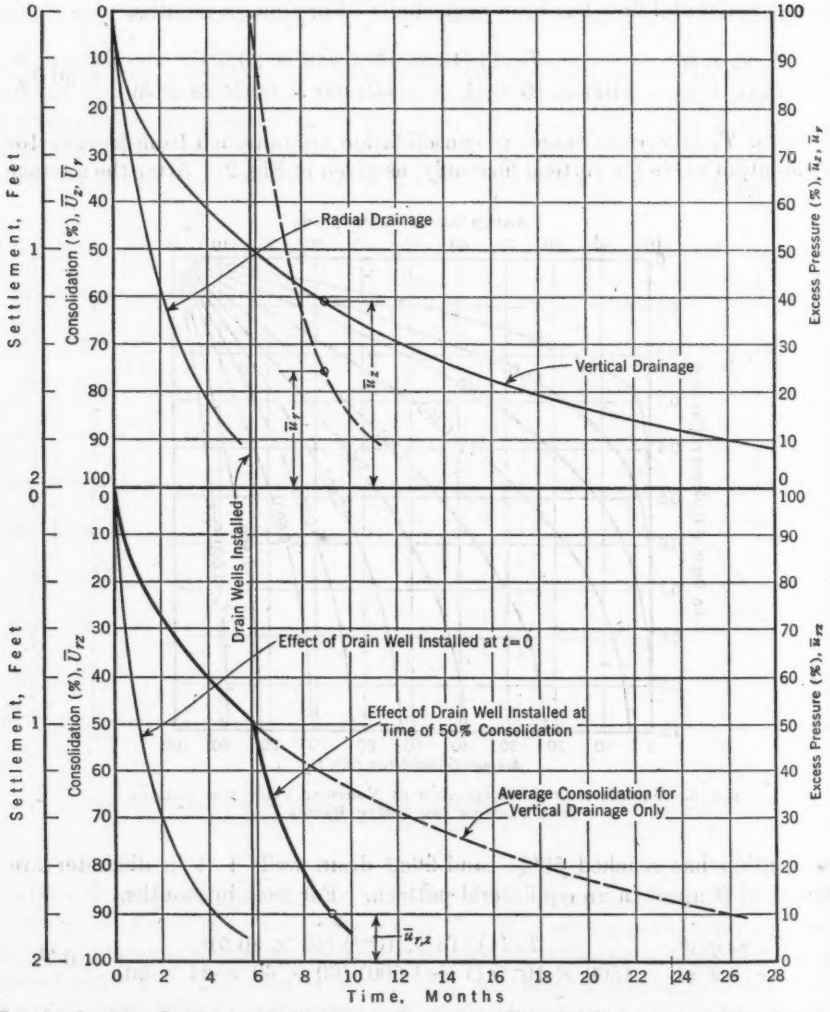

FIG. 6.—AVERAGE CONSOLIDATION RATES FOR RADIAL AND VERTICAL DRAINAGE IN CLAY CYLINDER IN WHICH $D = H$, $n = 40$, AND $k_h = 5\,k_v$, SHOWING EFFECT OF DRAIN WELLS INSTALLED WHEN LOAD IS APPLIED AND WHEN 50% CONSOLIDATION HAS TAKEN PLACE

For wells installed before the load has been added, the solid curve for $\bar{u}_r$ shown in Fig. 6 may be used. The effect of drain wells installed when 50% consolidation had been obtained and before application of load is shown in Fig. 6. Where $k_h \geqq 5\,k_v$, the effect of wells is obvious.

The determination of excess pressure at a point on the bottom of the clay layer and at the outer edge of the influence zone of a well nine months after load is applied is as follows: $Z\% = 40/40 = 1.0$, and $T_v = 9/30 = 0.3$. From Fig. 5, u_z for a relative depth of $Z\% = 1$ and $T_v = 0.3$ is 61%. The value of u_r at the outer surface of the zone of influence (for $n = 40$) is obtained from Fig. 4 at $T_h = \frac{9 - 5.85}{6} = 0.525$, and is 26%. Therefore the excess pressure at this point, which, incidentally, consolidates at the slowest rate of all, by Eq. 2, is

$$u_{r,z} = \frac{u_r \times u_z}{u_0} = \frac{26\% \times 61\%}{100\%} = 16\%$$

Case of Equal Vertical Strain.—For consolidation by radial flow with free strains permitted, the soil adjacent to the well consolidates and compresses faster than soil farther away from the drain. This difference in rate of consolidation develops differential settlement of the upper surface of the soil mass and shear strains within the mass. For free strains, it was assumed that these effects did not influence the redistribution of load to the soil nor the rate of consolidation. It is obvious, however, that these effects will redistribute the load to some extent, depending on the amount of arching developed in the material above the compressible soil. The extreme case would be where the arching process redistributes the load to the consolidating soil so that all vertical strains are equal and no differential settlement develops. This condition is rather severe. It can be obtained in the laboratory by use of a rigid loading platform, and probably is approached in the field if the ratio of H to d_s is large.

Using Eq. 1*b*, the rate at which water is expelled from a differential soil cube is:

$$\frac{\partial Q}{\partial t} = -\left[\frac{k_h}{\gamma_w}\left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) + \frac{k_v}{\gamma_w}\frac{\partial^2 u}{\partial z^2}\right] 2\,\pi\,r\,dr\,dz \quad \ldots\ldots\ldots (4)$$

The volume lost at any point is:

$$-\frac{\partial V}{\partial t} = \frac{-\,\partial e}{1 + e}\left(\frac{2\,\pi\,r\,dr\,dz}{\partial t}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (5a)$$

which is the same at any depth for any given time. At any depth, z, the relation between the average hydrostatic excess pressure, $\bar{u}$, and the average intergranular pressure, $\bar{p}$, is $\bar{u} + \bar{p} = p_f$, in which p_f is the final intergranular pressure upon completion of consolidation. Thus, $-\partial\bar{u} = \partial\bar{p}$, $\partial e = a_v\,\partial\bar{u}$, and Eq. 5*a* may be rewritten as:

$$\frac{-\,\partial V}{\partial t} = \frac{-\,a_v\,\partial\bar{u}}{(1 + e)\,\partial t}\,2\,\pi\,r\,dr\,dz \quad \ldots\ldots\ldots\ldots\ldots\ldots (5b)$$

Equating Eqs. 4 and 5*b* and simplifying,

$$c_h\left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) + c_v\frac{\partial^2 u}{\partial z^2} = \frac{\partial\bar{u}}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6a)$$

in which c_h is a coefficient of consolidation for horizontal flow and is equal to

$\frac{k_h\,(1+e)}{a_v\,\gamma_w}$. For radial flow only, this becomes:

$$c_h\left(\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}\right)=\frac{\partial \bar{u}}{\partial t}\qquad(6b)$$

A solution for this expression is—

$$u_r=\frac{4\,\bar{u}}{d^2_e\,F(n)}\left[r^2_e\log_e\left(\frac{r}{r_w}\right)-\frac{r^2-r^2_w}{2}\right]\qquad(7)$$

—in which

$$\bar{u}=u_0\,\epsilon^{\lambda}\qquad(8)$$

ϵ is the base of natural logarithms,

$$\lambda=\frac{-8\,T_h}{F(n)}\qquad(9)$$

and

$$F(n)=\frac{n^2}{n^2-1}\log_e(n)-\frac{3\,n^2-1}{4\,n^2}\qquad(10)$$

The initial distribution of the hydrostatic excess pressure is not uniform (see

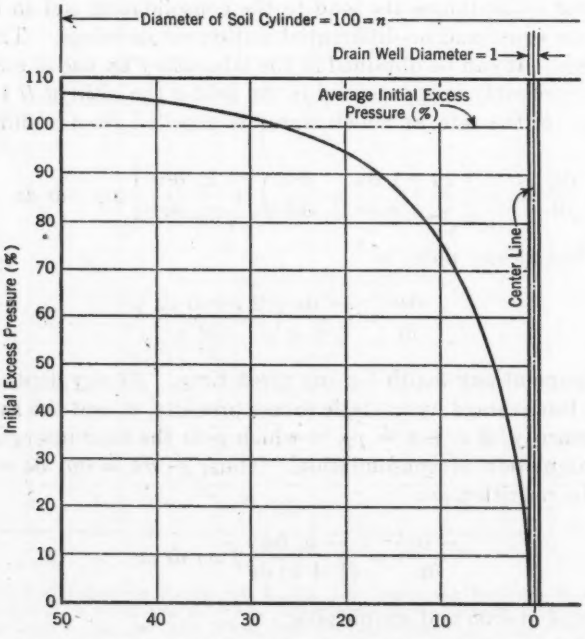

FIG. 7.—DISTRIBUTION OF INITIAL EXCESS PORE-WATER PRESSURE IN SOIL CYLINDER WITH $n = 100$

Fig. 7); but with passage of time the distribution approaches that for the free strain case (see Fig. 8).

Effect of Peripheral Smear

If drain wells are installed by driving cased holes and then backfilling as the casing is withdrawn, the driving and pulling of the casing will distort and

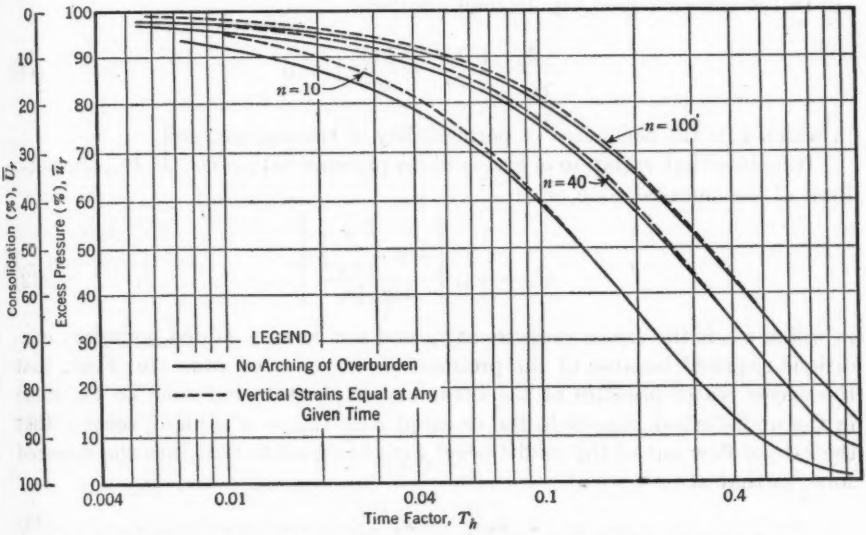

Fig. 8.—Comparison of Average Consolidation Rates in Clay Cylinders by Radial Drainage Only for Various Values of n Under Conditions of Equal Vertical Strains at Any Given Time and of No Arching of Overburden

remold the adjacent soil. In varved soils the finer and more impervious layers will be dragged down and smeared over the more pervious layers, resulting in a zone of reduced permeability in the soil adjacent to the well periphery. This distortion of the soil is very similar to that which occurs during sampling operations (see Fig. 9). It is not difficult to visualize a far greater smear from the use of a heavy casing or hollow mandrel with a bottom plug which displaces entirely the soil at the location of a drain well.

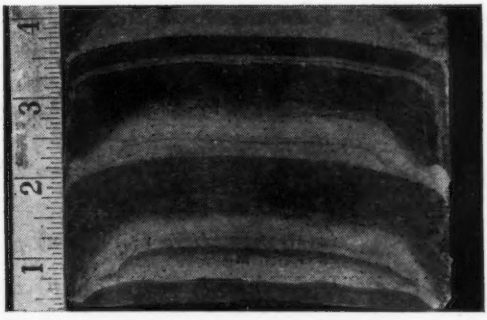

Fig. 9.—Distorted or Smeared Varved Clay Sample Taken with Early Massachusetts Institute of Technology Type 4¼-In.-Diameter Sampler

The remolded or smeared zone creates additional resistance which must be overcome by the excess water being expelled. This, in turn, will retard the rate of consolidation. The smeared zone will not be of constant thickness nor will it be homogeneous as far as soil properties are concerned. For this analysis, however, to obtain solutions for smear effect it is assumed that the

thickness of the smeared zone is constant and homogeneous. It is further assumed that because the remolded zone is adjacent to the drain well and will consolidate very fast its consolidation can be ignored and the zone treated as an incompressible material.

In the smeared zone Eq. 1*b* then becomes:

$$\frac{-k_s}{\gamma_w}\left(\frac{1}{r}\frac{\partial u'}{\partial r}+\frac{\partial^2 u'}{\partial r^2}\right)=0 \qquad (11)$$

in which k_s is the coefficient of permeability of the smeared soil.

It follows that variation of excess water pressure between r_w and r_s, the outer limit of the smeared zone, is:

$$u'_r = u'_{rs}\left[\frac{\log_e\left(\frac{r}{r_w}\right)}{\log_e(s)}\right] \qquad (12)$$

in which u'_{rs} is the excess pressure at r_s, and $s = r_s/r_w$. Added boundary conditions required because of the presence of the smeared zone are: First, that the excess water pressure at the boundary of the smeared zone be the same in the undisturbed zone as in the smeared zone ($u_{rs} = u'_{rs}$); and, second, that the rate of flow out of the undisturbed zone be equal to that into the smeared zone, so that at r_s:

$$k_h\frac{\partial u}{\partial r}=k_s\frac{\partial u'}{\partial r} \qquad (13)$$

Case of Free Strain.—In consolidation by radial flow to a central drain well with a smear zone at its periphery, Eq. 1*b* reduces to—

$$c_h\left(\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}\right)=\frac{\partial u}{\partial t} \qquad (14)$$

—from which a solution for u_r, for uniform initial excess pore-water pressure distribution, subject to the previously noted boundary conditions, is:

$$u_r = u_0\sum_{\alpha_1,\alpha_2,\alpha_3\cdots}^{\alpha=\infty}\frac{-\frac{2}{s\,\alpha}U_1(\alpha s)\,U_0\left(\frac{\alpha r}{r_w}\right)\epsilon^{\mu}}{\frac{4}{\pi^2\alpha^2 s^2}-U^2_0(\alpha s)-U^2_1(\alpha s)} \qquad (15)$$

in which

$$U_0(\alpha s) = J_0(\alpha s)\,Y_1(\alpha n) - J_1(\alpha n)\,Y_0(\alpha s) \qquad (16a)$$

$$U_1(\alpha s) = J_1(\alpha s)\,Y_1(\alpha n) - J_1(\alpha n)\,Y_1(\alpha s) \qquad (16b)$$

$$U_0\left(\frac{\alpha r}{r_w}\right) = J_0\left(\frac{\alpha r}{r_w}\right)Y_1(\alpha n) - J_1(\alpha n)\,Y_0\left(\frac{\alpha r}{r_w}\right) \qquad (16c)$$

The functions $J_0(\ )$ and $J_1(\ )$ are Bessel functions of the first kind of zero and first order, respectively, and $Y_0(\ )$ and $Y_1(\ )$ are Bessel functions of the second kind of zero and first order, respectively, and α, α_1, α_2, α_3, $\cdots$, are the

roots of

$$\frac{k_s\, U_0\,(\alpha\, s)}{k_h\, \alpha\, s \log_e (s)} + U_1\,(\alpha\, s) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

and

$$\mu = -\,4\, n^2\, \alpha^2\, T_h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

The average excess pore-water pressure between r_s and r_e is:

$$\bar{u}_r = u_0 \sum_{\alpha_1, \alpha_2, \alpha_3 \cdots}^{\alpha=\infty} \frac{U^2_1\,(\alpha\, s)\, \epsilon^{\mu}}{\alpha^2\,(n^2 - s^2)\left[\dfrac{4}{\pi^2\, \alpha^2\, s^2} - U^2_0\,(\alpha\, s) - U^2_1\,(\alpha\, s)\right]} \ldots\ldots (19)$$

If consolidation occurs by both radial and vertical flow, an approximate solution for the excess water pressure is:

$$u_{r,z} = \frac{u_r\, u_z}{u_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

in which u_r is found by Eq. 15; the average excess pressure for the entire body between r_e and r_s is:

$$\bar{u}_{r,z} = \frac{\bar{u}_r\, \bar{u}_z}{u_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

in which $\bar{u}_r$ is obtained from Eq. 19, and values of u_z and $\bar{u}_z$ are obtained from Figs. 2 and 5. Eqs. 20 and 21 are approximate because the effect of vertical flow in the smeared zone has been neglected. However, for reasonable values of k_h and k_v this effect is believed to be negligible.

Case of Equal Vertical Strain.—Solution of Eq. 6*b* for consolidation by radial flow to a central drain well with a smeared zone at its periphery is—

$$u_r = \bar{u}_r \frac{\left[\log_e\left(\dfrac{r}{r_s}\right) - \dfrac{r^2 - r^2_s}{2\, r^2_e} + \dfrac{k_h}{k_s}\left(\dfrac{n^2 - s^2}{n^2}\right)\log_e (s)\right]}{\nu} \ldots\ldots (22)$$

in which

$$\nu = F\,(n, s, k_h, k_s)$$

$$= \left[\frac{n^2}{n^2 - s^2}\log_e\left(\frac{n}{s}\right) - \frac{3}{4} + \frac{s^2}{4\, n^2} + \frac{k_h}{k_s}\left(\frac{n^2 - s^2}{n^2}\right)\log_e (s)\right] \ldots. (23)$$

and $\bar{u}_r$ can be expressed as—

$$\bar{u}_r = u_0\, \epsilon^{\xi} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

—in which

$$\xi = \frac{-\,8\, T_h}{\nu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

For combined radial and vertical flow, approximate solutions may be obtained using Eqs. 22 and 24 with u_z and $\bar{u}_z$ obtained as noted previously.

For purposes of studying the effect of smear in retarding the average rate of consolidation, Eq. 24 is preferred to Eq. 19 because of its simplicity. The effect of smear is indicated in Fig. 10. For a greater ratio of k_h/k_v the difference is still larger.

Effect of Well Resistance

The foregoing developments are for ideal drain wells of infinite permeability offering no resistance to flow up the well. Actually, head losses will occur due to resistance of well backfill material to flow. If the flow is large, or if the well area is small, then the back pressure due to well resistance will be high.

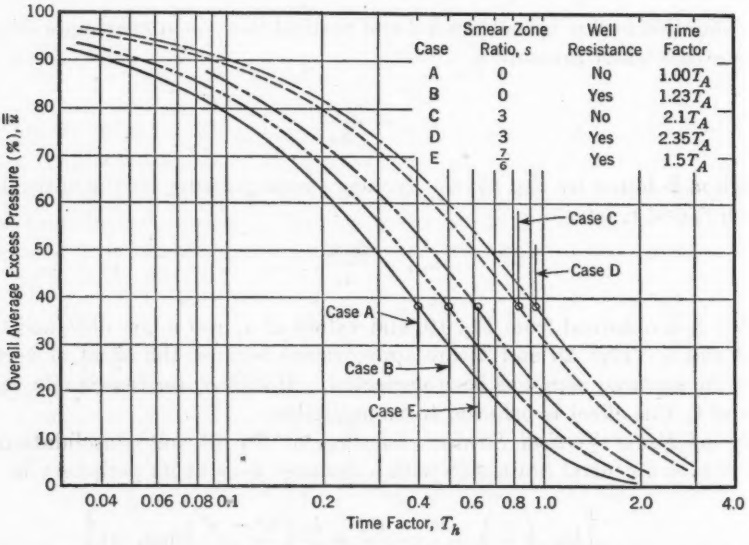

Fig. 10.—Effect of Smear and Well Resistance on Equal Strain Consolidation by Radial Flow to Drain Wells in Which $n = 60$, $k_w/k_h = 20{,}000$, $d_s/H = \frac{60}{80}$, and $k_h/k_s = 10$

On the other hand, if the flow is small, as for shallow deposits and tight deposits, or, if the well area is large, then the resistance of the well to flow will be small. A very pervious filling would be ideal from the standpoint of minimizing well resistance but filter action is needed with sufficiently small voids in the backfill to prevent "inwash" of the surrounding soil from clogging the well. This need is especially required for cohesionless silts and for varved soils in which the more pervious part of the varves is fine sand or coarse silt. For clays the filter requirements are less certain.

A solution for the effect of well resistance on the rate of consolidation for free strain has not been developed. A solution has been developed for equal vertical strain, with or without smear, where k_v of the consolidating soil is equal to zero; that is, $\frac{\partial u}{\partial z} \neq 0$, but because $k_v = 0$ no vertical flow exists.

A solution of Eq. 6*b* for $u_{r,z}$ at any point where smear and well resistance are involved is—

$$u_{r,s} = \bar{\bar{u}}_z \left\{ \frac{f(z)}{\nu} \left[\log_e \frac{r}{r_s} - \frac{r^2 - r^2_s}{2\,r^2_e} + \frac{k_h}{k_s} \left(\frac{n^2 - s^2}{n^2} \right) \log_e (s) \right] + 1 - f(z) \right\} \ldots (26)$$

—in which $\bar{\bar{u}}_z$, the average excess pressure between r_e and r_s at depth z, is:

$$\bar{\bar{u}}_z = u_0\, \epsilon^{\xi f(z)} \ldots (27)$$

$$f(z) = \frac{\epsilon^{\beta(z-2H)} + \epsilon^{-\beta z}}{1 + \epsilon^{-2\beta H}} \ldots (28)$$

and

$$\beta = \left[\frac{2\,k_h\,(n^2 - s^2)}{k_w\,r^2_e\,\nu} \right]^{\frac{1}{2}} \ldots (29)$$

in which k_w is the permeability of the well backfill.

The over-all average excess pressure over the entire soil mass between r_e and r_s and between 0 and $z = H$ is:

$$\bar{\bar{u}} = \int_0^H \bar{\bar{u}}_z\, dz \ldots (30)$$

This may be solved approximately by graphical integration or by use of Simpson's rule. In this solution horizontal flow in the drain well backfill has been ignored. Where there is no smear $F\,(n, s, k_h, k_s)$ reduces to $F(n)$. An example of the effect of well resistance is given in Fig. 10.

Acknowledgments

Mathematical solutions presented in this paper, except as otherwise noted, were developed by the writer for use in special design studies made by the Providence District, U. S. Engineer Department. The writer wishes to express his appreciation to Prof. W. Prager, Brown University, Providence, who checked the derivation of Eqs. 2 and 3, and to Kenneth S. Lane, Assoc. M. ASCE, formerly head, Providence District Soils Laboratory, and now head, Garrison (N. Dak.) District Soils Laboratory, United States Engineer Department, for his valuable aid and encouragement.

APPENDIX I. NOTATION

The following letter symbols, adopted for use in this paper and for the guidance of discussers, conform essentially to "Soil Mechanics Nomenclature," ASCE *Manual of Engineering Practice No. 22*, prepared by a Committee of the Soil Mechanics and Foundations Division, and adopted by the Society in April, 1941. In addition, the symbols of linear concepts are defined by Fig. 1.

A = constant of integration;

a_v = coefficient of compressibility = $-\frac{de}{dp}$;

B = constant of integration;

C = constant of integration;

c = coefficient of consolidation:

c_h = for horizontal flow = $\frac{k_h (1 + e)}{a_v \gamma_w}$;

c_v = for vertical flow = $\frac{k_v (1 + e)}{a_v \gamma_w}$;

d = diameter:

d_e = of zone of influence;

d_s = of remolded zone;

d_w = of drain well;

e = void ratio of soil;

H = depth of clay mass from a free-draining horizontal surface to an impervious one;

i = hydraulic gradient;

k = coefficient of permeability:

k_h = in horizontal direction;

k_s = of remolded zone;

k_v = in vertical direction;

k_w = of well backfill;

n = a ratio = $r_e/r_w = d_e/d_r$;

p = intergranular pressure:

p_f = final;

$\bar{p}$ = average;

$\bar{p}_z$ = average, at depth z;

Q = rate of flow of water;

r = radius, a coordinate in the cylindrical system:

r_e = of zone of influence;

r_s = of remolded zone;

r_w = of drain well;

S = spacing of drain wells;

s = a ratio = r_s/r_w;

T = time factor:

T_h = for radial flow = $\frac{c_h t}{d^2_e}$;

T_v = for vertical flow = $\frac{c_v t}{H^2}$;

t = time of consolidation:

t_h = due to horizontal flow;

t_v = due to vertical flow;

u = excess pore-water pressure:

u_0 = initial uniform;

u_r = due to radial flow;

u_{rs} = due to radial flow at outer limit of remolded zone;

$u_{rs,z}$ = due to radial flow at outer limit of remolded zone and depth z;
u_{rz} = due to radial flow only at depth z;
$u_{r,z}$ = due to combined radial and vertical flow;
$u_{w,z}$ = in drain well at depth z;
u_z = due to vertical flow;
u'_r = due to radial flow in the smeared zone at point r;
u'_{rs} = due to radial flow on remolded side of remolded zone boundary;

$\bar{u}$ = average excess pore-water pressure:
$\bar{u}_r$ = due to radial flow;
$\bar{u}_{r,z}$ = throughout entire mass, due to combined radial and vertical flow;
$\bar{u}_z$ = due to vertical flow;
$\bar{u}_z$ = in undisturbed zone at depth z, due to radial flow with well resistance;
$\bar{u}$ = in entire undisturbed soil mass, due to radial flow with well resistance;

V = volume of soil mass;
x = a coordinate in the rectangular system;
y = a coordinate in the rectangular system;
$Z\%$ = relative depth $= \frac{z}{H} = \frac{\text{depth } z}{\text{depth of clay mass}}$;
z = a coordinate in both the rectangular and the cylindrical systems;
α = root of Eq. 17;
β = a substitution factor defined by Eq. 29;
γ_w = unit weight of water;
ϵ = base of natural logarithms;
θ = a substitution factor defined by Eq. 39;
λ = a substitution factor defined by Eq. 9;
μ = a substitution factor defined by Eq. 18;
ν = a substitution factor defined by Eq. 23;
ξ = a substitution factor defined by Eq. 25;
$\bar{\sigma}_z$ = total average stress at depth z; and
ψ = a coordinate in the cylindrical system.

APPENDIX II. DEVELOPMENT OF SOLUTIONS

Case of Free Strain—No Well Resistance Nor Peripheral Smear

Derivations of Eqs. 2 and 3 are given elsewhere,[11,12,13,14] as is also proof of their validity.[10] Therefore, these derivations and proofs are not presented here.

Case of Free Strain with Peripheral Smear

Assume a cylindrical body of fine-grained, saturated compressible soil with an outer impervious boundary and a central drain having no resistance to flow

($k_w = \infty$). Also, assume that the horizontal permeability of the soil mass is k_h while that in a vertical direction, k_v, is zero. Further assume that the central drain is surrounded by a zone of remolded soil whose permeability is k_s, and which is bounded by the well surface, r_w, and by the outer radius of the remolded zone, r_s. The outer radius of the cylindrical body of undisturbed soil is r_e. It is further assumed that the cylindrical body is loaded so that an excess pore-water pressure $u_r = f(r)$ is induced at time zero and that during the process of consolidation vertical strains do not redistribute the load in the soil mass. The remolded soil is assumed to undergo no consolidation.

The boundary conditions are:

(1) When $t = 0$, $u_r = f(r)$ for $r_e > r > r_s$.

(2) No flow occurs accross the impervious surface at the outer boundary r_e of the soil cylinder $\left(\frac{\partial u_r}{\partial r} = 0\right)$.

(3) No vertical flow takes place because $k_v = 0$.

(4) The flow of water through the remolded zone surrounding the drain, for $r_s > r > r_w$, is given by Eq. 12.

(5) The seepage to the interface between the undisturbed and remolded soils at r_s is equal to the seepage away (Eq. 13).

The basic partial differential equation in the undisturbed zone ($r_e > r > r_s$) is expressed by Eq. 6*b*, a solution for which is

$$u_r = \left[A\, J_0\left(\frac{\alpha r}{r_w}\right) + B\, Y_0\left(\frac{\alpha r}{r_w}\right) \right] \epsilon^{\mu} \ldots\ldots\ldots\ldots (31)$$

—in which A and B are constants of integration. At r_e the second boundary condition is operative and

$$\frac{\partial u_r}{\partial r} = \frac{-\alpha}{r_w} [A\, J_1(\alpha n) + B\, Y_1(\alpha n)]\, \epsilon^{\mu} = 0 \ldots\ldots\ldots\ldots (32)$$

Therefore,

$$A\, J_1(\alpha n) + B\, Y_1(\alpha n) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

and

$$B = -A \left[\frac{J_1(\alpha n)}{Y_1(\alpha n)} \right] \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

Eq. 31 may be written, therefore, as

$$u_r = A \left[J_0\left(\frac{\alpha r}{r_w}\right) - \frac{J_1(\alpha n)}{Y_1(\alpha n)}\, Y_0\left(\frac{\alpha r}{r_w}\right) \right] \epsilon^{\mu} = A\, V_0\left(\frac{\alpha r}{r_w}\right) \epsilon^{\mu} \ldots (35)$$

The excess pressure in the remolded zone ($r_s > r > r_w$) may be expressed as

$$u'_r = \frac{\log_e\left(\frac{r}{r_w}\right)}{\log_e(s)}\, V_0(\alpha s)\, A\, \epsilon^{\mu} \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

Also, at r_s,

$$k_s \frac{\partial u'_{rs}}{\partial r} = \frac{k_s}{r_s \log_e (s)} V_0 (\alpha s) A \epsilon^{\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

and

$$k_h \frac{\partial u_{r,s}}{\partial r} = - k_h \frac{\alpha}{r_w} \left[J_1 (\alpha s) - \frac{J_1 (\alpha n)}{Y_1 (\alpha n)} Y_1 (\alpha s) \right] A \epsilon^{\mu}$$

$$= - k_h \frac{\alpha}{r_w} V_1 (\alpha s) A \epsilon^{\mu} \quad \ldots\ldots (38)$$

Equating Eqs. 37 and 38, simplifying, and substituting,

$$\frac{k_s}{\alpha k_h s \log_e s} = \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

$$\theta V_0 (\alpha s) + V_1 (\alpha s) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

If α is a root of this equation, the fifth boundary condition is satisfied and the roots are infinite. Therefore,

$$u_r = \sum_{\alpha_1, \alpha_2, \alpha_3, \ldots}^{\alpha=\infty} A V_0 \left(\frac{\alpha r}{r_w} \right) \epsilon^{\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

is also a solution of Eq. 6*b*, provided A can be so determined that, when $t = 0$,

$$f(r) = \sum_{\alpha_1, \alpha_2, \alpha_3, \ldots}^{\alpha=\infty} A V_0 \left(\frac{\alpha r}{r_w} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

to satisfy the first boundary condition.

To solve for A, multiply both sides of Eq. 42 by $r V_0 \left(\frac{\alpha r}{r_w} \right) dr$ and integrate from r_s to r_e. Noting that $V_1 (\alpha n) = 0$, and if $\alpha \neq \alpha'$,

$$\int_{r_s}^{r_e} A V_0 \left(\frac{\alpha r}{r_w} \right) V_0 \left(\frac{\alpha' r}{r_w} \right) r \, dr$$

$$= \frac{+ A r_w^2}{\alpha^2 - (\alpha')^2} \left[- \frac{\alpha' r}{r_w} V_0 \left(\frac{\alpha r}{r_w} \right) V_1 \left(\frac{\alpha' r}{r_w} \right) + \frac{\alpha r}{r_w} V_1 \left(\frac{\alpha r}{r_w} \right) V_0 \left(\frac{\alpha' r}{r_w} \right) \right]_{r_s}^{r_e}$$

$$= \frac{+ A r_w^2}{\alpha^2 - (\alpha')^2} [\alpha' s V_0 (\alpha s) V_1 (\alpha' s) - \alpha s V_1 (\alpha s) V_0 (\alpha' s)]$$

$$= \frac{- A r_w^2 s}{\alpha^2 - (\alpha')^2} [\alpha' \theta' V_0 (\alpha s) V_0 (\alpha' s) - \alpha \theta V_0 (\alpha s) V_0 (\alpha' s)] = 0 \quad \ldots\ldots (43)$$

Therefore, all the right-hand terms of Eq. 42 when multiplied and integrated as previously noted vanish except where $\alpha = \alpha'$, and

$$\int_{r_s}^{r_e} f(r) V_0 \left(\frac{\alpha r}{r_w} \right) r \, dr = \int_{r_s}^{r_e} A V_0^2 \left(\frac{\alpha r}{r_w} \right) r \, dr \quad \ldots\ldots\ldots\ldots (44)$$

Thus,

$$A = \frac{\int_{r_s}^{r_e} f(r)\, V_0\left(\frac{\alpha r}{r_w}\right) r\, dr}{\int_{r_s}^{r_e} V^2{}_0\left(\frac{\alpha r}{r_w}\right) r\, dr} \quad \ldots\ldots (45)$$

If $f(r) = u_0$,

$$u_0 \int_{r_s}^{r_e} V_0\left(\frac{\alpha r}{r_w}\right) r\, dr = u_0 \left[\frac{r\, r_w}{\alpha} V_1\left(\frac{\alpha r}{r_w}\right)\right]_{r_s}^{r_e} = \frac{-u_0\, r_s\, r_w\, V_1(\alpha s)}{\alpha} \quad \ldots (46)$$

and

$$\int_{r_s}^{r_e} V^2{}_0\left(\frac{\alpha r}{r_w}\right) r\, dr = \left\{\frac{r^2}{2}\left[V^2{}_0\left(\frac{\alpha r}{r_w}\right) + V^2{}_1\left(\frac{\alpha r}{r_w}\right)\right]\right\}_{r_w}^{r_e}$$

$$= \frac{r_e^2}{2} V^2{}_0(\alpha n) - \frac{r_s^2}{2} V_0(\alpha s) - \frac{r_s^2}{2} V^2{}_1(\alpha s) \quad \ldots\ldots (47)$$

Therefore,

$$A = \frac{-\frac{2\, u_0}{s\, \alpha} U_1(\alpha s)\, Y_1(\alpha n)}{\frac{4}{\pi^2\, \alpha^2\, s^2} - U^2{}_0(\alpha s) - U^2{}_1(\alpha s)} \quad \ldots\ldots (48)$$

in which

$$U_0(\alpha n) = \frac{-2}{\pi\, \alpha\, n} \quad \ldots\ldots (49a)$$

$$U_0\left(\frac{\alpha r}{r_w}\right) = V_0\left(\frac{\alpha r}{r_w}\right) Y_1(\alpha n) \quad \ldots\ldots (49b)$$

$$U_1\left(\frac{\alpha r}{r_w}\right) = V_1\left(\frac{\alpha r}{r_w}\right) Y_1(\alpha n) \quad \ldots\ldots (49c)$$

Substituting Eq. 48 in Eq. 41 for the initial uniform excess pressure, $u_0 = f(r)$:

$$u_r = u_0 \sum_{\alpha_1, \alpha_2, \alpha_3 \ldots}^{\alpha=\infty} \frac{-\frac{2}{s\, \alpha} U_1(\alpha s)\, U_0\left(\frac{\alpha r}{r_w}\right) \epsilon^{\mu}}{\frac{4}{\pi^2\, \alpha^2\, s^2} - U^2{}_0(\alpha s) - U^2{}_1(\alpha s)} \quad \ldots\ldots (50)$$

which is Eq. 15.

The average excess pressure in the cylinder of compressible soil between r_e and r_s is found by integrating Eq. 50 and dividing by the area. Thus,

$$\bar{u}_r = \int_{r_s}^{r_e} \frac{2\, \pi\, r\, u_r\, dr}{\pi\, (r_e^2 - r_s^2)} \quad \ldots\ldots (51)$$

Substitution of Eq. 50 in Eq. 51, followed by integration and simplification produces the following, which is Eq. 19 of the main paper:

$$\bar{u}_r = u_0 \sum_{\alpha_1, \alpha_2, \alpha_3 \cdots}^{\alpha=\infty} \frac{U^2_1(\alpha s)\, \epsilon^{\mu}}{\alpha^2 (n^2 - s^2) \left[\frac{4}{\pi^2 \alpha^2 s^2} - U^2_0(\alpha s) - U^2_1(\alpha s) \right]} \quad \ldots (52)$$

Case of Equal Vertical Strain with or Without Well Resistance or Peripheral Smear

Assume a cylinder of soil with conditions similar to those of the previous case except that the well backfill has a finite permeability, k_w, which furnishes resistance to flow of expelled water up the well bore. Further assume that all vertical strains at any depth z are equal. This means that there are no shearing strains and consequently no differential settlement of the upper surface. Because of this last condition the same amount of water is expelled from all differential soil cubes at a given depth for any given time. All flow in the well bore is assumed to be vertical.

The boundary conditions are then as follows:

(1) Initial uniform load placed on upper surface of cylinder.

(2) No flow across the impervious surface at outer boundary of soil cylinder $\left(\frac{\partial u}{\partial r} = 0 \right)$.

(3) No vertical flow in soil cylinder because $k_v = 0$.

(4) Flow of water through the remolded zone surrounding the drains is, for $r_s > r > r_w$,

$$\frac{1}{r} \frac{\partial u'}{\partial r} + \frac{\partial^2 u'}{\partial r^2} = 0 \quad \ldots (53)$$

for conditions of no consolidation in this zone.

(5) Seepage to the interface between the remolded soil and undisturbed soil is equal to the seepage away $\left(k_h \frac{\partial u}{\partial r} = k_s \frac{\partial u'}{\partial r} \right)$.

(6) At r_s the excess pressures on both sides of the interface are equal; that is, $u_{rs,z} = u'_{rs,z}$.

(7) Excess pressure in the drain well is zero at $z = 0$.

(8) At base of drain well vertical flow ceases; that is, $\partial u_{w,z}/\partial z = 0$ at $z = H$.

(9) The radial flow from the soil into the well at depth, z, is equal to the increase in flow up the well:

$$2 \pi r_w k_s \frac{\partial u'_{rw,z}}{\partial r} dz = - \pi r^2_w k_w \frac{\partial^2 u_{w,z}}{\partial z^2} dz \quad \ldots (54)$$

Because the vertical strains are all equal at any given depth, z, the amount of water expelled from any small soil mass is the same throughout the entire mass at any given time. Therefore the amount of water passing any radial

surface in the zone $r_e > r > r_s$ is proportional to the volume of the cylinder between that radial surface and the external boundary. Thus,

$$\frac{\partial Q}{\partial t} = k\, i\, A' = \frac{-k_h}{\gamma_w}\frac{\partial u}{\partial r}\, 2\,\pi\, r\, dz = C_1\,(r^2_e - r^2)\,\pi\,\frac{dz}{\partial t} \quad \ldots\ldots (55)$$

in which i is the hydraulic gradient; A' is the area of flow; and C, with appropriate subscripts, represents integration constants. Therefore,

$$\frac{\partial u}{\partial r} = \frac{C_1\,\gamma_w}{2\,k_h\,\partial t}\,\frac{r^2_e - r^2}{r} = C_2\left(\frac{r^2_e - r^2}{r}\right) \quad \ldots\ldots (56)$$

and the pressure distribution at any depth, z, at time t is:

$$u_{r,z} = C_2 \int (r^2_e - r^2)\,\frac{dr}{r} = C_2\left[\, r^2_e \log_e (r) - \frac{r^2}{2}\,\right] + C_3 \quad \ldots\ldots (57)$$

The excess pressure, $u'_{r,z}$, in the remolded zone by condition (4) is:

$$u'_{r,z} = (u'_{rs,z} - u_{w,z})\,\frac{\log_e\left(\dfrac{r}{r_w}\right)}{\log_e (s)} + u_{w,z} \quad \ldots\ldots (58)$$

in which $u_{rs,z}$ is the excess pressure at r_s and depth z and $u_{w,z}$ is the excess pressure in the drain well at depth z and condition (6) requires that $u_{rs,z} = u'_{rs,z}$ at the interface. Equating Eq. 57 with Eq. 58, at r_s,

$$C_2\left[\, r^2_e \log_e (r_s) - \frac{r^2_s}{2}\,\right] + C_3 = u'_{rs,z} \quad \ldots\ldots (59)$$

In addition, condition (5) requires that, at r_s, $k_h\,\dfrac{\partial u}{\partial r} = k_s\,\dfrac{\partial u'}{\partial r}$. Therefore,

$$k_h\, C_2\left(\frac{r^2_e - r^2_s}{r_s}\right) = k_s\,\frac{(u'_{rs,z} - u_{w,z})}{r_s \log_e (s)} \quad \ldots\ldots (60)$$

so that

$$u'_{rs,z} = u_{rs,z} = \frac{k_h}{k_s}\, C_2\,(n^2 - s^2)\, r^2_w \log_e (s) + u_{w,z}$$

$$= C_2\left[\, r^2_e \log_e (r_s) - \frac{r^2_s}{2}\,\right] + C_3 \quad \ldots\ldots (61)$$

This makes

$$C_3 = C_2\left[\,\frac{k_h}{k_s}\,(n^2 - s^2)\, r^2_w \log_e (s) - r^2_e \log_e (r_s) + \frac{r^2_s}{2}\,\right] + u_{w,z} \quad \ldots\ldots (62)$$

so that substitution in Eq. 61 gives:

$$u_{r,z} = C_2\left[\, r^2_e \log_e\left(\frac{r}{r_s}\right) - \left(\frac{r^2 - r^2_s}{2}\right) + \frac{k_h}{k_s}\,(n^2 - s^2)\, r^2_w \log_e (s)\,\right] + u_{w,z} \quad \ldots\ldots (63)$$

The average excess pressure between r_e and r_s is:

$$\bar{\bar{u}}_z = \frac{1}{\pi\,(r_e^2 - r_s^2)} \int_{r_s}^{r_e} u_{r,z}\, 2\,\pi\, r\, dr$$

$$= C_2\, r_e^2 \left[\frac{n^2}{n^2 - s^2} \log_e \left(\frac{n}{s} \right) - \frac{3}{4} + \frac{s^2}{4\,n^2} + \frac{k_h}{k_s} \left(\frac{n^2 - s^2}{n^2} \right) \log_e (s) \right] + u_{w,z}$$

$$= C_2 \frac{d_e^2}{4} F\,(n, s, k_h, k_s) + u_{w,z} \ldots\ldots (64)$$

in which $F\,(n, s, k_h, k_s) = \nu$, which is defined by Eq. 23.

In accordance with condition (9) and Eq. 60, at r_w:

$$\frac{2\,k_s}{r_w\,k_w} \frac{\partial u'_{r,z}}{\partial r} = -\frac{\partial^2 u_{w,z}}{\partial z^2} = \frac{2\,k_s}{r_w^2\,k_w} \frac{(u'_{rs,z} - u_{w,z})}{\log_e (s)} = 2\,C_2 \frac{k_h}{k_w} (n^2 - s^2) \ldots (65)$$

However, from Eq. 64:

$$C_2 = \frac{4\,(\bar{\bar{u}}_z - u_{w,z})}{d_e^2\,F(n, s, k_h, k_s)} \ldots\ldots (66)$$

and Eq. 65 may be written as,

$$\frac{-\,\partial^2 u_{w,z}}{\partial z^2} = \frac{8\,k_h\,(n^2 - s^2)\,(\bar{\bar{u}}_z - u_{w,z})}{k_w\,d_e^2\,F(n, s, k_h, k_s)} = \beta^2\,(\bar{\bar{u}}_z - u_{w,z}) \ldots\ldots (67)$$

When $t = 0$, $\bar{\bar{u}}_z = u_0$ and Eq. 67 becomes—

$$\frac{-\,\partial^2 u_{w,z}}{\partial z^2} = \beta^2\,(u_0 - u_{w,z}) \ldots\ldots (68)$$

—a solution for which is:

$$u_0 - u_{w,z} = C_4\,\epsilon^{\beta z} + C_5\,\epsilon^{-\beta z} \ldots\ldots (69)$$

At $z = 0$, $u_{w,z} = 0$ (condition (7)) so that $u_0 = C_4 + C_5$; and, at $z = H$, $\frac{\partial u_{w,z}}{\partial z} = 0$ (condition (8)), so that:

$$\frac{\partial u_{w,z}}{\partial z} = +\,\beta\,C_4\,\epsilon^{\beta H} - \beta\,C_5\,\epsilon^{-\beta H} = 0 \ldots\ldots (70)$$

Substituting these in Eq. 69 gives:

$$u_0 - u_{w,z} = u_0 \frac{\epsilon^{\beta(z-2H)} + \epsilon^{-\beta z}}{1 + \epsilon^{-2\beta H}} = u_0\,f(z) \ldots\ldots (71)$$

Thus, at $t = 0$,

$$u_{w,z} = u_0\,[1 - f(z)] \ldots\ldots (72)$$

Also, at $t = 0$,

$$\bar{\bar{u}}_z = u_0 = \frac{C_2 d_e^2}{4} F(n, s, k_h, k_z) \quad (73)$$

therefore,

$$C_2 = \frac{4 u_0 f(z)}{d_e^2 F(n, s, k_h, k_z)} \quad (74)$$

The relationship between average excess pressure $\bar{\bar{u}}_z$ and average soil pressure $\bar{p}_z$ is:

$$\bar{\bar{u}}_z + \bar{p}_z = \bar{\sigma}_z \quad (75)$$

in which $\bar{\sigma}_z$, the total average stress, is a constant for any fixed loading. Therefore,

$$\partial \bar{\bar{u}}_z = - \partial \bar{p}_z \quad (76)$$

and, because all strains are equal,

$$\partial e = - a_v \partial p_z = \partial e_z = a_v \partial \bar{\bar{u}}_z \quad (77)$$

The volume change then at any depth z between r and r_e is:

$$- \frac{\Delta \partial V}{\partial t} = - \frac{\partial e}{1 + e} (r_e^2 - r) \frac{\pi \, dz}{\partial t} = - \frac{a_v \partial \bar{\bar{u}}_z}{1 + e} (r_e^2 - r^2) \frac{\pi \, dz}{\partial t} \quad (78)$$

and the total rate of volume change at depth z for time t is:

$$\frac{- \partial V}{\partial t} = \frac{- a_v \partial \bar{\bar{u}}_z}{1 + e} (r_e^2 - r_z^2) \pi \frac{dz}{dt} \quad (79)$$

From Eqs. 55 and 56, the total water flowing into the well at depth z is:

$$\frac{\partial Q}{\partial t} = \frac{k_h}{\gamma_w} C_2 \frac{r_e^2 - r_z^2}{r_z} 2 \pi r_z \, dz \quad (80)$$

which is equal to Eq. 79; and, thus, when $t = 0$,

$$C_2 = \frac{- a_v \partial \bar{\bar{u}}_z \gamma_w}{2 (1 + e) k_h \partial t} = \frac{4 u_0 f(z)}{d_e^2 F(n, s, k_h, k_z)} \quad (81)$$

and

$$\frac{\partial \bar{u}_z}{\partial t} = - \frac{u_0 \, 8 f(z)}{F(n, s, k_h, k_z)} \left[\frac{(1 + e) k_h}{a_v \gamma_w d_e^2} \right] = - \frac{u_0 \, 8 f(z) T_h}{F(n, s, k_h, k_z) t} = \frac{\xi u_0 f(z)}{t} \quad (82)$$

If it is assumed that

$$\bar{\bar{u}}_z = u_0 \epsilon^{\xi f(z)} \quad (83)$$

then,

$$\frac{\partial \bar{\bar{u}}_z}{\partial t} = \frac{\xi\, u_0\, f(z)}{t}\, \epsilon^{\xi f(z)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (84)$$

and, at $t = 0$,

$$\frac{\partial \bar{\bar{u}}_z}{\partial t} = -\frac{u_0\, 8\, f(z)}{F(n, s, k_h, k_s)} \left[\frac{(1+e)\, k_h}{a_v\, \gamma_w\, d_e^2} \right] \ldots\ldots\ldots\ldots\ldots (85)$$

which will be recognized as Eq. 82. From Eq. 64 excess pressure in the drain well is:

$$u_{w,z} = \bar{\bar{u}} - \frac{C_2\, d_e^2\, \nu}{4} = \bar{\bar{u}}_z + \frac{a_v\, \gamma_w}{2\,(1+e)\, k_h} \frac{d_e^2\, \nu}{4} \frac{\partial \bar{\bar{u}}_z}{\partial t}$$

$$= \bar{\bar{u}}_z - \frac{a_v\, \gamma_w}{(1+e)\, k_h} \frac{d_e^2\, \nu}{8\, \nu} \frac{u_0\, 8\, f(z)\, (1+e)\, k_h}{a_v\, \gamma_w\, d_e^2}\, \epsilon^{\xi f(z)}$$

$$= \bar{\bar{u}}_z - u_0\, \epsilon^{\xi f(z)} = \bar{\bar{u}}_z\, [1 - f(z)] \ldots (86)$$

From Eqs. 37 and 38 the excess pressure at any point within the undisturbed soil is:

$$u_{r,z} = \bar{\bar{u}}_z \left\{ \frac{f(z)}{F(n, s, k_h, k_s)} \right.$$

$$\left. \times \left[\log_e \left(\frac{r}{r_s} \right) - \frac{r^2 - r_s^2}{2\, r_e^2} + \frac{k_h}{k_s} \left(\frac{n^2 - s^2}{n^2} \right) \log_e (s) \right] + 1 - f(z) \right\} \ldots (87)$$

The average excess pressure, $\bar{\bar{\bar{u}}}$, throughout the entire undisturbed soil mass is:

$$\bar{\bar{\bar{u}}} = \frac{1}{H} \int^H \bar{\bar{u}}_z\, dz \ldots\ldots\ldots\ldots\ldots\ldots\ldots (88)$$

which may be solved by such approximate method as Simpson's rule. For no smear but with well resistance, $F(n, s, k_h, k_s)$ reduces to $F(n)$ as given in Eq. 10. For smear but no well resistance, $f(z)$ becomes equal to 1.

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PAPERS

RIVER INFILTRATION AS A SOURCE OF GROUND WATER SUPPLY

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SYNOPSIS

The characteristics of a ground water supply based on the infiltration of water from a near-by stream are discussed. The effect of changes in river stage upon yield, as well as the effect of changes in river temperature on well field yields, drawdowns, and water temperatures, are revealed through the data presented. The absence of pathogenic bacteria from the ground water despite the contamination of the source of water is shown.

The paper describes a method of testing a well field of horizontal collectors to determine the firm yield of the field under the worst probable conditions of river stage and temperature.

The construction of a horizontal collector is described and the production record and operating characteristics of a field of seven such units are given in detail. The paper is based on more than four years of careful observation of the water supply of the Indiana Ordnance Works near Charlestown, Ind., and was written while the writer worked for the United States Geological Survey.

INTRODUCTION

An adequate supply of water is becoming more important as new industries develop which depend on large quantities of water for cooling purposes and in manufacturing processes. The producers of foods, synthetics of all kinds, and precision machinery are among the newer industrial water consumers. The problem of obtaining large quantities of satisfactory water at a minimum cost is a difficult one, even under the best of circumstances. If it is available, ground water is particularly desirable because of its cheapness and its relatively uniform quality.

This paper summarizes the data and conclusions obtained from a study of the ground water supply of the Indiana Ordnance Works. Since the well field

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by November 1, 1947.

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is composed of seven horizontal collectors, a brief section is devoted to their construction. However, the greater part of the paper contains an analysis of data obtained during more than four years of continuous well field operation and includes the details of a test conducted to determine the capacity of the water production system.

Location and Description of Well Field

The well field of the Indiana Ordnance Works is situated on the Indiana shore of the Ohio River about 12 miles north of Louisville, Ky., between the river and the upland on a narrow flood-plain terrace composed primarily of gravel, although covered by a layer of silt and top soil. The geology is not significantly different from any one of a hundred terraces along the Ohio River.

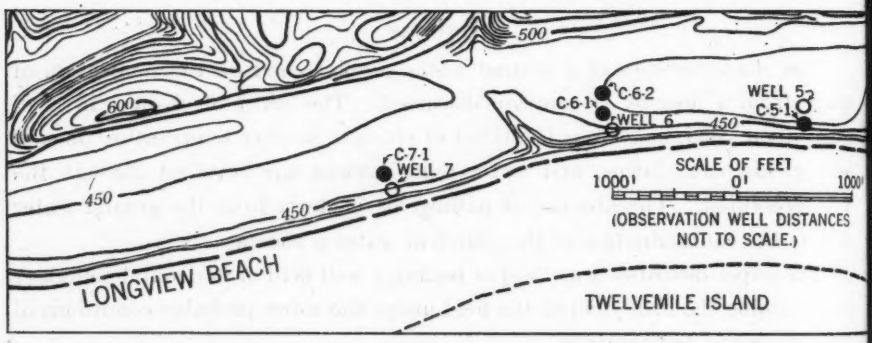


Fig. 1.—Well Field of t

The log of observation well C–6–1, which is 30 ft from collector 6 (see Fig. 1) is typical:

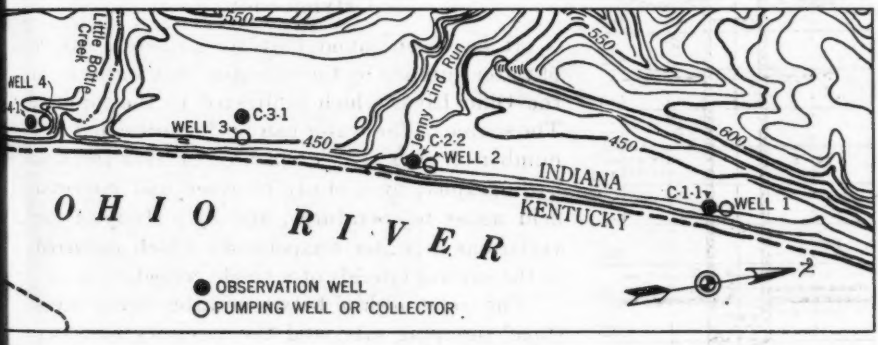
Depth (ft)	Description
0 to 18.5	Top soil
18.5 to 50	Gravel boulders and sand, mixed
50 to 65	Pea gravel
65 to 72	Sand and fine gravel
72 to 93	Fine gravel (screen set at 93 ft)

The bedrock floor, which is about 100 ft below the land surface at the river's edge, continues at that elevation inland from 200 ft to 500 ft. On the basis of test drilling inland there seems to be a sharp rise in the rock floor which flattens off about 30 ft or 40 ft below the land surface and meets limestone bluffs as much as 1,500 ft inland. On this basis alone the gravel area would seem to be a relatively large one. Actually, the width of the aquifer from which the collectors draw water ranges from about 200 ft to 500 ft, extending inland from the river's edge. Fig. 1 is a map of the Indiana Ordnance Works field which shows the collector locations. Production wells are plotted to scale but observation wells are spotted diagrammatically. The gravels extend into the

river an undetermined distance, but gravel companies dredge gravel in the reach opposite the well field from about 500 ft to 1,500 ft offshore.

Construction of a Collector

A collector is a caisson in the lower end of which are one or more tiers of portholes. The portholes are spaced uniformly in each tier. After the caisson (usually 13 ft in inside diameter) reaches the requisite depth, a thick concrete plug is poured on the bottom to keep out water and extraneous material. After the plug has hardened, lengths of slotted well casing 8 in. in diameter are jacked hydraulically into the surrounding water bearing formation. The boring head of each slotted pipe lateral has holes in it enabling the returning water to carry fines with it to the central caisson. This flow of water washes



Indiana Ordnance Works

out the silt and fine sand of the formation, leaving a natural gravel pack around the slotted pipe.

After a predetermined number of pipes, with a total length of from 1,000 ft to 1,200 ft, have been pushed out, all the portholes are closed by gate valves and the pipes are flushed one by one until the water coming from each one is free from sand. At the Indiana Ordnance Works the amount of sand removed from each collector during the pushing of pipe was between 50 cu yd and 100 cu yd. Fig. 2 shows a plan and section of collector 4 of the Indiana Ordnance Works which is a typical unit. The position of the laterals, relative to the Ohio River and to the bottom of the caisson, is indicated. Figs. 3 and 4 are photographs taken during the construction of similar collectors.

Collector Field Operation

The collector field consists of seven horizontal collectors spaced at intervals of from 1,300 ft to 2,000 ft within 100 ft of the river's edge at normal pool stage. In each unit two pumps were installed each capable of delivering 3,500 gal per min. Thus, the field had a pumping capacity of approximately 49,000 gal per min or about 70 mgd. The field was first pumped in April, 1941, and was in continuous operation until October, 1945.

From April, 1941, to April, 1945, inclusive, a period of forty-nine months of continuous operation, a total of 58.3 billion gal was pumped at an average rate of 39.7 mgd. The highest monthly pumping rate occurred in September, 1944, when an average of 51.2 mgd was pumped. The lowest monthly pumping rate during routine plant operation occurred in November, 1941, when 28.1 mgd were pumped. The days of greatest water production were September 15, 1942, when 55,975,000 gal were pumped and June 26, 1944, when 55,742,000 gal were pumped. No data are available for peak hourly water supply, but undoubtedly the field has been pumped at a rate of more than 60 mgd for short periods. The average daily water production for each month of plant operation is shown in Fig. 5.

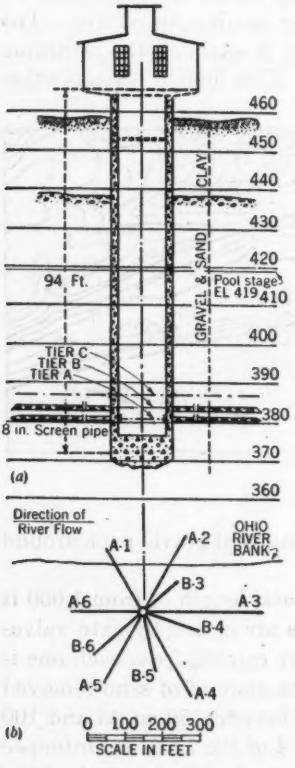


FIG. 2.—COLLECTOR NO. 4, FIG. 1

HYDROLOGY

It has been found that the principal source of water pumped by the collectors was water from the Ohio River which infiltrated to the aquifer. The source of the water can be demonstrated in a number of ways: By comparison of river and well hydrographs; by a study of river and collector field water temperatures; and by a study of the variations in water temperature which occurred in the several laterals of a single collector.

The relationship between water level, river stage, pumping rate, and the viscosity of water is indicated by a study of river and observation well hydrographs and pumpage.

For example, the Ohio River was approximately at pool stage, El. 420, throughout the months of January, 1944, and June, 1944. In January the average daily pumpage from the field was about 34 mgd; in June it was nearly 49 mgd. At collector 6 the average daily pumpage in January was slightly more than 5 million gal; in June it was slightly more than 9 million gal; yet, despite a constant pool stage and an increase in pumpage in the entire field, the average water level elevation in observation well C–6–2 (200 ft inland from collector 6) rose from 399.5 ft in January to 402 ft in June.

The only major variable factor not heretofore mentioned is temperature. In January the average collector water temperature was 59° F and in June it was 55° F. However, the river water temperature averaged 35° F in January and 78° in June. For rough computations, the average of the river and collector temperatures in January was 47° F; in June, 67° F. The kinematic viscosity of water changes from 1.38 to 1.01 centipoises in that range—a decline of more than 35%. This decline in water viscosity evidently accounts for an apparently contradictory condition: Same river level; more water pumped;

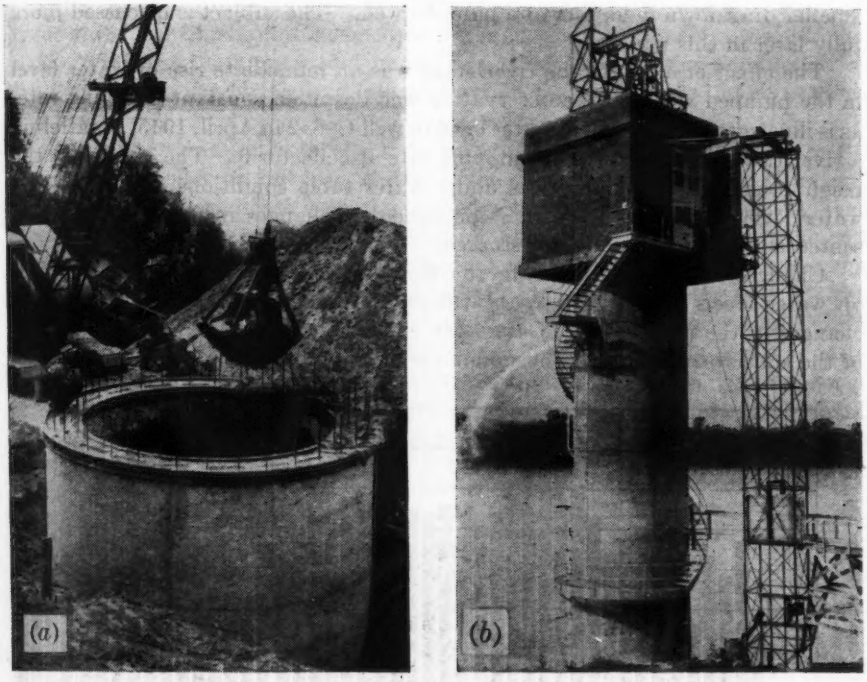


FIG. 3.—CONSTRUCTION VIEWS
(*a*) Sinking the Caisson
(*b*) Completed Collector Undergoing 3,000 Gal per Min Test

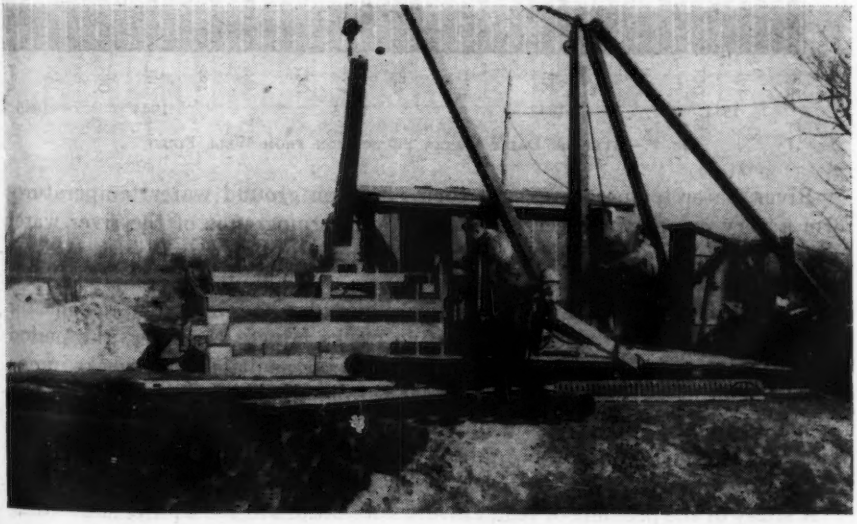


FIG. 4.—LOWERING SLOTTED PIPE INTO CAISSON FOR INSTALLATION AS RADIAL COLLECTORS

smaller drawdown in aquifer and pumped well. The subject is discussed more fully later in this paper.

The effect of a rise in the river stage was an immediate rise of water level in the pumped well and the observation well despite a constant pumping rate, as is illustrated by the rise in water level in well C–6–2 in April, 1945, paralleling a river rise despite a constant pumping rate at collector 6. The aquifer in the neighborhood of collector 6 was under water table conditions as long as the water level was below El. 420. Consequently, changes in water level represented a gain, or loss, in water storage.

Changes in the pumping rate at collector 6 also had an immediate effect on water levels in well C–6–2, regardless of the trend of water levels due to changes in the river stage. Detectable changes in water level in the vicinity of the collector were caused by minor variations in pumping rates.

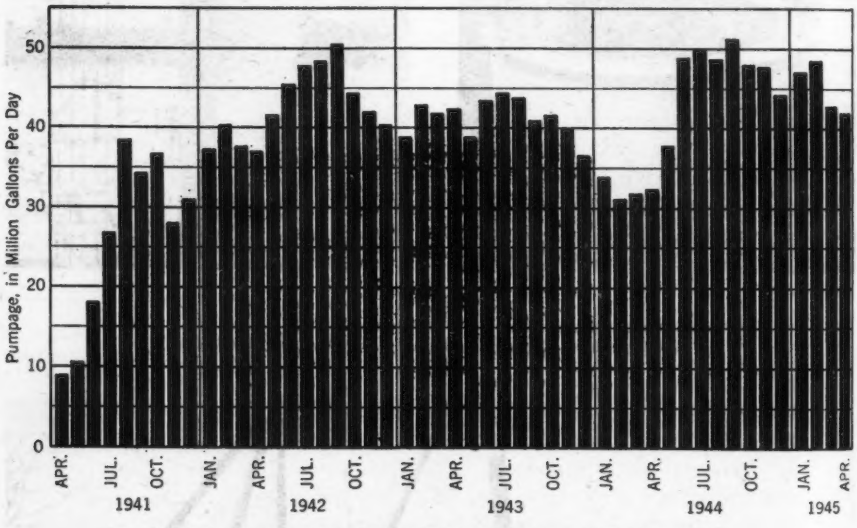


FIG. 5.—AVERAGE DAILY WATER PRODUCTION FROM WELL FIELD

River water temperatures and their effect on ground water temperatures form a very complex subject. Fig. 6 shows thermographs of the river water and of the water discharged by several of the collectors into one of the two main reservoirs of the plant. A study of the available data indicated that a seasonal trend in river temperatures was followed by a similar trend in collector water temperatures from eight to twelve weeks later. However, the period of lag has differed from collector to collector, being as little as six weeks in one and as much as fourteen weeks in another. The lag at any particular unit appeared to depend on the operation schedule and on the interference of near-by units and the geology of the aquifer, as well as on the abruptness of river temperature changes.

A study of the problem of temperature fluctuations in the aquifer shows that there were several factors involved: (1) The mixing of ground water with in-

filtrated river water when the field was first pumped; (2) the admixture of river waters of different temperatures while en route to the collector; (3) the heat storage of the aquifer and underlying rocks; and (4) the conduction of heat upward to the surface and laterally within the aquifer due to temperature gradients.

The water coming from the river entered the aquifer not only directly opposite the collector, but also upstream and downstream from it. Thus, different hydraulic gradients were established between the collector and the river depending upon the point of infiltration of the water. The nearer to the collector that infiltration occurred, the steeper the gradient and the greater the water velocity, all other things being equal. As a result, water that left the river a long time ago reached the collector by one path at the same time as water that left the river more recently reached the well along another path. Thus, an integrating process went on in the aquifer to produce water warmer than the river temperature lows and cooler than the river temperature highs. During certain times of the year the temperature differential between river and collector water was from as much as 18° F to 20° F.

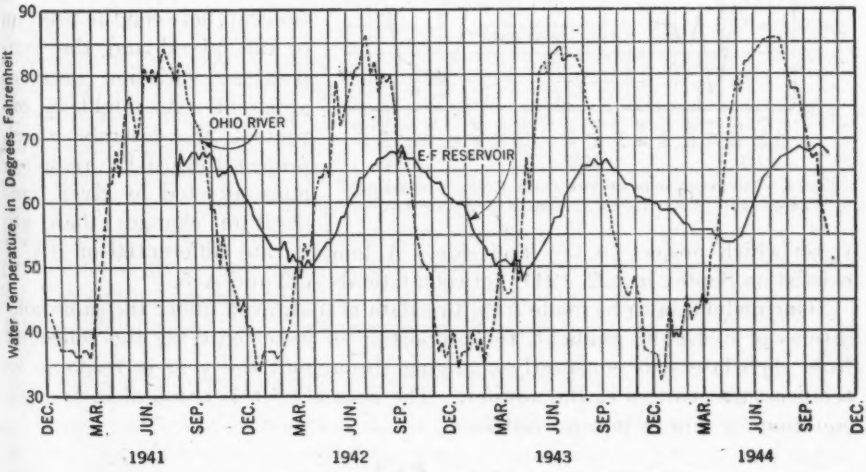


FIG. 6.—THERMOGRAPHS OF OHIO RIVER WATER AND PUMPED WATER ENTERING A RESERVOIR

The foregoing description is, however, an oversimplification of the actual physical process involved. Superimposed on this idealized mechanism is the effect of temperature on the viscosity of the water and consequently on the manner of flow in the aquifer. The viscosity difference increases the percentage of warmer water entering the well as compared to the idealized case. In the summertime, when the river is warm, water coming from the reach opposite a well will have the advantage of decreased viscosity plus the steeper hydraulic gradient as compared to older (and colder) water already infiltrated from other reaches of the river. On the other hand, in the winter, when river temperatures are low, the viscosity factor will tend to increase the relative velocity of water infiltrated from the river at an earlier date and from greater distances as

compared to the newer and colder water approaching the well along the direct path. This condition and others, such as heat flow within water and heat storage by the aquifer and its underlying bedrock, tend to make the exact mathematical analysis extremely involved for even the simplest example of flow to a well.

Temperature Characteristics of Infiltrated Water

The units at the Indiana Ordnance Works are composed of eight or ten laterals (actually horizontal wells) converging to a central caisson. In 1943, thermocouples were installed on eight of the laterals in collector 4 and arrangements were made to obtain periodic temperature readings. Fig. 7 gives typical thermographs.

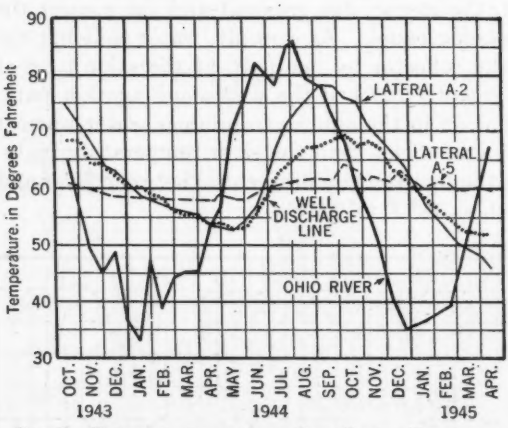


Fig. 7.—Thermographs of the Ohio River, Discharge Line from Well 4, and Laterals A–2 and A–5

When these thermographs and the layout of the laterals in Fig. 2 are studied, the relationship between the position of the lateral and the temperature variation becomes apparent. As might be expected, the laterals which project toward the river are more affected by river temperature changes than are those which project to the land side. A temperature differential of $16\frac{1}{2}$° F existed on September 22, 1944, between laterals A–2 and A–5.

One deduction to be made from this data is that, even under the same conditions of hydraulic gradient, the proportion of water entering the collectors from each lateral is constantly changing throughout the year in response to temperature changes in the aquifer. The reason for this is found in Darcy's equation for flow in porous media:

$$Q = \frac{P\,i\,A}{\mu} \qquad (1)$$

in which Q is the quantity of flow; P is the permeability of the materials composing the aquifer; i is the hydraulic gradient; A is the cross-sectional area perpendicular to the direction of flow; and μ is the viscosity of the fluid. Ignoring other factors, in the range of temperatures shown in Fig. 7 from 46° F to 78° F, the coefficient of viscosity ranges from 1.39 to 0.88 centipoises. In other words, the coefficient of viscosity of 78° F is only 63% of the coefficient at 46° F.

Sanitary Quality of Collector Water

The bacteriological quality of the raw water from each collector has been periodically tested. Although the 37° C plate count has almost always shown

the presence of bacteria in the water obtained from the collectors, further tests with lactose broth incubated for 24 hours and 48 hours have rarely shown the presence of gas forming bacteria. For example, between August 1, 1944, and April 1, 1945, a typical eight-month period which included a long period of high water during which the river overflowed parts of the collector field area, a total of two hundred and twenty samples was tested. Of these, a total of eight samples (10-milliliter portions) formed gas in lactose broth and only two confirmed.

The collectors that produced the gas forming water samples were retested as soon as possible. No gas forming organisms were found. Had the aquifer been generally contaminated by the source of recharge, the retesting undoubtedly would have revealed gas forming organisms. After a study of the entire four-year record it was concluded that the contaminated samples were probably due to minor errors in sampling technique.

Considering the polluted character of the Ohio River next to the field, it was further concluded that harmful bacteria were removed or killed in the sands and gravels between the river and the collectors. How this is accomplished is worthy of investigation, because tests by the United States Public Health Service have shown that some gas forming bacteria survive the complete treatment of filter plants[2] making it necessary to resort to break-point chlorination. Among some of the possibilities that have been suggested are: Adsorption of pathogenic bacteria by the particles of silt, clay, sand, and gravel (note the use of diatomaceous earth in the latest techniques of filtration); death of pathogenic bacteria due to change in environment or lack of food; filtration by the fines at the bottom of the river; and elimination of pathogenic bacteria by action of such organisms as plankton or ground bacteria. However, until a systematic study is undertaken which leads to a workable hypothesis the facts are that coliform organisms rarely, if ever, were found in the water pumped by the collector field of the Indiana Ordnance Works.

Will Silt Clog the Intake Area?

In comparing the infiltration of river water to an aquifer and the passage of water through a slow or rapid sand filter, there appears to be only one major point of similarity—the fact that water must pass through a porous medium. In a filter all the available water must pass through, and deposit all its fines on, the surface of the filter. In the infiltration area of an aquifer the principal movement of water is along the river bottom in planes roughly perpendicular to the direction of infiltration. The fines, therefore, do not remain on the surface of the area as they would if raw river water were led to sand beds without prior sedimentation. The fines are continually removed by the river and replaced temporarily by material coming downstream—that is, the bed is constantly scoured and built up. Moreover, occasional periods of increased discharge put the fines back into suspension, leaving only the heavier sands and gravels on the river bottom. Evidently river infiltration to an adjacent aquifer

[2] "Studies of the Efficiency of Water Purification Processes," by H. W. Streeter, *Public Health Bulletin No. 172*, U. S. Public Health Service, Washington, D. C., May, 1927.

should not be expected immediately upstream from a dam as the deposition of silt and fines will occur and preclude infiltration.

The reason sand beds in a filter plant must be scraped or backwashed is that a gelatinous layer of fine silt and organic matter forms on the filter bed surface. The efficiency of the filtration process probably depends on the existence of this layer. The filter plant operator wages a constant struggle to maintain a certain amount of the material on the sand bed, yet to prevent it from sealing the filter completely. Periodically this layer is removed by scraping or backwashing. As far as can be determined, no such continuous layer has ever formed on the surface of the infiltration area supplying the Indiana Ordnance Works well field.

There is no evidence that permanent silting occurred during the period of collector field operation. All the data indicate that the production capacity of the field remained unchanged during the entire period of operation.

Calibration of the Collector Field and Prediction of Firm Yield

The field had been operating successfully for about a year when, for various reasons, it was decided to make pumping tests to determine the magnitude of interference between the units and the probable minimum yield of the system when operating at full capacity.

Since the plant was in full production at the time, the testing had to be performed without interfering with plant operations and no water could be pumped to waste.

It was planned to operate the field, or most of the field, the same way save for one difference in pumping rate in one collector. Most of the time this was not feasible so all variations in plant water demand had to be met at the end of the field farthest from the point of principal interest. Thus, while tests were being run on unit 1, variations in plant load if any were to be taken by units 7 and 6, respectively.

Presumably, after allowing for river stage fluctuations and water temperatures, all differences in drawdowns in collectors 1, 2, 3, and possibly 4, would be due to the change in the pumping rate of collector 1.

A systematic scheme of pumping the field was prepared and the testing was started with the fullest cooperation of the War Department and the E. I. du Pont de Nemours Company, the plant operators. The tests lasted for nearly four months, during which time a number of pumping schedules were nullified because of operating demands. Eventually, however, seven reasonably satisfactory pumping schedules were carried through and temperature and water level data were obtained as needed.

From previous experience it had been found that about four days of pumping at a constant rate were sufficient to establish a steady flow of ground water in the aquifer, so each schedule was followed for four days. Water levels within each caisson could be determined by automatic air-line water level recorders. However, since there was an observation well within 30 ft of each caisson, water levels were also determined there by steel tape measurements. It was found that the drawdowns in these observation wells were substantially the same as those in the respective production wells. As it was easier to secure accurate

measurements in the observation wells, and as these wells fluctuated less due to slight variations in pumpage, the water level elevations of the observation wells were used in the computations.

During the test period, collector water temperatures were changing from 60° F at the start of the period to 70° F at the end. In order that the data might be comparable, the drawdowns were corrected to a standard temperature of 68.5° F at the collector where the temperature was measured. The corrections were made on the basis that, in steady state flow through porous media, the drawdown is inversely proportional to the viscosity—that is, a well pumping 10 mgd at a temperature of 60° F with a drawdown of 10 ft would have had a drawdown of only 8.9 ft at the standard temperature, 68.5° F. To illustrate the method of operation and the type of data obtained, an operation schedule is summarized in Table 1. All the usable data obtained during the pumping tests are given in Table 2.

TABLE 1.—Sample Operation Schedule for Test of Well Field at Indiana Ordnance Works

Date	Collector No.						
	1	2	3	4	5	6	7
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(a) Pumpage, in Million Gallons per Day							
7-21-42	0.713	4.795	5.035	4.763	9.688	10.070	9.843
7-22-42	0.583	4.717	4.925	4.653	9.454	9.850	9.571
7-23-42	1.873	4.749	4.937	4.056	9.512	9.804	9.583
7-24-42	1.322	4.808	5.009	4.879	9.078	9.811	9.564
Total	4.491	19.069	19.906	18.351	37.732	39.535	38.561
Average	(1.10)	(4.75)	(5.00)	(4.60)	(9.45)	(9.90)	(9.65)
(b) Collector Water Temperature, in Degrees Fahrenheit							
Average	65	65	62	63	61	62	60
(c) Water Level Elevations[a]							
7-21-42	412.69[b]	401.79[b]	400.76	401.06	395.53	398.25	406.58
7-22-42	414.09	402.81	400.91	401.11	395.48	399.56	406.51
7-23-42	413.31	403.33	400.93	402.06	396.40	399.43	406.24
7-24-42	414.61	403.76	401.12	401.11	395.35	399.64	406.39
Average	(414.00)	(403.30)	(400.93)	(401.34)	(395.68)	(399.47)	(406.43)
(d) Drawdown,[c] in Feet							
Observed	5.45	16.15	19.52	18.11	23.77	19.98	13.02
Corrected	5.22	15.45	17.90	16.84	21.50	18.34	11.58

[a] In observation wells. [b] Omitted from average because of antecedent conditions. [c] Mean river elevation: 419.45.

By a process of comparison the drawdown of each collector and its interference with all others, for a pumping rate of 10 mgd, was finally obtained. This was checked by recomputing the elevations at each collector taken from

each pumping schedule using observed temperatures and river levels. These elevations were compared with the observed elevations; and, when a significant discrepancy appeared, the cause for the discrepancy was sought. Sometimes too much weight had been given to the poorer data. Minor corrections were then made on a trial basis. Finally, Table 3 was obtained, which summarizes the drawdowns caused at each production unit by any single collector pumping 10 mgd at a temperature of 68.5° F. Table 3 can be regarded as a mathematical model of the collector field in a state of steady flow.

TABLE 2.—CONSOLIDATED DATA SHEET FOR ALL USABLE PUMPING SCHEDULES

Schedule No.	Collector No. 1	2	3	4	5	6	7
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(*a*) AVERAGE PUMPAGE, IN MILLION GALLONS PER DAY							
1.....	1.10	4.75	5.00	4.60	9.46	9.90	9.65
3.....	6.80	9.25	4.80	6.75	9.40	9.90	0
3[a].....	9.25	4.10	4.75	5.35	9.40	9.80	4.10
4.....	9.40	4.75	4.95	5.70	9.65	5.15	9.70
5.....	9.20	4.70	4.75	4.95	5.05	9.80	9.50
7.....	4.95	5.30	4.80	5.05	9.55	9.70	9.45
8A.....	9.55	9.60	0.10	5.20	9.70	9.80	9.60
(*b*) AVERAGE DAILY DRAWDOWN, IN FEET[b]							
1.....	5.22	15.45	17.90	16.84	21.50	18.34	11.58
3.....	14.42	26.20	18.40	22.00	22.62	17.58	2.63
3[a].....	21.50	19.13	18.81	19.76	23.32	18.83	6.97
4.....	21.78	19.70	18.60	19.76	23.31	13.88	11.84
5.....	22.01	19.55	18.84	18.62	16.30	19.43	12.26
7.....	14.58	17.48	19.28	20.11	24.28	20.34	12.75
8A.....	24.36	29.35	11.04	19.24	24.58	20.85	14.27

[a] Variant. [b] Corrected to 68.5° F.

These tests furnish the factual basis for comparing the performance of the collector field with that at any future date. They will enable any theories as to apparently reduced collector field yields to be answered by a similar test. Any silting or reduced infiltration capacity of the aquifer would result in a reduction in water level elevations throughout the field as compared to the 1942 standard, under the same conditions of pumping, after corrections for temperature have been made.

The test results, moreover, can be usefully applied in predicting minimum collector field yields and in making sure that the installed pump capacity in each unit matches its capacity.

TABLE 3.—MATHEMATICAL MODEL OF COLLECTOR FIELD, GIVING SPECIFIC DRAWDOWN, IN FEET PER 10 MGD, CAUSED BY PUMPING EACH COLLECTOR

Collector No.	Collector Being Pumped 1	2	3	4	5	6	7
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1.....	20.0	5.5	2.0	....	....	....	....
2.....	6.5	22.0	6.0	2.5	....	....	....
3.....	3.2	4.3	24.0	5.0	1.5	....	....
4.....	1.9	2.1	4.3	22.3	2.8	1.1	....
5.....	....	1.4	2.2	4.3	17.6	2.7	1.7
6.....	....	....	....	1.8	2.8	14.5	3.4
7.....	....	....	....	....	1.4	2.4	10.8

The first steps in predicting minimum collector yields were: (1) To test the ability of the mathematical model to predict drawdowns on the basis of observed pumping rates, water temperatures, and river levels; and (2) to compare such predictions with the observed drawdowns. If these data were reasonably accurate, the model might then be used in predicting

minimum yields; but, as minimum yields are to be expected during low pool stages in the winter and early spring, the mathematical model also had to be tested under these conditions.

Since the temperature distribution in the aquifer is so complex, it would be on the safe side to use the lowest measured water temperatures during any period. In the summertime the lowest temperatures in the aquifer are near the water collector; and, as long as river water temperatures are above collector water temperatures, this condition must exist. The original calibration tests were conducted under such conditions; hence, temperature corrections were based on the temperature of water discharged by the collectors.

However, in the winter when river temperatures are low, theoretically the temperature to use would probably be that existing somewhere between the river and the collectors. There appears to be no logical reason why the arithmetic average of the river and collector temperatures should be used when the temperature distribution in the aquifer is so complex. In any event, over a period of from thirty to sixty days of constant operation in the winter, the governing temperature would seem to approach the average river temperature during that period as a limit. A study of river temperature records since 1910 furnished by the Louisville Water Company shows a normal range in temperature for the winter period from 32° F to 43° F, with occasional freakish days above 43° F.

A complete record of pumpage at the Indiana Ordnance Works since April, 1941, has been available to the writer. It might, therefore, appear to be a simple matter to find a number of three-day or four-day periods of pumping at a constant rate which would serve to test the mathematical model. However, because of the man-power shortage and the confidence of the plant operators in the permanence of the supply, the systematic measurements in the observation wells which had been made at weekly intervals in 1941 and 1942 were decreased in frequency until, in 1945, only one measurement was made a month. The constantly changing water demand of the Ordnance Plant further tended to decrease the probability of the water level measurements occurring at the end of a period of constant pumping. However, a careful study of all the data revealed six periods between August, 1942, and December, 1944, during which the requisite conditions were approximately satisfied. Of these, three occurred in the summer and three in the winter.

Table 4 summarizes the water levels computed by the mathematical model, the observed water levels, and the difference between them.

All errors preceded by a minus (−) sign are on the unsafe side—that is, the observed water elevation was lower than the computed water elevation. Some of the errors result from major pumping variations, as on August 20, 1943, when the pumping rate at collector 1 changed considerably during the selected period. However, other large errors are not so readily explained and may result from temporary curtailment or increase in pumpage due to discharge-line pressure changes just prior to making the water level measurements.

A study of the data shows that the mathematical model of the ground water supply system works reasonably well when discharge temperatures are used in the summer and average river temperatures are used in the winter.

After the mathematical model was tested, the percentages of maximum errors on the unsafe side for each collector in numerical order were found to be 14.3, 5.8, 2.4, 3.1, 10.6, 16.2, and 14.0, respectively.

It is of interest to note that all the maximum observed errors on the unsafe side occurred during the period dated August 20, 1943, when the temperature factor favored large yields.

TABLE 4.—COMPARISON OF COMPUTED AND OBSERVED WATER LEVELS

Date	Water level elevation	Collector No.							River stage
		1	2	3	4	5	6	7	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
8-21-42...	Observed...	395.0	399.5	400.0	401.0	403.2	398.8	406.0	419.7
	Computed..	398.0	399.4	399.7	401.3	403.0	400.0	405.4	....
	Error.....	−3.0	0.1	−0.3	0.3	0.2	−1.2	0.6	....
8-20-43...	Observed...	399.3	399.2	399.1	400.7	402.0	396.5	406.0	419.9
	Computed..	402.4	400.4	399.6	401.3	403.9	400.3	407.8	....
	Error.....	−3.1	−1.2	−0.5	−0.6	−1.9	−3.8	−1.8	....
12-24-43..	Observed...	398.3	395.4	393.1	393.1	395.3	393.9	400.8	419.8
	Computed..	398.2	392.9	391.2	393.1	394.2	393.4	400.7	....
	Error.....	0.1	2.5	1.9	0.0	1.1	0.5	0.1	....
1-7-44....	Observed...	398.0	394.8	392.8	395.7	396.9	398.5	402.8	420.3
	Computed..	398.3	392.8	391.6	395.2	396.2	400.1	402.2	....
	Error.....	−0.3	2.0	1.2	0.5	0.7	−1.6	0.6	....
7-21-44...	Observed...	397.3	400.2	403.3	401.0	398.0	397.4	406.2	420.0[a]
	Computed..	399.7	400.3	399.0	401.1	397.2	399.2	406.2	....
	Error.....	−1.4	−0.1	4.3	−0.1	0.8	−1.8	0.0	....
12-16-44..	Observed...	393.2	396.1	395.2	397.2	390.7	389.4	401.4	420.3
	Computed..	390.7	392.0	391.5	393.1	391.2	392.0	400.0	....
	Error.....	2.5	4.1	3.7	4.1	−0.5	−2.6	1.4	....

[a] Estimated.

The final step in determining the firm yield of the collector field was to use the construction data as a basis for computing yields. The maximum drawdown in each unit is governed by the elevation of the uppermost tier of installed laterals and the contractor's instructions stated that a minimum of 3 ft of water was to be maintained above the uppermost tier. These specifications were used as the basis for computations.

The firm yield of the field was found by trial, using the mathematical model corrected to a temperature of 35° F. This temperature is believed to be the average that can be expected to occur for fairly long periods during a normal winter. Pool elevation is taken as 419.0 ft in computing allowable drawdown. A significantly lower pool elevation can be expected only when wickets of the Louisville Dam are lowered to break up ice jams, a truly infrequent occurrence. The first step is to compute Table 5, assuming a pumpage of 10 mgd from each well. Using Cols. 9 and 10, Table 5, as a basis, a first approximation of firm water is obtained by multiplying the ratio between the available drawdown and the computed drawdown by 10 mgd. Data thus obtained are placed in the

mathematical model and a new set of drawdowns is computed and similarly compared with the available drawdowns. This procedure is repeated and new values of pumpage are obtained for each collector until the computed drawdown and available drawdown coincide. After repeating this process three

TABLE 5.—TRIAL SPECIFIC DRAWDOWNS, IN FEET, CAUSED BY PUMPING EACH COLLECTOR AT 10 MGD WHEN THE TEMPERATURE IS 35° F AND THE POOL STAGE IS 419.0

Collector No.	Collector Being Pumped							Computed drawdown	Maximum drawdown available
	1	2	3	4	5	6	7		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1.....	33.8	9.3	3.4	....	....	....	....	46.5	42.0
2.....	11.0	37.1	10.1	4.2	....	....	....	62.4	39.0
3.....	5.4	7.3	40.6	8.5	2.5	....	....	64.3	33.0
4.....	3.2	3.5	7.3	37.7	4.7	1.9	....	58.3	36.0
5.....	....	2.4	3.7	7.3	29.7	4.6	2.9	50.6	38.0
6.....	....	....	....	3.0	4.7	24.5	5.7	37.9	43.0
7.....	....	....	....	....	2.4	4.1	18.3	24.8	48.0

times, the firm yields of Table 6 were obtained based on the assumption that the whole field is pumped at the same time. Pump installation to at least the indicated capacity is required in each collector to obtain the maximum well field yield under adverse circumstances.

Additional data on river temperature probabilities and a considerable amount of statistical analysis will be required to establish design temperatures for collectors or wells obtaining water from river infiltration. Incidentally, this branch of hydrology may also be valuable to engineers interested in protecting levees from dangerous underseepage; a levee that may be safe from dangerous underseepage during a February or March flood might be unsafe from an equal river stage occurring in May, June, or July. The phenomena, it should be noted, are fundamentally identical.

TABLE 6.—FIRM YIELD WITH ENTIRE FIELD PUMPING

Collector No.	Yield (mgd)
1.....	10.0
2.....	5.8
3.....	4.3
4.....	5.9
5.....	6.8
6.....	10.5
7.....	22.5
Total....	65.8

CONCLUSIONS

Study of the water supply of the Indiana Ordnance Works has demonstrated that nearly all the water pumped has originated in the Ohio River and has infiltrated to the aquifer. Water collectors or wells deriving water from such a source will furnish water whose temperature variation follows the variation in temperature of the river water, but with a lag and reduction in amplitude fixed by the hydrology of the aquifer and the operation of the field.

At the Indiana Ordnance Works, the temperature of the water pumped from the collector field as a whole varied from 49° F to 70° F, although individual units furnished water whose temperature was as much as 2° F above or

below this range. The range in temperature is determined by the temperature variations of the source of water, the distance to the recharge area, the landward extent and transmissibility of the aquifer, the rates and durations of discharge, the heat storage of the aquifer and movement of heat within it, and the water contribution from the land side of the aquifer.

Furthermore, the temperature of water discharged by each lateral of a unit is primarily determined by its relationship to the source of recharge. At the Indiana Ordnance Works, a temperature differential between 15° F and 20° F was found to exist during certain times of the year in water entering a single caisson from individual laterals.

Temperature of the water has a profound affect on the minimum quantity of water, or firm water, available from units whose maximum drawdown is fixed. The design temperature for computing the firm water approaches the lowest average river temperature prevailing over a one-month or a two-month period.

The firm yield of an existing collector field deriving its water from river infiltration can be determined by methods which have been described in this paper. Such tests are valuable in so far as they furnish the operator with a quantitative method to detect any deterioration in the collectors or the infiltration capacity of the aquifer. They are also useful in balancing installed pump capacity with collector productivity. The firm yield of a field must be known in order that additions to water supply capacity can be logically planned.

The bacteriological quality of the water discharged by the collectors at the Indiana Ordnance Works was extremely good. Gas forming organisms were rarely found in the water and it is believed that the gas formers as well as the few coliform organisms which were found by bacteriological tests were the results of careless sampling technique, despite the heavy pollution of the Ohio River. Tastes and odors were apparently absent from the water discharged by the collectors, although no systematic attempt was made to detect them. Natural purification of polluted water by aquifers undoubtedly merits more study than it has received in the past.

Acknowledgments

Nearly all the data collected during the four-year period covered by this report were collected by the personnel of the Indiana Ordnance Works. That the thousands of measurements of water levels, pumpages, temperatures, and bacterial quality could be correlated at all is tribute enough to the industry and ability of these men and women.

The assistance of the War Department and the E. I. du Pont de Nemours Company is acknowledged with thanks. Special mention should be made of the capable and cooperative commanding officers of the Indiana Ordnance Works during the period: Brig.-Gen. R. E. Hardy and Col. Kelly H. Lewis, his

successor. Captains A. L. Ahlers, E. E. Keiser, and A. J. Klemka and their civilian aides, including John E. Ulrich and the late L. G. Couter, also rendered valuable assistance.

The plant managers of the Indiana Ordnance Works—J. F. Daly, R. F. Boltz, and M. L. Valentine—cooperated fully to facilitate obtaining the required information. Power superintendents F. M. Acker and C. C. Hagerty, as well as members of the power and water staff, including C. F. Parcels, G. R. Calhoun, S. G. Ferris, and C. M. Fair, were most obliging and helpful. Members of the Wilmington (Del.) office of the E. I. du Pont de Nemours Company, especially E. B. Showell and W. J. Sloan, were most cooperative in facilitating the study.

E. W. Silitch, M. ASCE, and A. D'Audiffret of the Ranney Method Water Supplies, Inc., rendered substantial assistance by furnishing engineering information and construction data as well as some of the photographs used in this article.

The writer desires to thank the various members of the Division of Ground Water of the United States Geological Survey for criticism and supervision as well as encouragement in conducting the study described in this paper. In particular, thanks are due to O. E. Meinzer, formerly geologist-in-charge of the Division of Ground Water, under whose general supervision the work was done; the late D. G. Thompson, geologist; and L. K. Wenzel and C. E. Jacob, Assoc. M. ASCE, hydraulic engineers, whose criticism has been invaluable.

Acknowledgment is also made to the Director, Geological Survey, United States Department of the Interior, for permission to publish the material contained in this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

UNDERGROUND CONDUITS—AN APPRAISAL OF MODERN RESEARCH

BY M. G. SPANGLER,[1] M. ASCE

SYNOPSIS

The increasingly successful application of scientific principles of mechanics to the analysis of soil has characterized engineering development during the first two score years of the twentieth century. A number of problems that involve the soil as an integral part of the finished structure and which a generation or two ago were considered by many to be too complex to be subject to orderly classification and solution are yielding to the onslaught of organized research.

One notable example of problems in this category concerns the design of underground structures such as conduits, sewers, drains, culverts, water mains, and gas lines. Conduits of this general type have been used by mankind for at least 3,000 years, being found among the earliest examples of the practice of the engineering arts; but not until about 1910 was serious effort directed toward the development of a rational method for determining the magnitude and character of the loads to which underground conduits were subjected in service due to the earth overburden and other load sources. In those early days, Anson Marston, Past-President and Hon. M. ASCE, then director of the Engineering Experiment Station at Iowa State College in Ames, inaugurated a research program which has been continuously active under his direction. The purpose of this paper is to review Dean Marston's theory of loads on, and supporting strengths of, underground conduits and to record the experimental background, together with working values of many of the constants required for the practical application of the theory.

LOAD CLASSIFICATION OF UNDERGROUND CONDUITS

Underground conduits, in general, may be divided into two main classes on the basis of construction conditions under which they are installed—that is (1) ditch conduits and (2) projecting conduits.

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **November 1, 1947.**

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(1) "Ditch conduits" are structures installed and completely buried in narrow ditches in relatively passive or undisturbed soil. Examples of this class of conduits are sewers, drains, and water mains (see Fig. 1(*a*)).

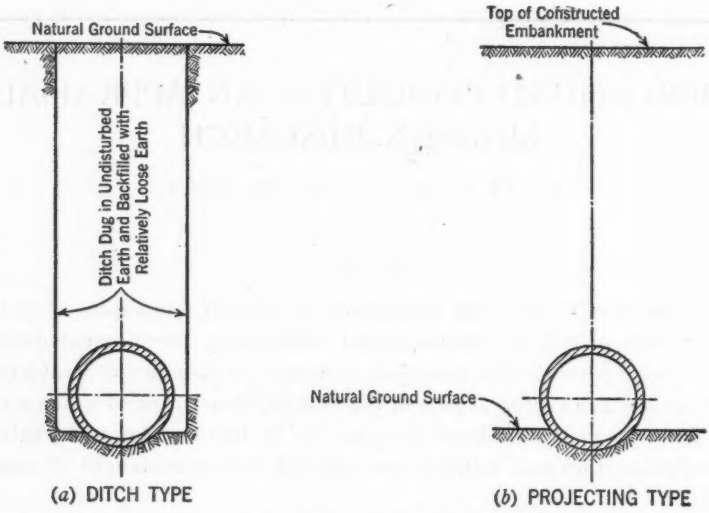

Natural Ground Surface
Ditch Dug in Undisturbed Earth and Backfilled with Relatively Loose Earth
(a) DITCH TYPE
Top of Constructed Embankment
Natural Ground Surface
(b) PROJECTING TYPE

FIG. 1.—ESSENTIAL ELEMENTS OF TYPICAL CONDUITS

(2) "Projecting Conduits" are structures installed in shallow bedding with the top of the conduit projecting above the surface of the natural ground, and then covered with an embankment, as shown in Fig. 1(*b*). Railway and highway culverts are good illustrations of this class of conduits. Conduits installed in ditches wider than about two or three times their maximum horizontal breadth may also be treated as projecting conduits.

NOTATION

The letter symbols in this paper are defined where they first appear, in text or by illustration, and are assembled for reference in Appendix I. Discussers are requested to conform to this usage.

LOADS ON DITCH CONDUITS

When a conduit is placed in a ditch not wider than about two or three times its outside breadth, and covered with earth, the backfill will tend to settle downward. This downward movement or tendency for movement of the soil in the ditch above the pipe produces vertical frictional forces or shearing stresses along the sides of the ditch which act upward on the prism of soil within the ditch and help to support the backfill material. Assuming the cohesion between the backfill material and the sides of the ditch to be negligible, the magnitude of these vertical shearing stresses is equal to the active lateral pressure exerted by the earth backfill against the sides of the ditch, multiplied

by the tangent of the angle of friction between the two materials. This assumption of negligible cohesion is justified because: (*a*) Even when the ditch is dug in, and backfilled with, cohesive material, considerable time must elapse before effective cohesion between the backfill material and the sides of the ditch can develop after backfilling; and (*b*) the assumption of no cohesion yields the maximum probable load on the conduit. The maximum load may develop at any time during the life of the conduit due to heavy rainfall or other causes which may eliminate or greatly reduce cohesion between the backfill and the sides of the ditch.

Let Fig. 2 represent the section of a unit length of ditch conduit and consider a horizontal element of the fill material having a height dH_p at a distance H_p below the ground surface. Equating[2] the vertical forces acting on this element:

$$P + dP + 2\,K\,\mu'\,\frac{P}{b_d}\,dH_p = P + \gamma\,b_d\,dH_p \quad \ldots\ldots (1a)$$

This is a linear differential equation, the solution for which is

$$P = \gamma\,b_d^2\,\frac{1 - e^{-\alpha H_p}}{2\,K\,\mu'} \quad \ldots\ldots (1b)$$

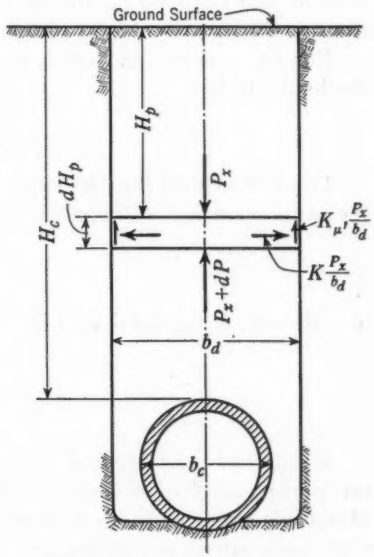


FIG. 2.—FORCE DIAGRAM FOR A DITCH CONDUIT

in which P equals the total vertical pressure on a horizontal plane in the backfill, in pounds per linear foot; μ' is the coefficient of sliding friction between the fill materials and the sides of the ditch; b_d is the width of ditch at the top of the conduit; H_p is the height of the ground surface above any horizontal plane in the backfill; γ is the unit weight of filling material, in pounds per cubic foot; e is the base of natural logarithms; and, to simplify typography:

$$\alpha = \frac{2\,K\,\mu'}{b_d} \quad \ldots\ldots (2a)$$

The ratio K of active lateral pressure to vertical pressure, is defined as

$$K = \frac{\sqrt{\mu^2 + 1} - \mu}{\sqrt{\mu^2 + 1} + \mu} \quad \ldots\ldots (2b)$$

in which μ is the coefficient of internal friction of the filling material. The coefficient μ' may be equal to or less than μ, but it cannot be greater.

When $H_p = H_c$, Eq. 1*b* gives the total vertical pressure within the ditch at the level of the top of the pipe. The proportion of this total pressure that

[2] "The Theory of Loads on Pipes in Ditches and Tests of Cement and Clay Drain Tile and Sewer Pipe," by A. Marston and A. O. Anderson, *Bulletin No. 31*, Iowa Eng. Experiment Station, Ames, Iowa, 1913, p. 32.

will be carried by the conduit will depend upon the relative rigidity of the conduit and of the fill material between the sides of the conduit and the sides of the ditch. In the case of very rigid pipes, such as burned clay, concrete, or heavy cast-iron pipe, the side fills may be relatively compressible and the pipe itself will carry practically all the load, P. On the other hand, if the pipe is a relatively flexible, thin-walled pipe and the side fills are thoroughly tamped in at the sides of the pipe, the stiffness of the side fills may approach that of the conduit and the load on the structure will be reduced by the amount of load the side fills are capable of carrying.

For the case of rigid ditch conduits with relatively compressible side fills, the load will be:

$$W_c = C_d \gamma b_d^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

For the case of flexible pipes and thoroughly compacted side fills having the same degree of stiffness as the pipes, the load will be:

$$W_c = C_d \gamma b_c b_d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3b)$$

in which W_c is the total load on the conduit, and:

$$C_d = \frac{1 - e^{-\alpha H_c}}{2 K \mu'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3c)$$

Eq. 3*a* has been verified many times by experiment and by observation of the performance of actual sewers and drains. The results of two of Dean Marston's early load experiments on rigid pipes are shown in Fig. 3, together with loads calculated by Eq. 3*a*.

Evaluation of Eqs. 3*a* and 3*b* may be simplified by the use of Fig. 4 in which values of C_d for various values of H_c/b_d have been plotted for several kinds of filling materials having different coefficients of internal friction.

The width of ditch, b_d, is the actual width of a normal, parallel-sided ditch. In case the ditch is constructed with sloping sides, or the conduit is placed in a subditch at the bottom of a wider trench, experiments[3] have shown that the width of ditch at or slightly below the top of the pipe is the proper width to use when determining the load.

These ditch conduit formulas (Eqs. 3*a* and 3*b*), with proper selection of the physical factors involved, give the maximum loads to which any particular conduit may be subjected in service. On the other hand, because of the development of cohesion, any particular conduit may escape the maximum load for a long time, sometimes until its removal for other causes than load failure. Experiments and field observations show that the load on a conduit at the time the fill is completed is usually less than it will be at some later time. This condition accounts for the fact that sewers and other conduits which have been observed to be structurally sound immediately upon completion are sometimes found to be cracked some months or years later.

[3] "Loads on Pipe in Wide Ditches," by W. J. Schlick, *Bulletin No. 108*, Iowa Eng. Experiment Station, Ames, Iowa, 1932.

Loads on Projecting Conduits

Projecting conduits, as defined and as the name implies, are placed with their tops projecting some distance above the natural ground surface. They may be of any shape, such as circular, rectangular, or elliptical; may be made of any material, such as concrete, burned clay, cast iron, corrugated metal, or wood; and may possess any degree of rigidity from the very rigid concrete pipes and monolithic box culverts to the very flexible, lightweight corrugated metal pipes.

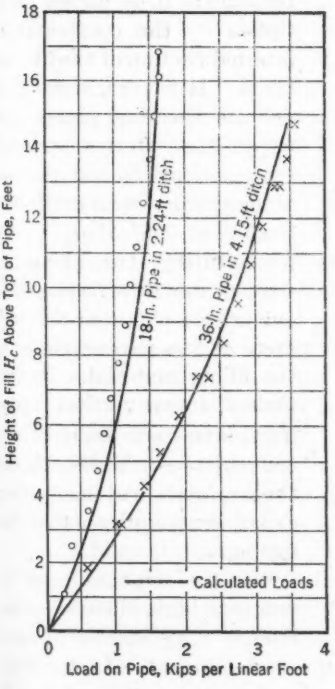

Height of Fill H_c Above Top of Pipe, Feet
18-In. Pipe in 2.24-ft ditch
36-In. Pipe in 4.15-ft ditch
Calculated Loads
Load on Pipe, Kips per Linear Foot

Fig. 3.—Measured Loads on Ditch Conduits

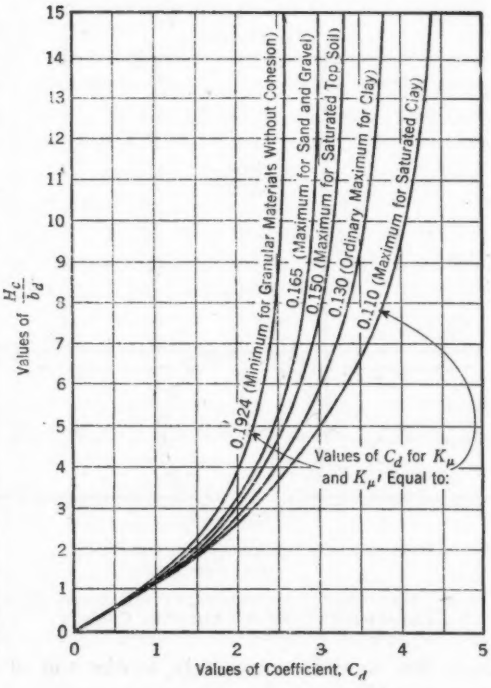

Values of $\frac{H_c}{b_d}$
0.1924 (Minimum for Granular Materials Without Cohesion)
0.165 (Maximum for Sand and Gravel)
0.150 (Maximum for Saturated Top Soil)
0.130 (Ordinary Maximum for Clay)
0.110 (Maximum for Saturated Clay)
Values of C_d for K_μ and $K_{\mu'}$ Equal to:
Values of Coefficient, C_d

Fig. 4.—Computation Diagram for Ditch Conduits

As a basic case for studying the action of an embankment over this type of conduit, consider first a rigid structure resting on an earth foundation which settles under the load the same amount as the natural ground adjacent to the structure (see Fig. 5). Then in the embankment built over the conduit, there are three masses or prisms of soil, one of which, called the "interior prism," is directly over the conduit between the vertical planes tangent to the sides. The other two masses of soil, called the "exterior prisms," are those on each side of the structure adjacent to the tangent vertical planes.

It is evident that the height of the interior prism will be less than that of the exterior prisms by the amount which the conduit projects above the natural ground. In accordance with the well-known phenomenon that a high

prism of soil will settle more than a lower prism, there is a tendency for the exterior prism to settle more than the interior prism and for friction forces or shearing stresses to be exerted along the tangent vertical planes bounding the interior prism. These shearing stresses are transmitted to the conduit, making the load on the structure greater than the weight of the interior prism of soil. The magnitude of the shearing stresses (again neglecting cohesion) is assumed to be equal to the active lateral pressure at these planes, multiplied by the coefficient of internal friction of the fill material. It is recognized that definite shearing planes between the interior and exterior prisms of soil do not actually exist in an earth embankment and that, in all probability, the shearing stresses are transferred from one prism to another through more or less narrow zones of the filling material. Nevertheless, actual vertical shearing planes are assumed for convenience in developing the theory, and load measuring experiments indicate that the assumption is valid.

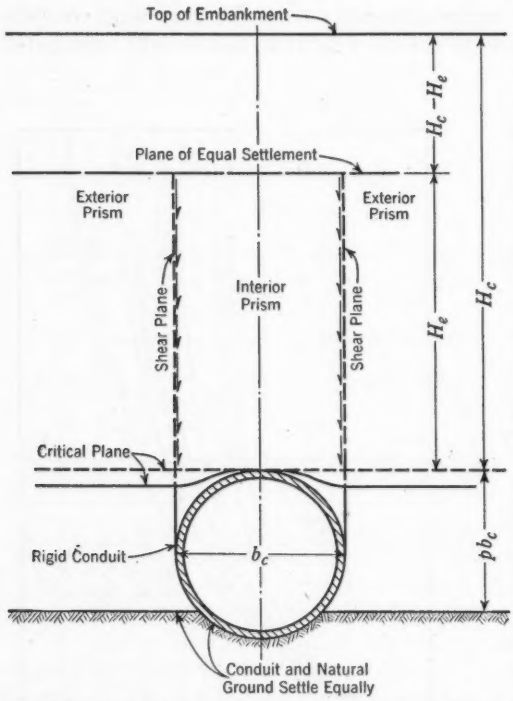


FIG. 5.—BASIC CASE FOR CONSIDERING THE ACTION OF AN EMBANKMENT OVER A PROJECTING CONDUIT

If the embankment is not very high, these shearing stresses may extend upward from the conduit completely to the top of the embankment. In the case of higher fills, the shearing stresses will not extend to the surface, but will terminate at some horizontal plane between the top of the conduit and the top of the embankment, known as the "plane of equal settlement." The distance between the top of the conduit and this plane is called the "height of equal settlement, H_e."

The plane of equal settlement is defined as the horizontal plane in the embankment at and above which the settlements of the interior and exterior prisms of soil are equal. Above the plane of equal settlement there is no tendency for relative movement between the three adjacent prisms and no shearing stresses are generated along the boundaries of the interior prism above this plane, whereas, below it, relative movements do have a tendency to occur and shearing stresses are developed.

A plane of equal settlement in this basic case results from the transfer of pressure, by shear, from the exterior prisms to the interior prism. Since the

vertical deformation of a prism of material due to its own weight is a function of the height as well as characteristic of the material, normally the summation of deformations from the bottom of a prism upward will be at a greater rate in a high prism than in a lower one if the prisms act independently of one another. In the case of projecting conduits, however, the exterior prisms of soil transfer a part of their vertical pressure to the interior prism and the rate of summation of vertical deformations will be reduced in the exterior prisms and increased in the interior prism because of this stress transfer. Therefore, the total summation of deformations in the interior prism will approach that in the exterior prisms, and the height at which the deformations become equal is the height of equal settlement.

The existence of a plane of equal settlement was first announced by Dean Marston in 1922[4] on the basis of pure mathematical reasoning and a formula for evaluating its height was developed at that time. Since then the actual existence of such a plane has been demonstrated by measurements of the settlements and of the loads on a number of experimental conduits.

This basic settlement situation is practically always modified by two additional factors which must be considered in the development of a general theory of fill loads. The first of these is the settlement of the undisturbed subgrade under the exterior prisms adjacent to the conduit; and the second, the settlement of the top of the conduit. The settlement of the subgrade adjacent to the conduit augments the downward movement of the two exterior prisms of earth. The settlement of the top of the conduit (which is equal to the sum of the settlement of its foundation and the distortion or shortening of its vertical dimension) has the tendency to neutralize this action by reducing the relative movement between the interior and exterior prisms. Indeed, if the conduit is sufficiently flexible, if it is placed on a very yielding foundation, or if both conditions hold true, the top of the conduit may settle enough to permit the interior prism to move downward a greater amount than the exterior prisms. When this occurs, the direction of the induced shearing stresses is reversed and the stresses are subtractive from the weight of the prism of earth over the conduit. The rate of summation of settlements in relation to height above the conduit for the interior prism is less than that in the exterior prisms due to the reduced pressure. Nevertheless, there still may be a plane of equal settlement, even if the direction of relative movements is reversed, because the total settlement is augmented by the settlement of the top of the conduit, which causes the entire interior prism to move downward.

In this connection it is convenient to define a "critical plane" which is the horizontal plane in the fill material at the level of the top of the conduit at the beginning of construction of the embankment and before settlements have begun to develop. With this definition in mind, the aforementioned facts concerning direction of the induced shearing stresses may be simplified by stating that, when the critical plane settles more than the top of the conduit, the shearing stresses act downward on the interior prism and that, when it settles less, they act upward.

[4] "Second Progress Report to the Joint Concrete Culvert Pipe Committee," by Anson Marston, April 7, 1922 (mimeographed).

A neutral or transition case occurs when the top of the conduit settles downward an amount just equal to the settlement of the critical plane. When this case occurs, the plane of equal settlement is right at the top of the conduit and coincident with the critical plane; the interior and exterior prisms of soil move downward equally throughout their full height and no shearing stresses are induced, with the result that the load on the structure is equal to the weight of the prism of soil directly over it. Some typical settlement situations affecting loads on projecting conduits are illustrated in Fig. 6. (The solid

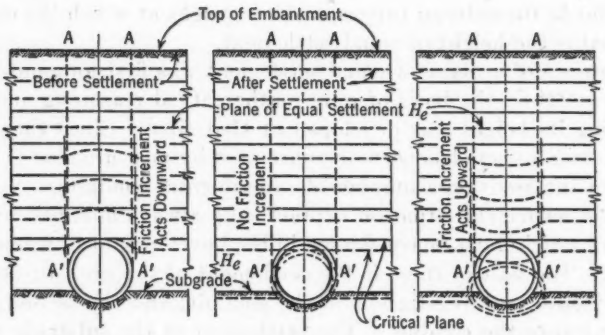


FIG. 6.—TYPICAL SETTLEMENT CONDITIONS AFFECTING LOADS ON PROJECTING CONDUITS

horizontal lines represent imaginary planes of embankment before settlement. The broken lines represent the same planes after settlement.[5])

In the mathematical analysis of loads on projecting conduits, the net effect of all these settlement factors, both as to magnitude and direction of the relative movements of the three prisms of soil, is combined into an abstract ratio known as the "settlement ratio," which is defined as the ratio of the difference between the settlement of the critical plane and the top of the conduit, to the deformation of the fill material adjacent to the conduit within the vertical distance between the top of the conduit and the natural ground line. Dean Marston defined this ratio on the basis of the settlements of the various elements which are produced by that section of the fill above the plane of equal settlement.

For purposes of calculating the external vertical loads on projecting conduits, the field conditions affecting the loads are conveniently grouped into four subclassifications based on the magnitude of the settlement of the interior prism relative to that of the exterior prisms and, therefore, the direction of the shearing stresses, and the height of the embankment in relation to the height of equal settlement. These four conditions are: (1) The complete projection condition, (2) the incomplete projection condition, (3) the complete ditch condition, and (4) the incomplete ditch condition.

[5] "Investigation of Loads on Three Cast Iron Pipe Culverts Under Rock Fills," by M. G. Spangler, *Bulletin No. 104*, Iowa Eng. Experiment Station, Ames, Iowa, 1931, p. 26, Fig. 11.

They are defined as follows:

(1) The "complete projection condition" prevails when the top of the conduit settles less than the critical plane and when the height of the embankment is less than the theoretical height of equal settlement.

(2) The "incomplete projection condition" prevails when the top of the conduit settles less than the critical plane and when the height of the embankment is greater than the height of equal settlement.

(3) The "complete ditch condition" prevails when the top of the conduit settles more than the critical plane and when the height of the embankment is less than the theoretical height of equal settlement.

(4) The "incomplete ditch condition" prevails when the top of the conduit settles more than the critical plane and when the height of the embankment is greater than the height of equal settlement.

Mathematical Derivation of Load Formulas for Projecting Conduits

Dean Marston defines[4,6] the various settlements, y, as the "* * * increments due to the addition at or above the height of equal settlement of any [the same for all] incremental uniform layer of fill materials." A settlement ratio is defined as the ratio of the difference between the settlement of the critical plane and the top of the conduit of the fill adjacent to the conduit within the height $\rho\, b_c$ (Fig. 5) caused by the fill material above the plane of equal settlement.[6a] In other words:

$$\delta = \frac{(y_m + y_g) - (y_f + y_c)}{y_m} \ldots\ldots\ldots\ldots\ldots\ldots (4a)$$

in which y_g is the settlement of the embankment subgrade adjacent to the conduit, in feet; y_f is the settlement of the flow line of the conduit; y_c is the deflection of the conduit—that is, the shortening of its vertical dimension; and y_m is the deformation of the filling material adjacent to the conduit within the height $\rho\, b_c$ in feet—that is:

$$y_m = \frac{(H_c - H_e)\,\gamma}{E_f}\,\rho\, b_c \ldots\ldots\ldots\ldots\ldots\ldots (4b)$$

in which E_f is the modulus of deformation of the filling material. The settlement of the "critical plane," in this notation is $y_m + y_g$ and the settlement of the top of the conduit is $y_f + y_c$. In Eq. 4*a*, δ is negative for the "incomplete" ditch condition and positive for the "incomplete" projection condition.

Considering first the complete ditch and projection conditions (Fig. 7(*a*)),[6a] in which $H_c < H_e$, the vertical forces on any thin horizontal element of the interior prism may be equated as follows:

$$P + dP = P + \gamma\, b_c\, dH_p \pm 2\, K\, \mu\, \frac{H_p}{b_c}\, dH_p \ldots\ldots\ldots\ldots (5a)$$

[6] "The Theory of External Loads on Closed Conduits in the Light of the Latest Experiments," by Anson Marston, *Bulletin No. 96*, Iowa Eng. Experiment Station, Ames, Iowa, 1930.

[6a] In Figs. 5 and 7, change the dimension pb_c to ρb_c.

The solution of this equation is

$$P = \gamma\, b_c^2 \frac{e^{\pm \beta H_p} - 1}{\pm 2\, K\, \mu} \quad \ldots\ldots (5b)$$

in which μ is the coefficient of internal friction of the filling material, and to simplify typography (compare Eq. 2*a*):

$$\beta = \frac{2\, K\, \mu}{b_c} \quad \ldots\ldots (5c)$$

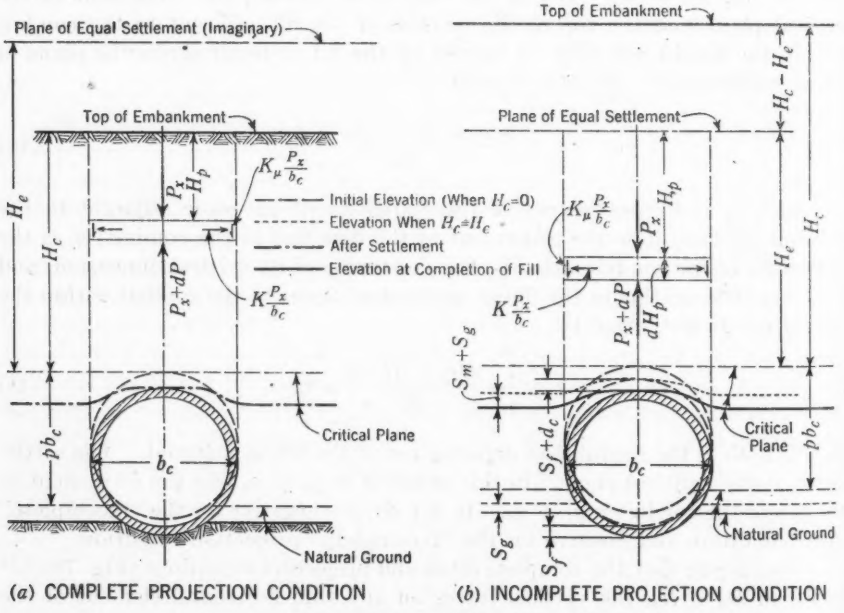
At the top of the conduit, $P = W_c$ and $H_p = H_c$; therefore:

$$W_c = C_c\, \gamma\, b_c^2 \quad \ldots\ldots (6a)$$

in which the load coefficient for projecting conduits is represented by:

$$C_c = \frac{e^{\pm \beta H_c} - 1}{\pm 2\, K\, \mu} \quad \ldots\ldots (6b)$$

in which H_c is the height above the top of a conduit. In Eq. 6*b* the plus signs are used for the complete projection condition and the minus signs for the complete ditch condition.



(*a*) COMPLETE PROJECTION CONDITION (*b*) INCOMPLETE PROJECTION CONDITION

FIG. 7.—FORCES ACTING ON A PROJECTING CONDUIT

Considering the incomplete ditch and the incomplete projection conditions (Fig. 7(*b*)),[6a] in which $H_c > H_e$, and again equating the vertical forces on a thin horizontal element:

$$P + dP = P + \gamma\, b_c\, dH_p \pm 2\, K\, \mu \frac{P}{b_c} dH_p \quad \ldots\ldots (7a)$$

When $H_p = 0$, $P = (H_c - H_e)\,\gamma\,b_c$ and the solution of this differential equation (Eq. 7*a*) is:

$$P = \frac{\gamma\,b_c^2}{2\,K\,\mu} \pm \frac{\gamma\,b_c^2}{2\,K\,\mu}\,e^{\pm\beta H_p} + (H_c - H_e)\,\gamma\,b_c\,e^{\pm\beta H_p} \ldots\ldots\ldots (7b)$$

At the top of the conduit, $P = W_c$ and $H_p = H_e$; therefore, in Eq. 6*a*, the load coefficient C_c becomes:

$$C_c = \frac{e^{\pm\beta H_e} - 1}{\pm 2\,K\,\mu} + \left(\frac{H_c}{b_c} - \frac{H_e}{b_c}\right) e^{\pm\beta H_e} \ldots\ldots\ldots\ldots\ldots (8)$$

As before, the plus signs are applicable to the incomplete projection condition and the minus signs apply to the incomplete ditch condition.

To derive an expression for evaluating H_e, the following two conditions are assumed:

a. The internal friction in the embankment materials distributes the infinitely small increments or decrements of pressure from shear into the interior prism below the plane of equal settlement in such manner that their effect on settlement may be assumed to be substantially the same as for uniform vertical pressure; and

b. The internal friction in the embankment materials distributes the infinitely small decrements or increments of pressure from shear into the exterior prisms so completely that their effect on settlement may be neglected.

Equating the settlements in the exterior prisms and the settlements in the interior prism caused by the fill material above the plane of equal settlement:

$$\frac{(H_c - H_e)\,\gamma}{E_f}\,H_e + y_m + y_g = \int_0^{H_e} \frac{(H_c - H_e)\,\gamma}{E_f}\,e^{\pm\beta H_p}\,dH_p + y_f + y_c \ldots (9)$$

In Eq. 9, using Eqs. 4:

$$e^{\pm\beta H_e} \mp 2\,K\,\mu\,\frac{H_e}{b_c} = \pm 2\,K\,\mu\,\delta\,\rho + 1 \ldots\ldots\ldots\ldots (10)$$

In the three quantities of Eq. 10 the $+$, $-$, and $+$ signs are used for the projection condition; and the $-$, $+$, and $-$ signs, respectively, are used for the ditch condition.

Eq. 10 must be solved by trial, since it contains H_e as an exponent of e, the base of natural logarithms (see Eq. 5*c*).

As in the case of ditch conduits, the solution of these various expressions for loads on projecting conduits is made easy by the construction of a load coefficient diagram,[7,8] as shown in Fig. 8. For selected values of the "settlement" ratio and the "projection" ratio ($\delta\,\rho$) and given values of H_c/b_c the proper value of the coefficient C_c is read from the curves and substituted in Eq. 6*a*. Comparisons between actual weighed loads in carefully conducted experiments and loads computed by Dean Marston's load theory are shown in Fig. 9.

[7] "The Theory of External Loads on Closed Conduits in the Light of the Latest Experiments," by Anson Marston, *Bulletin No. 96*, Iowa Eng. Experiment Station, Ames, Iowa, 1930, p. 14, Fig. 3.

[8] "The Supporting Strength of Rigid Pipe Culverts," by M. G. Spangler, *Bulletin No. 112*, Iowa Eng. Experiment Station, Ames, Iowa, 1933, p. 46, Fig. 18.

Several alternate methods of determining the value of H_e have been investigated, including an analysis in which the settlement ratio was based on the total settlements of the various elements of the fill and of the conduit caused by the total height of embankment. The loads obtained by this alternate analysis were nearly the same as those obtained by Dean Marston's original analysis, being somewhat less for high values of $\frac{H_e}{b_e}$ and $\delta\ \rho$.

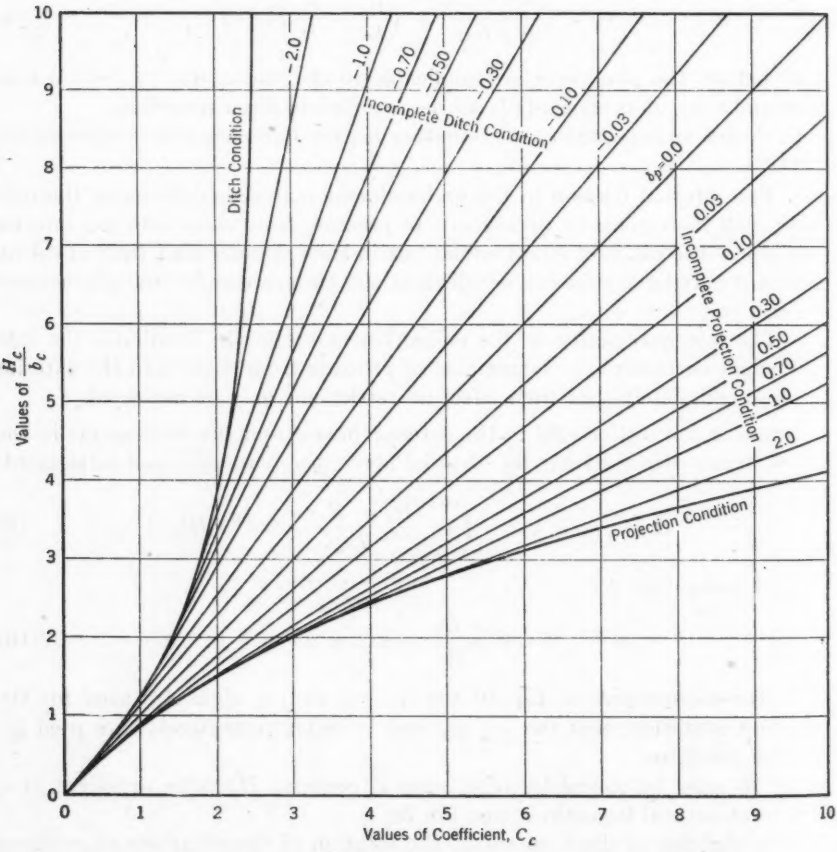


FIG. 8.—COMPUTATION DIAGRAM FOR EXTERNAL LOADS ON CONDUITS ($K\ \mu = 0.1924$)

Working values of μ, the coefficient of internal friction of the embankment material, may be determined by soil tests in a particular case. However, Dean Marston has shown that the variation in load on a projecting conduit is relatively small for wide variations of the coefficient of internal friction. For example, when this coefficient varies from about 0.3 to 1.0, corresponding to angles of friction from about 17° to 45°, the product $K\ \mu$ varies from about 0.17 to 0.1924 and back to 0.17. Since the load is dependent upon the prod-

uct $K\mu$ and not on μ alone, a value of $K\mu = 0.19$ may reasonably be chosen as a safe working value for most cases of projecting conduit loads.

Time Effect on Loads

To study the effect of time on the loads produced on culverts by earth embankments, three test culverts supported on weighing devices were built and loaded with a 15-ft embankment late in the fall of 1927, and observations of the loads on these structures have been made periodically since that date.

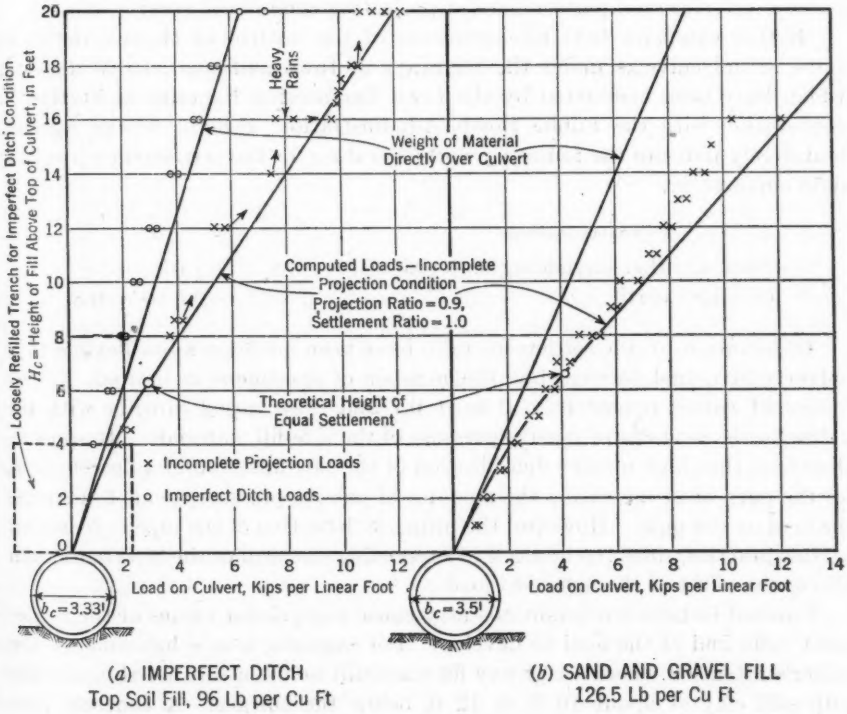

Loosely Refilled Trench for Imperfect Ditch Condition
H_c = Height of Fill Above Top of Culvert, in Feet
Heavy Rains
Weight of Material Directly Over Culvert
Computed Loads – Incomplete Projection Condition
Projection Ratio = 0.9, Settlement Ratio = 1.0
Theoretical Height of Equal Settlement
× Incomplete Projection Loads
o Imperfect Ditch Loads
Load on Culvert, Kips per Linear Foot
b_c = 3.33'
b_c = 3.5'
(a) IMPERFECT DITCH
Top Soil Fill, 96 Lb per Cu Ft
(b) SAND AND GRAVEL FILL
126.5 Lb per Cu Ft

Fig. 9.—Comparison Between Actual Weighed Loads on Projecting Conduits

This load history shows that the maximum load was reached in April of the following spring after the fill was completed, after which the load fell off somewhat, reaching a low point about 10% below the maximum in the following October and November. Since that date, the load has fluctuated between these approximate limits, reaching the maximum for each year early in the spring and the minimum late in the fall.

Eq. 10 for H_e has been derived for the assumed case in which the fill material has a uniform modulus of deformation throughout; and it is emphasized that the value of H_e is independent of the value of this modulus. In other words, even if the fill material is thoroughly compacted during placement

and a high value of the compression modulus is attained, the direction of the relative settlements of the interior and exterior prisms and the height of equal settlement will not be affected thereby.

The settlement ratio is wholly rational in the theoretical analysis of loads on projecting conduits. However, except in unusual cases it will not be practicable to predict the value to use in the design of a given structure on the basis of individual study and analysis of the soil at a proposed site. It has been deemed advisable and appropriate, therefore, to consider the settlement ratio as a semi-empirical factor and to determine proper design values on the basis of the observed performance of projecting conduits in service.

Rather extensive field measurements of the settlement characteristics of some actual culverts under the highways of Iowa and southern Minnesota, which have been conducted by the Iowa Engineering Experiment Station in cooperation with the Public Roads Administration, Federal Works Agency, tentatively indicate the following working values for the settlement ratio—for rigid culverts on:

Foundation material	s
Rock or other unyielding material	+ 1.0
Ordinary earth	+ 0.5 to + 0.8

Observations of the settlement ratio have been made on a few flexible type culverts in actual service, but the number of specimens is limited. These measured values ranged from 0 to + 0.7 and they varied directly with the values of the modulus of passive pressure of the side-fill materials. It appears, therefore, that high quality densification of the side fills around a flexible pipe, for the purpose of increasing the modulus of passive pressure, tends to increase the load on the pipe. However, the ultimate deflection of the pipe is decreased by this practice, since the increase in supporting strength of the pipe more than offsets the effect of the increased load.

Unusual installation conditions may cause exceptional values of the settlement ratio and of the load to develop. For example, a case has come to the writer's attention where a highway fill was built across a salt marsh underlain with stiff clay at about 10 ft or 12 ft below the surface. A concrete pipe culvert was built prior to construction of the fill. To reduce the expected settlement of the culvert and to increase its supporting strength, the pipes were laid in a well-designed concrete cradle, which in turn was supported on piles driven into the clay stratum. Shortly after the fill was completed this culvert collapsed, even though it was constructed of excellent reinforced concrete pipe. Although no settlement data are available, it seems probable that the fill material at the sides (the "exterior prisms") settled a very large amount due to the soft spongy character of the marsh bed, whereas the culvert resting on piles driven to the stiff clay could not settle. Under these conditions the critical plane probably settled much more than the top of the culvert, causing a very high positive value of the settlement ratio and a very high load to develop.

Imperfect Ditch Conduits

In the early days of Dean Marston's researches on conduit loads, he was impressed by the very high loads that may develop on projecting conduits when the conditions are such that large shearing forces add to the weight of the prism of soil directly over the conduit, and he strove to devise a method of construction that would reduce or eliminate these shearing forces, or would possibly reverse their direction so that they would act benevolently as in the case of ditch conduits. With this objective in mind, he developed the imperfect ditch method of construction in which the soil on both sides and above the conduit for some distance above its top is thoroughly compacted by rolling, tamping, or any other suitable method. Then a ditch is constructed in this compacted fill by removing the prism of material directly over the conduit. The ditch is refilled with very loose compressible material, after which the embankment is completed in a normal manner.

The purpose of this method of construction is accomplished by creating a condition wherein it is certain that the prism of material directly over the conduit will settle more than the adjacent prism; the ditch in the artificially compacted material must be deep enough and the refilling material must be loose enough to insure this action. Straw or other highly compressible material may be used as part of the ditch backfill to augment the settlement of the interior prism. The efficacy of this method of construction was demonstrated by a series of experiments conducted by Dean Marston from 1919 to 1921, in which the load on a conduit was measured, first, when the fill was constructed in the ordinary manner for a projecting conduit, and, second, when the fill was constructed by the imperfect ditch method. The loads measured in these experiments are shown in Fig. 9(*a*).

Surface Loads

In addition to external loads imposed by the filling material around and above underground conduits, these structures are also subjected to loads resulting from highway, railway, or airplane traffic or from other types of loads applied at the surface and transmitted through the soil to the underground structure. Such loads are of major importance when a conduit is placed under a trafficway with a relatively shallow covering of earth.

John H. Griffith, M. ASCE, was among the first to suggest the applicability of the Boussinesq solution[9] for the distribution of stress in a semi-infinite elastic solid to various problems of stress distribution in soils in a report prepared for a subcommittee on soils of the United States Bureau of Standards of which he was chairman. Extensive experiments[10,11] on both ditch and projecting conduits subsequent to this suggestion have shown that a concentrated surface load, such as a truck wheel, is transmitted through the soil covering to the underground structure substantially in accord with the Boussinesq solution.

[9] "Revised Report of Subcommittee on Soils," U. S. Bureau of Standards, *Proceedings*, ASCE, Vol. XLVI, 1920, pp. 916–941.

[10] "The Causes of Structural Failures of Sewers," by Anson Marston and M. G. Spangler, *Proceedings*, Am. Soc. of Municipal Engrs., Vol. 38, 1933, p. 236.

[11] "Experimental Determinations of Static and Impact Loads Transmitted to Culverts," by M. G. Spangler, Clyde Mason, and Robley Winfrey, *Bulletin No. 79*, Iowa Eng. Experiment Station, Ames, Iowa, 1926.

From these facts, the load on an underground conduit due to a concentrated surface load may be expressed as:

$$W_P = \frac{1}{L} F_i C_t P_0 \quad \ldots\ldots (11)$$

in which W_P is the average load on an underground conduit due to a concentrated surface load, in pounds per linear foot; P_0 is a concentrated surface load, in pounds; F_i is an impact factor; L is the length of the conduit, in feet; and C_t is a coefficient for load on an underground conduit due to a concentrated surface load.

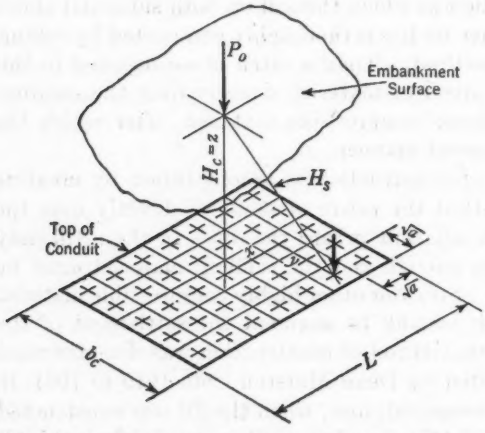


FIG. 10.—COMPUTATION OF THE CONCENTRATED SURFACE LOAD TRANSMITTED TO AN UNDERGROUND CONDUIT

The coefficient C_t may be calculated by dividing the area of the top of the conduit or its horizontal projection, $L\,b_c$, into a number of small areas as illustrated in Fig. 10, and summing up the total pressure on all these small areas, according to the formula:

$$C_t = a \sum \frac{3}{2\pi} \frac{H_c^3}{H_s^5} \quad \ldots (12a)$$

in which a is the area of an element, in square feet; H_c is the vertical height from the top of the conduit to the embankment surface, in feet; and H_s is the slant height, in feet, from the center of each element to the point of application of the load; that is: $H_s = \sqrt{x^2 + y^2 + z^2}$, in feet.

Early in 1929 D. L. Holl integrated this expression for C_t and obtained:

$$C_t = 1 - \frac{2}{\pi} \left\{ \left[\sin^{-1} H_c \sqrt{\frac{\frac{L^2}{4} + \frac{b_c^2}{4} + H_c^2}{\left(\frac{L^2}{4} + H_c^2\right)\left(\frac{b_c^2}{4} + H_c^2\right)}} \right] - \frac{\frac{L\,b_c\,H_c}{4}}{\sqrt{\frac{L^2}{4} + \frac{b_c^2}{4} + H_c^2}} \left(\frac{1}{\frac{L^2}{4} + H_c^2} + \frac{1}{\frac{b_c^2}{4} + H_c^2} \right) \right\} \quad \ldots\ldots (12b)$$

Eq. 12*b* can be evaluated[12] readily by the table prepared by N. M. Newmark,[13] M.ASCE, for determining stresses in the soil beneath uniformly loaded rectangular foundations.

[12] "A Method of Computing Live Loads Transmitted to Underground Conduits," by M. G. Spangler and Richard L. Hennessy, 1946 Annual Meeting, Highway Research Board.

[13] "Simplified Computation of Vertical Pressures in Elastic Foundations," by Nathan M. Newmark, *Circular No. 24*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1935.

Values of C_t for conduits 3 ft long of various widths, b_c, are shown[14] in Fig. 11 for various heights of fill up to 10 ft. When fill loads and surface loads are combined to obtain the design load on an underground conduit, the minimum load will be found to occur when the height of fill is relatively low over the top of the structure.

The impact factor, F_i, is equal to unity when the surface load is static. When it is moving, as in the case of truck or airplane wheels, the value of F_i may vary widely depending on speed of the vehicle, vibratory action, wing uplift, and most important, on roughness characteristics of the roadway surface. Experiments[9] have indicated design values of F_i ranging from 1.5 to 2.0 for highway traffic on an unsurfaced roadway.

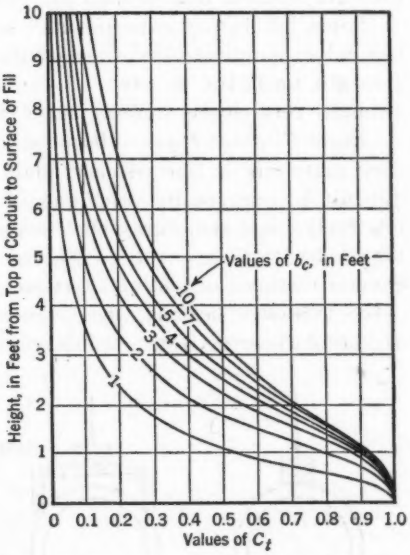


FIG. 11.—CONCENTRATED SURFACE LOAD COEFFICIENTS FOR UNDERGROUND (L = 3 Ft)

Professor Griffith gave extensive consideration to problems of the transmission of stress through soils and the applicability of the Boussinesq classical solution to them. In 1929 he developed a generalized expression[15] for the intensity of vertical pressure in a soil mass due to a concentrated surface load in the form:

$$P_z = \frac{\nu}{2\pi} P_0 \frac{\cos^{\nu+2}\theta}{z^2} \quad \ldots\ldots (13)$$

in which P_z is the intensity of pressure at a point in the soil mass; P_0 is the concentrated load applied at a point on the surface; θ is the angle formed with the vertical by the radius vector from the point of application of the surface load to the point considered; z is the vertical distance from the surface to the point; and ν is a disposable parameter, often referred to as the "concentration" factor or "dispersion" factor.

Professor Holl[16] has utilized Eq. 13 in the solution of a number of problems dealing with the transmission of surface loads through soil masses.

THE SUPPORTING STRENGTH OF UNDERGROUND CONDUITS

Underground conduits are constructed in a wide variety of shapes and of many different structural materials. In general, the load on a conduit is independent of its shape and the material of which it is made. In contrast, the supporting strength of a conduit is intimately dependent upon its shape and the kind and quality of material of which it is made. The remainder of this

[14] "The Theory of External Loads on Closed Conduits in the Light of the Latest Experiments," by Anson Marston, *Bulletin No. 96*, Iowa Eng. Experiment Station, Ames, Iowa, 1931, p. 19, Fig. 5.

[15] "Dynamics of Earth and Other Macroscopic Matter," by John H. Griffith, *Bulletin No. 117*, Iowa Eng. Experiment Station, Ames, Iowa, 1934.

[16] "Stress Transmission in Earths," by D. L. Holl, *Proceedings*, Highway Research Board, National Research Council, Vol. 20, 1940, p. 709.

paper will be devoted to a discussion of the supporting strength of a number of commonly used types of conduits.

Monolithic Arch and Box Culverts or Sewers.—These monolithic types of reinforced concrete structures may be designed satisfactorily by any current procedures for analysis of rigid frame structures. Numerous measurements of the distribution of the vertical load on both rectangular and circular shaped culverts due to the earth overburden have indicated that it is essentially uniformly distributed over the width of the conduit and may be so considered for design purposes. Culverts may or may not be subjected to active lateral pressure on all or part of their side-wall areas, depending on the local situation.

Some laboratory experiments[17] on the supporting strength of monolithic box culverts indicate that the actual supporting strength exceeds the calculated strength until the concrete of the top slab cracks, after which the actual strength very closely approximates the calculated strength.

Rigid Circular Pipes.—Rigid circular pipes, which are usually pre-cast of such materials as burned clay, plain or reinforced concrete, or cast iron, are difficult to analyze by principles of mechanics; and, since they are usually relatively small structures, their supporting strength can be most easily determined by testing a representative group of specimens in the laboratory. Several methods of testing short sections of circular pipes have been devised[18] —the two-edge bearing, the three-edge bearing, the sand bearing, and the Minnesota bearing tests—the details of which are shown in Fig. 12. Of these,

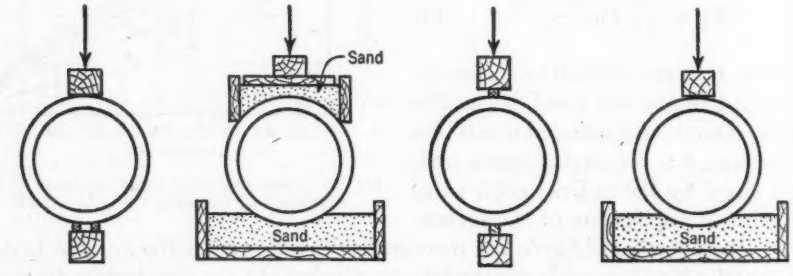


FIG. 12.—TYPICAL BEARINGS FOR LABORATORY TESTS OF PIPES

the three-edge bearing test (Fig. 12(*a*)) is the simplest and most easily performed. At the same time it gives accurate and uniform results, and for these reasons it is widely used in tests to determine the strength of pipes, although some engineers prefer the sand bearing test (Fig. 12(*b*)) because of the wider distribution of both the applied load and reaction.

The test load and reaction on the pipe are distributed differently in each of these types of tests and the ultimate load or supporting strength will like-

[17] "The Supporting Strength of Reinforced Concrete Box Culverts," by George C. Pagels, thesis presented to Iowa State College, Ames, Iowa, in 1935 in partial fulfilment of the requirements for the degree of Master of Science.

[18] "The Supporting Strength of Rigid Pipe Culverts," by M. G. Spangler, *Bulletin No. 112*, Iowa Eng. Experiment Station, Ames, Iowa, 1933, p. 13, Fig. 5.

wise be different. It is convenient to express the supporting strength of a pipe in terms of its strength when tested by the three-edged bearing method, taken as unity, by a load factor or strength ratio which is defined as the ratio of the strength of a pipe under any stated condition of loading to its three-edge bearing test strength. The following values of load factor or strength ratio for the other three test methods shown in Fig. 12 are applicable:

Test	Load factor
Sand bearing	1.5
Two-edge bearing	1.0
Minnesota bearing	1.1

Likewise when a pipe is placed and loaded in an actual field installation, the loads and reactions will be distributed much differently than in the test loading; and, in certain cases, lateral pressures will be exerted against the sides of the pipe thereby increasing its ability to support vertical loads.

A wide variety of bedding conditions affecting the load and reaction distribution and the lateral pressure situation may be encountered in sewerage, water works, and culvert construction practice; and a wide variety of supporting strengths of a given conduit may be obtained simply by varying the installation conditions. It is feasible to establish and define classifications of bedding conditions covering a range of practical attainments and determine a load factor for each classification which, when multiplied by the three-edge bearing laboratory strength of the pipe, will give the safe supporting strength for pipes installed in accordance with the definition of that classification.

Ditch Conduit Beddings

For ditch conduits used in sewerage and drainage construction, the following bedding classifications have been defined[19,20,21] and are illustrated in Fig. 13.

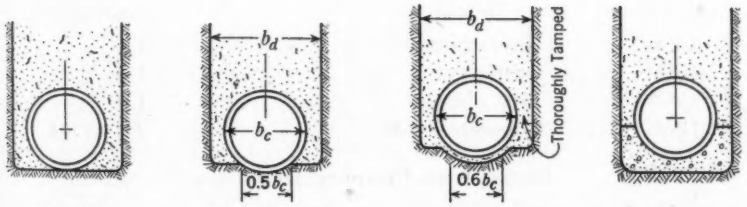


Fig. 13.—Typical Bedding Specifications for Sewers and Other Ditch Conduits

Bedding conditions for cast-iron pipe in water works construction will be discussed subsequently.

[19] "The Supporting Strength of Sewer Pipe in Ditches and Methods of Testing Sewer Pipe in Laboratories to Determine Their Ordinary Supporting Strength," by Anson Marston, W. J. Schlick, and H. F. Clemmer, *Bulletin No. 47*, Iowa Eng. Experiment Station, Ames, Iowa, 1917.

[20] "Supporting Strength of Drain Tile and Sewer Pipe Under Different Pipe-Laying Conditions," by W. J. Schlick, *Bulletin No. 57*, Iowa Eng. Experiment Station, Ames, Iowa, 1920.

[21] "Concrete Cradles for Large Pipe Conduits," by W. J. Schlick and James W. Johnson, *Bulletin No. 80*, Iowa Eng. Experiment Station, Ames, Iowa, 1926.

1. "Impermissible" bedding (Fig. 13(*a*)) is that method of bedding ditch conduits in which little or no care is exercised to shape the foundation to fit the lower part of the conduit exterior and to refill all spaces under and around the conduit.

2. "Ordinary" bedding (Fig. 13(*b*)) is that method of bedding ditch conduits in which the conduit is placed with "ordinary" care in an earth foundation shaped to fit the lower part of the conduit exterior with reasonable closeness for a width of at least 50% of the conduit breadth. The remainder of the conduit is surrounded to a height of at least 0.5 ft above its top by granular materials, shovel placed and shovel tamped to fill completely all spaces under and adjacent to the conduit—all under the general direction of a competent engineer.

3. "First-class" bedding (Fig. 13(*c*)) is that method of bedding ditch conduits in which the conduit is set on fine granular materials in an earth foundation carefully shaped to fit the lower part of the conduit exterior for a width of at least 60% of the conduit breadth. The remainder of the conduit is entirely surrounded to a height of at least 1.0 ft above its top by granular materials carefully placed by hand to fill, completely, all spaces under and adjacent to the conduit. The fill is tamped thoroughly on each side and under the conduit as far as practicable in layers not exceeding 0.5 ft thick—all under the direction of a competent engineer represented by a competent inspector constantly present during the operation.

4. "Concrete-cradle" bedding is that method of bedding conduits in which the lower part of the conduit exterior is set in plain or reinforced concrete, of suitable thickness, under the lowest part of the conduit exterior and extending upward on each side of the conduit for a greater or less proportion of its height.

The load factor for each of these bedding classifications has been determined experimentally to be:

Figure No.	Bedding	Load factor
13(*a*)	Impermissible	1.1
13(*b*)	Ordinary	1.5
13(*c*)	First class	1.9
13(*d*)	Concrete cradle	2.2 to 3.4

Projecting Conduit Beddings

In culvert construction practice, rigid pipes are often installed as projecting conduits and the fill material may exert an active lateral pressure against the sides of the pipes, which tends to increase the supporting strength of the structure. The supporting strength of a projecting conduit, therefore, is a function of both the distribution of the vertical load and the reaction on the pipe, and the magnitude and distribution of the active lateral pressure on sides which are exposed to the embankment filling material.

Because of the large number of possible combinations of reaction distributions and effective lateral pressure distributions, the experimental determination of load factors for projecting conduits has been supplemented by analytical studies of the stress situation in pipe rings under various combinations of loads

and lateral pressures.[22] By definition, the load factor is

$$F_p = \frac{R_c}{R_{eb}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14a)$$

in which F_p is the load factor; R_c is the supporting strength of the pipe in any stated loading condition; and R_{eb} is the three-edge bearing supporting strength of the pipe.

Except for the normal variations in physical properties of the material of which culvert pipes are made, it is sufficiently accurate to assume that the outer fiber stress at the point and at the time of rupture will be the same whether the pipe is loaded in a testing machine or in an actual embankment. Therefore, the thin ring elastic theory and the flexure formula may be used to express the outer fiber stress in terms of the loads on the pipe in the field and laboratory loadings; and, by equating these expressions, the load factor is readily determined. When the field loading situation is such that the pipe cracks first at the bottom, this process leads to the formula:

$$F_p = \frac{1.431}{X_P - X_A \kappa_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14b)$$

in which X_P is a parameter which is a function of the distribution of the vertical load and vertical reaction; X_A is a parameter which is a function of the area of the vertical projection of the pipe on which the active lateral pressure of the fill material acts; and κ_t is the ratio of the total lateral pressure to the total vertical load.

When the load and reaction situation causes the pipe to crack first at the top (which is usually the case when pipes are bedded in concrete cradle), X'_P and X'_A should be substituted for X_P and X_A in Eq. 14*b*. Values of the parameters X_A and X'_A for various projection ratios are as follows:

Projection ratio, p	Values of X_A	Values of X'_A
0	0	0.150
0.3	0.217	0.743
0.5	0.423	0.856
0.7	0.549	0.811
0.9	0.655	0.678
1.0	0.638	0.638

As in the case of ditch conduits, it is convenient to name and define several classes of bedding conditions for projecting conduits and to determine a value of X_P or X'_P for each class. These classes of bedding are defined as follows:[23]

1. "Impermissible projection bedding" is that method of bedding projecting conduits in which little or no care is exercised either to shape the foundation surface to fit the lower part of the conduit exterior or to fill all spaces under and around the conduit with granular materials. This type of bedding also includes the case of conduits on rock foundations in which an earth cushion is

[22] "The Supporting Strength of Rigid Pipe Culverts," by M. G. Spangler, *Bulletin No. 112*, Iowa Eng. Experiment Station, Ames, Iowa, 1933.

[23] *Ibid.*, p. 15, Fig. 6.

provided under the conduit, but is so shallow that the conduit, as it settles under the influence of vertical load, approaches contact with the rock.

2. "Ordinary projection bedding" is that method of bedding projecting conduits under embankments in which the conduit is bedded with "ordinary" care in an earth foundation shaped to fit the lower part of the conduit exterior with reasonable closeness for at least 10% of its over-all height. The remainder of the conduit is surrounded by granular materials, placed by shovel to fill all spaces completely under and adjacent to the conduit—all under the general direction of a competent engineer. In the case of rock foundations, the pipes are bedded on an earth cushion, having a thickness under the pipe of not less than 0.5 in. per foot of height of fill, with a minimum allowable thickness of 8 in.

3. "First-class projection bedding" is that method of bedding projecting conduits, having a projection ratio not greater than 0.70, in which the conduit is carefully bedded on fine granular materials in an earth foundation carefully shaped to fit the lower part of the conduit exterior for at least 10% of its over-all height. The earth filling material is thoroughly rammed and tamped, in layers not more than 6 in. deep, around the conduit for the remainder of the lower 30% of its height—all under the direction of a competent engineer, represented by a competent inspector constantly present during the operation. In the case of rock foundations, the pipes are bedded in an earth cushion having a depth as provided under ordinary projection bedding.

4. "Concrete-cradle projection bedding" is that method of bedding conduits in which the lower part of the conduit exterior is bedded in a cradle, constructed of 2,000-lb concrete, or better, having a minimum thickness under the pipe of one fourth of the nominal internal diameter and extending up the sides of the pipe for a height equal to one fourth of the outside diameter. The values of X_P for these bedding classes are as follows:

Type of projection bedding	Value of X_P
Impermissible	1.310
Ordinary	0.840
First class	0.707
Concrete cradle (X'_P)	0.505

Working values of the load factor F_P, computed for rigid culvert pipes installed as projecting conduits on ordinary, first-class, and concrete-cradle beddings, with various projection ratios and a value of the settlement ratio of 0.7 are shown in Fig. 14. The general correctness of the foregoing analytical method of determining load factors has been established by a series of experiments in which ten pipe culverts of four kinds of material—plain concrete, reinforced concrete, burned clay, and cast iron—were loaded by earth embankments and their supporting strengths determined. Then samples of pipes representative of each of the ten culverts were tested in the laboratory by the three-edge bearing test and the ratios of the field and laboratory strengths were obtained.[24]

[24] "The Supporting Strength of Rigid Pipe Culverts," by M. G. Spangler, *Bulletin No. 112*, Iowa Eng. Experiment Station, Ames, Iowa, 1933, p. 53.

Concrete-Incased Clay Pipe

Circumstances may arise under which an engineer may wish to increase the strength of clay pipes by incasing them in concrete. Some extensive experiments[25] in which commercial vitrified salt-glazed clay pipe was incased in plain concrete and the combined structure tested in both sand and three-edge bearings have indicated that the test strength is only slightly greater than the sum of the individual supporting strengths of the clay pipe and of the incasement. The action of the incased pipe was very nearly that of two independent but concentric rings—the supporting strength of the combined structure being very much less than if the two materials had acted as a unit in resisting the stresses due to the test load. As a result of these experiments, computations have been made of the three-edge bearing supporting strength of A.S.T.M. (American Society for Testing Materials) standard sewer pipe when incased in concrete of various thicknesses. The results of these computations are shown in Fig. 15.

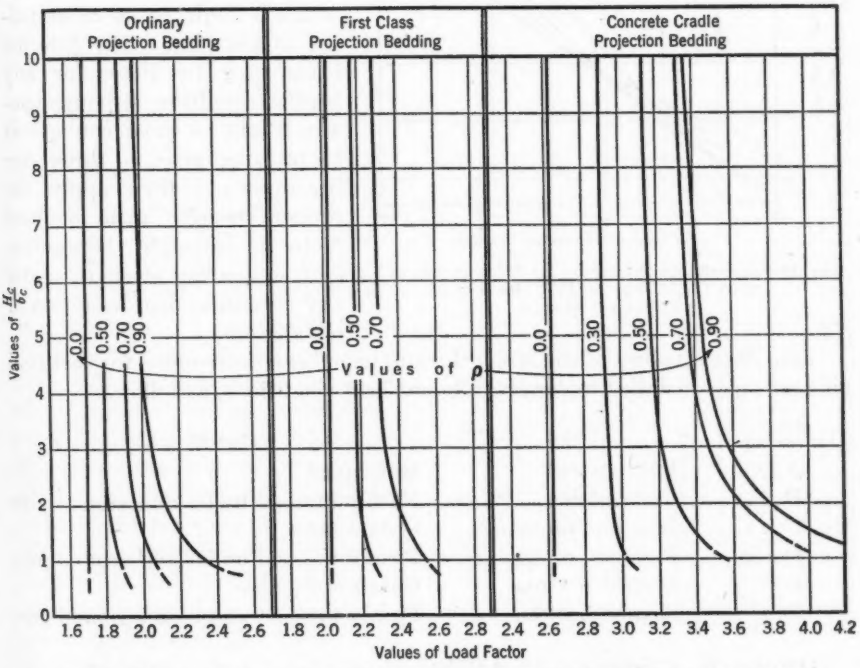


Fig. 14.—Working Values of the Load Factor for Projecting Conduits ($\delta = 0.70$)

The Supporting Strength of Cast-Iron Pipe When Subjected to Internal Pressure

When used for the transportation of water, gas, or other fluids, cast-iron pipe conduits not only are subjected to the ring stresses produced by the

[25] "Supporting Strength of Concrete-Incased Clay Pipe," by W. J. Schlick, *Bulletin No. 93*, Iowa Eng. Experiment Station, Ames, Iowa, 1929.

external loads due to backfilling and surface loads, but in addition must resist the stresses produced by the internal fluid pressure as well. Since 1939 completed studies[26, 27, 28] on the supporting strength of this kind of pipe under combined loads and pressures indicated an approximately parabolic relationship between the magnitude of the external load and the internal pressure which is required to cause failure of the pipe. The external loads may be those for any loading condition, although usually it will be most convenient to test the pipe in three-edge bearings and then apply the proper strength ratio or load factor to calculate the equivalent supporting strength under any particular field condition of installation.

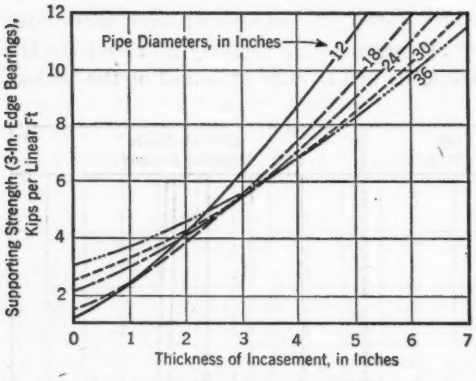


FIG. 15.—SUPPORTING STRENGTHS (AT 70° F) OF STANDARD A.S.T.M. SEWER PIPE, INCASED IN 3,000-LB CONCERTE

Safe design values of the strength ratio have been determined for cast-iron pit-cast pipes installed under six different field conditions, as follows:

Condition	Trench	Backfill
A	Flat bottom	Untamped
B	Flat bottom	Firmly tamped under and around pipe
C	Pipe laid on blocks	Untamped
D	Pipe laid on blocks	Firmly tamped under and around pipe
E	Shaped bottom	Untamped
F	Shaped bottom	Firmly tamped under and around pipe

Quoting W. J. Schlick,[29] M. ASCE:

> "The safe design values of the strength ratio for these field conditions are shown for nominal pipe diameters from 4" to 60" [in Fig. 16]. The values shown in this figure have been adopted by Sectional Committee A21 of the American Standards Association, with whom this research work was conducted on a cooperative basis. They are based directly upon the experimental results although those adopted for conditions B and F have been reduced from the experimental values in the belief that the tamped earth beddings in the laboratory tests were more carefully made and therefore more effective than is safe to assume for field conditions."

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[26] "A Proposed New Method for Determining Barrel Thicknesses of Cast Iron Pipe," by T. H. Wiggin, M. L. Enger, and W. J. Schlick, *Journal*, A.W.W.A., May, 1939, p. 841.

[27] "American Recommended Practice Manual for Computation of Strength and Thickness of Cast Iron Pipe" (A21.1), A.S.A., 1939.

[28] "Supporting Strength of Cast-Iron Pipe for Water and Gas Service," by W. J. Schlick, *Bulletin. No. 146*, Iowa Eng. Experiment Station, Ames, Iowa, 1940.

[29] *Ibid.*, p. 8, Fig. 2.

Flexible Pipes

In general, it may be stated that underground conduits derive their capacity to support the earth above them from two sources: First, the inherent strength of the pipe to resist external pressures; and, second, the lateral pressure of the soil at the sides of the pipe. In rigid pipes such as those made of concrete, cast iron, and burned clay, the inherent strength of the pipe is the predominant source of supporting strength.

In flexible pipes such as corrugated metal culverts and thin-walled steel conduits, the situation is reversed. The pipe itself has relatively little inherent strength, and a large part of its capability of supporting vertical load must be derived from the passive pressures induced as the sides move outward against the soil.

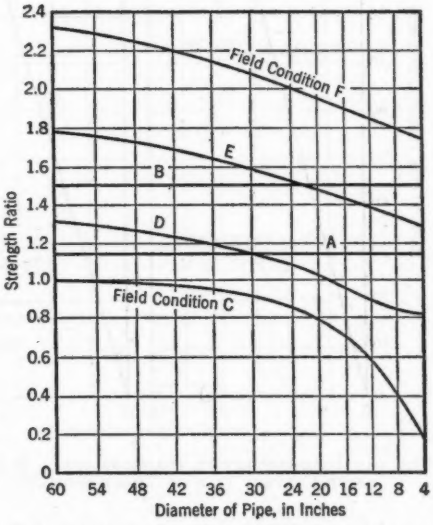

Fig. 16.—Working Values of Strength Ratio for Cast-Iron Pit-Cast Pipe in Six Field Conditions

Another major difference between the rigid types of conduits and the flexible types is that the latter fail by deflection rather than by rupture of the pipe walls, as do the former. The action of a flexible pipe under embankment loading is one of large deflection change unaccompanied by rupture of the pipe wall; and, for this reason, the design of the flexible types of conduits should be based upon the deflection of the ring rather than upon stress in the side walls as in the case of the rigid types. This statement is predicated on the assumption that the longitudinal joints in the flexible pipe are sufficiently strong to develop the pipe wall as a continuous ring.

Laboratory tests[30] of short sections of corrugated metal culvert pipes of various diameters and gages of metal have indicated that the elastic theory of flexure as applied to thin rings by Claude W. L. Filkins and Edwin J. Fort,[31] M. ASCE, is applicable to these flexible conduits, even if the deflections and changes in radius of the elastic ring are relatively very large. Therefore, if the loads and pressures acting on a flexible pipe are known or can be assumed confidently, the deflection of the pipe can be determined by this theory within the elastic limit of the pipe material. Typical results of laboratory measurements of diameter changes under three-edge loading conditions are shown in Fig. 17.

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[30] "The Structural Design of Flexible Pipe Culverts," by M. G. Spangler, *Bulletin No. 153*, Iowa Eng. Experiment Station, Ames, Iowa, 1941.

[31] "Stresses and Deflections in Circular Rings Under Various Conditions of Loading," by Claude W. L. Filkins and Edwin J. Fort, *Transactions*, Assn. of Civ. Engrs. of Cornell Univ., Ithaca, N. Y., Vol. IV, 1896, pp. 99–112.

A number of field loading experiments on corrugated metal pipe culverts, in which the vertical and lateral pressures and the deflections of the pipes were measured have led to the following conclusions regarding soil pressures and deflections of flexible conduits:

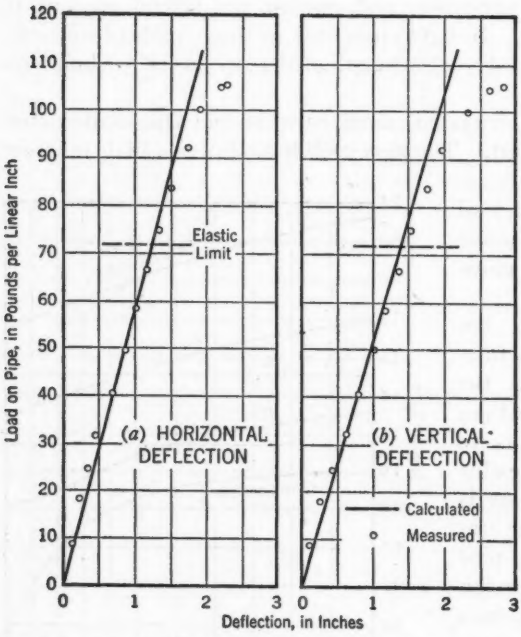

FIG. 17.—LOAD-DEFLECTION DIAGRAMS FOR A 42-IN. CORRUGATED CULVERT PIPE ON THREE-EDGE BEARINGS

1. The vertical load may be determined by Dean Marston's theory of loads on conduits; it is distributed approximately uniformly over the breadth of the pipe.

2. The vertical reaction is equal to the vertical load and is distributed approximately uniformly over the width of bedding of the pipe.

3. The horizontal pressure on each side of the pipe is distributed parabolically over the middle of 100° of the pipe, and the maximum unit pressure (which occurs at the ends of the horizontal diameter of the pipe) is equal to the modulus of passive resistance of the fill material multiplied by one half of the horizontal deflection of the pipe.

4. The pipe may continue to deform for some period of time after the maximum fill load has developed, due to continuing deformation of the side fills under lateral pressure. The assumed loading is shown graphically[32] in Fig. 18. The expression for horizontal deflection resulting from these hypotheses is:

$$\Delta = F_d \frac{F_k W_c r^3}{E I + 0.061 \epsilon r^4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which Δ is the horizontal deflection of the pipe, in inches; F_d is the deflection lag factor; F_k is a bedding constant, depending upon the bedding angle; W_c is the vertical load per unit length of the pipe, in pounds per inch; r is the mean radius of the pipe, in inches; E is the modulus of elasticity of the pipe material, in inches; I is the moment of inertia per unit length of cross section of pipe wall, in inches^4 per inch; and ϵ is the modulus of passive resistance of the enveloping soil, in pounds per square inch per inch.

[32] "The Structural Design of Flexible Pipe Culverts," by M. G. Spangler, *Bulletin No. 163*, Iowa Eng. Experiment Station, Ames, Iowa, 1941, p. 26, Fig. 17.

Values of the bedding constant, F_k, for various values of the bedding angle are:

Degrees	Bedding constant, F_k
0	0.110
15	0.108
22.5	0.105
30	0.102
45	0.096
60	0.090
90	0.083

The modulus of passive resistance of the side filling material is defined as the unit pressure developed as the side of a pipe moves outward a unit distance against the side fill. Little is known concerning the nature of this modulus. In the Rankine theory of lateral soil pressures, the limiting value of the ratio of passive horizontal pressure to vertical pressure which a granular soil without

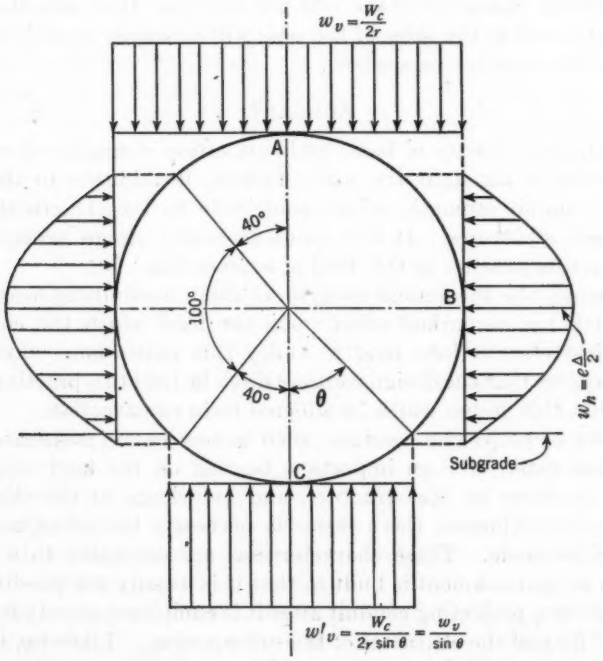

FIG. 18.—THE DISTRIBUTION OF PRESSURES AROUND A FLEXIBLE PIPE UNDER AN EARTH FILL

cohesion can develop is shown to be the reciprocal of the active pressure ratio. However, this theory does not give a clue as to the amount of movement required to develop the limiting value of passive pressure, and it would seem that the actual passive pressure may be any value less than the maximum, depending on soil characteristics and amount of movement of the sides of the pipe.

Several facts concerning the value of the modulus of passive resistance of side fills adjacent to experimental flexible culvert pipes have been observed. First, this modulus varies widely with soil characteristics. Measurements have indicated a value for a gravel material approximately three times as great as for a loam material. Also in the case of a silty clay soil material, the modulus of passive resistance was approximately doubled by hand tamping as compared to the value for the same material in a relatively loose, shovel-placed condition. The second fact that has been observed is that when a flexible pipe is installed without struts the horizontal movement of the sides of the pipe and the passive resistance pressure at the sides of the pipe increase in a linear relationship with the height of fill, indicating a constant value of the modulus of passive resistance pressure as the height of fill is built up.

Field observations of several flexible pipe culverts in service, completed in 1946, have indicated that the settlement ratio, the modulus of passive pressure, and the deflection lag factor are all intimately interrelated. These studies also indicate that the structural performance of this type of conduit is greatly influenced by the character of the side fills and that time and effort spent in compacting the soil at the sides of the pipe will be amply repaid in better performance of the structure as a whole.

Summary

Dean Marston's theory of loads and supporting strengths of underground conduits serves to highlight the many factors, in addition to the height of fill and the conduit strength, which contribute to the structural success or failure of such structures. It is a guide to sound design procedure and to good construction practice in this field of engineering.

For example, the theoretical analysis of ditch conduits indicates that the width of ditch has a marked effect upon the load which the conduit must carry and field observations tend to verify this indication. Good practice, therefore, requires that the design width of ditch be held to a practical minimum value and that this design width be adhered to in construction.

In the case of projecting conduits, such factors as the projection ratio and the settlement ratio have an important bearing on the load which the soil overburden produces on the structure, and knowledge of the characteristics of the site which influence these ratios is necessary before an accurate load analysis can be made. These characteristics are concealed to a very great extent when an embankment is built so that it is usually not possible to determine the load on a projecting conduit after it is completed merely by observing the height of fill and the character of the soil covering. Likewise, the bedding characteristics of a conduit, and the conditions governing the development of lateral earth pressures on the structure, both of which influence its field supporting strength, cannot be determined readily after construction is completed. It is not feasible, therefore, to base one's judgment of the safe height of fill which can be constructed over a given type of conduit in one location, on the height of fill which the same type of conduit may be successfully supporting in another location, unless it is known that all factors which influence load and supporting strength are the same in both locations.

APPENDIX. NOTATION

The following letter symbols, adopted for use in this paper and in its discussion, conform essentially with ASCE Manual of Engineering Practice No. 22, "Soil Mechanics Nomenclature":

a = an area element (see Fig. 10);
b = horizontal breadth or width, with subscripts denoting:
  c = conduit;
  d = width of ditch at top of conduit;
C = load coefficient, with subscripts denoting:
  c = projecting conduits;
  d = ditch conduits;
  t = load on the underground conduit transmitted by concentrated load on the surface;
E = modulus of elasticity of the pipe material; Young's modulus;
E_f = modulus of deformation of the filling material;
e = base of natural logarithms;
F = factor, with subscripts denoting:
  d = deflection lag;
  i = impact;
  k = bedding constant;
  p = load;
H = height of fill, with subscripts denoting:
  c = height to ground surface above top of conduit;
  e = height to the plane of equal settlement from the top of a pipe; this is the "height of equal settlement";
  p = height to top of backfill from any horizontal plane;
  s = slant height from the center of an elementary area in Fig. 10 to the point of application of a load;
I = moment of inertia per unit length of cross section of pipe wall;
K = ratio of lateral pressure at a point to vertical pressure; hydrostatic pressure ratio;
L = longitudinal length of the conduit;
P = total vertical pressure on a horizontal plane in the backfill, or within the interior prism:
  P_0 = the pressure of a concentrated load at the surface;
  P_z = total at any point distant z from the surface;
R = supporting strength or resistance of a pipe, with subscripts denoting:
  c = strength in any stated loading condition;
  eb = strength for the case of three-edge bearing;
r = mean radius of the pipe;
W = weight or total load, with subscripts denoting:
  c = load on an underground conduit;
  P = load transmitted by a concentrated surface force;
w = load per unit area, with subscripts denoting:

h = horizontal unit load;
v = vertical unit load;
X = a parameter symbol, denoting by subscripts a function of:
A = the area of the vertical projection of the pipe on which the active lateral pressure of the fill material acts;
P = the distribution of the vertical load and vertical reaction;
When the load and reaction situation (Eq. 14*b*) causes the pipe to crack first at the top the parameter symbol is primed, thus: X';
y = vertical deflection or vertical settlement, corresponding to Δ = horizontal deflection, the subscripts denoting:
c = deflection of the conduit or the shortening of its vertical dimension;
f = settlement of the flow line of the conduit;
g = settlement of embankment subgrade adjacent to the conduit;
m = deformation of the filling material adjacent to the conduit within the height $\rho\, b_c$;
z = distances along the z-axis, also vertical distance from the surface to a given point;
α = a substitution symbol involving b_d introduced to simplify typography (see Eq. 2*a*);
β = a substitution symbol involving b_c introduced to simplify typography (see Eq. 5*c*);
γ = unit weight of filling material;
Δ = horizontal deflection;
δ = a settlement ratio defined by Eq. 4*a*;
ϵ = modulus of passive resistance of the enveloping soil;
θ = angle formed with the vertical by the radius vector from the point of application of the surface load to the point considered (Eq. 13), also the angle to the intersection of a pipe trench with the ground surface (Fig. 18);
κ_t = ratio of total lateral pressure to total vertical load;
μ = coefficient of internal friction between fill materials; μ' = coefficient of sliding friction between the fill materials and the sides of the ditch;
ν = concentration factor or dispersion factor; a disposable parameter; and
ρ = projection ratio; ratio of the vertical distance between the top of the conduit and the natural ground surface adjacent to the conduit, to b_c; the conduit projection is $\rho\, b_c$.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SOME THOUGHTS ON ENGINEERING EDUCATION

Discussion

BY DONALD M. BAKER

DONALD M. BAKER,[46] M. ASCE.—In reading the discussions of this paper, one is reminded of the old Hindu story about the blind men who were taken to visit an elephant, and afterward asked to describe the animal. The man who felt the elephant's legs stated that an elephant was like a tree, the one who had felt his trunk described the animal as being like a huge snake, while the one who had felt his tail likened him to a thick rope.

Each discusser—practicing engineer, educator, or layman—discusses engineers, the practice of engineering, and engineering education from his contacts with them. Some have been wide and varied, have involved large numbers of engineers practicing many specialties in varied locations, while others, although extending over considerable periods of time, have been with limited groups and sometimes within limited areas.

Most of the engineering educators appear to have had in mind recent graduates, seeking employment following graduation or during the early years of their practice, or possibly to be thinking of the employers endeavoring to recruit young graduates to fill subprofessional or preprofessional positions in their organizations. Professor Brinker talks about maintaining student interest in courses and about preparing civil engineering graduates so that, following commencement, they may secure positions as instrumentmen rather than as rodmen, as draftsmen rather than as tracers. Professor Carpenter feels, and rightly so, that employers of young graduates should be convinced of the value of viewing such graduates as professionally trained men, and of affording them an opportunity to rise from subprofessional to professional status.

On the other hand, practicing engineers and others seem to have in mind the engineer in his more mature years, when he is practicing as a specialist or as a

NOTE.—This paper by Donald M. Baker was published in April, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1946, by E. S. Boalich, Russell C. Brinker, I. Oesterblom, Samuel T. Carpenter, Lynn Perry, and L. E. Grinter; October, 1946, by Robert O. Thomas, Clement C. Williams, N. W. Dougherty, H. A. Wagner, M. E. McIver, and Scott B. Lilly; December, 1946, by William A. Conwell, and A. Amirikian; February, 1947, by C. A. Dykstra, and Louis Balog; and May, 1947, by Henry B. Lynch, and Alfred R. Golzé.

[46] Partner, Ruscardon Engrs., Los Angeles, Calif.

"routineer," an engineering executive or administrator, a business man, or merely as a contented and worthwhile citizen.

Mr. Balog, when he sees the "highly educated designer" as a true representative of the civil engineering profession, apparently has in mind those young South American structural designers—twenty-five to thirty-five years old—who, he states, have developed a reinforced concrete design practice far superior to that of the level achieved in this country. In this assertion he ignores the efforts of those civil engineers, young and old, whose endeavors, in many instances involving more or less routine design, have resulted in many huge dams; water supply, flood control, irrigation, and power projects; highways; railroads; airfields; and port facilities; that in turn have done so much to establish the high standard of living prevalent in the United States. Perhaps Mr. Balog has in mind a definition of "design" which involves merely the use of formulas and charts to determine the dimensions of various members of a structure. Should not this word "design" include the conception, planning, arrangement, location, dimensions, use, and general economics of a structure or utility.

Mr. Wagner and Miss McIver, from their long and intimate experience with engineers of all ages, kinds, and capacities, and with large numbers of employers of engineers, have probably achieved a broader and deeper contact with these than is the case with most of the other discussers. It is significant, in view of this, that Mr. Wagner poses these extremely pertinent questions: "What is engineering?" "What is the engineering profession?" and "What is its scope?" When the answers to these questions are known with a reasonable degree of certainty, engineering societies and engineering educators will have a far sounder basis for their activities than they now have.

The writer throughout his paper and this discussion has referred to engineering as a profession, although for some considerable time there has been a question in his mind as to whether the practice of engineering involves the practice of a profession, or whether it involves a business in which a professional man is engaged. No attempt is made herein to answer this question; it is one which merits serious consideration by engineers, engineering societies, and engineering educators.

The statistical material used in this paper to present the composition and structure of the engineering profession, and the conditions that govern and prevail in actual practice, embrace, on a nationwide scale, a very large and representative segment of the engineering profession. Where derived data are based on sampling, the samples were of a size adequate to produce results of a quality sufficient to meet the purposes of this paper. Conclusions reached were substantially in agreement with independent studies made by Mr. Thomas and Miss McIver. This does not mean, however, that further work of this character is unnecessary. On the other hand, it is urgently needed, and the author would be the first to admit the inadequacy of his basic data. He submits, however, that the statistical material presented all points in the same direction, and, furthermore, conclusions reached coincide with experience.

National engineering societies during recent years have given far more attention to engineers, as against engineering, than was given in the past. This

may be due, in part, to the large additions of younger men to their membership and the demand by these younger engineers for a display of such interest. Activity along these lines can be expected to continue to increase, if for no other reason than that changed conditions growing out of wartime employment will require it. These activities, however, cannot be carried on with the highest degree of effectiveness until more knowledge is obtained concerning the answers to the three questions posed by Mr. Wagner. An expenditure by national engineering societies of the order of $500,000 in this field over a five-year period would yield handsome returns to their members, to industry, and to the nation at large.

The paper, although by its title pertaining to the broad field of engineering education, was limited primarily to a discussion of the engineering profession, the practice of engineering, educational curricula, and course content. The writer also made recommendations as to modification or changes in these last two for the purpose of producing young engineers who, as time goes on, can more adequately meet the demand for engineering services in the postwar market. Had the paper fully covered the subject, it should have included a discussion of engineering instructors, engineering instruction, and the problem, strongly and properly emphasized by Mr. Boalich, of how better to control the admission to engineering courses of students who possess those temperamental and other personal characteristics which fit them to follow engineering and to be content in its practice.

Mr. Lynch suggests that preparation for collegiate training in engineering should commence in high school. This suggestion has a great deal of merit, although it probably will not be adopted until such time as engineering schools, societies, and state registration boards have come to work more closely together toward a common objective. Although there still exists a widespread feeling among engineers that professional status may be obtained without a formal engineering education—due probably to the still large number of men claiming such status without holding a degree—the next generation of engineers will have a far smaller percentage of men within its ranks who have not had a formal engineering education. This situation has already occurred in medicine, dentistry, accounting, and law. The young man who makes up his mind in high school to follow professional engineering and begins then to shape his course and plans for the "long pull" will be the one who attains professional status in the highest sense of the words.

A number of discussers refer to the desirability of developing imagination in students. Mr. Oesterblom talks about the "creative urge," and very rightly decries the tendency to "train a flexible mind into a rigid form." Imagination and vision are personal traits. Some persons possess them in a greater degree than do others, but even in those who inherently lack them, they can be cultivated by inspirational instruction. This latter is extremely difficult to secure; and only too often those instructors who possess the ability to give it either leave teaching or are advanced rapidly to positions on the faculty where their contacts are with older students.

Professor Lilly has pointed out the crux of this problem in his statement, "* * * quality in the student must come from quality in the faculty."

It is customary in developing faculties in engineering, as well as in other schools of collegiate grade, to assign as instructors in freshman subjects recent graduates, if possible men who have achieved scholastic honors. If and when these younger instructors attain experience and seniority, they are assigned to sophomore courses and, ultimately, with greater seniority and possibly with increasing reputation derived from research and writing, may arrive at teaching junior and senior courses.

Freshman and sophomore students from eighteen to twenty years old are passing through the most impressionable and idealistic period of their adult life. Seldom do they know just why they enrolled in an engineering course, although many of them may think they do. During these formative first two years, their contact is almost entirely with immature and inexperienced instructors; and it is upon such instructors that the responsibility largely rests of providing inspired instruction at the time when it is most needed and most effective. Even after the student has reached his junior year, his contacts are seldom with instructors who have had experience in the practice of engineering in a responsible capacity and who are familiar with problems commonly met in engineering practice. With only such contacts as described, how is the student, during the first two years, to determine whether he is fitted to, or will want to follow, engineering as a career and be happy in so doing.

Perhaps the lack of student interest during the first two years of the engineering course referred to by Professor Brinker and others is due more to uninteresting instructors and unimpressive instruction than to course content. Inspirational instruction is as much a matter of the personality of the instructor as it is of his reputation in or knowledge of his subject. It is also more a matter of the manner in which he presents his subject than the content of his lectures. The writer recalls his course in descriptive geometry, a subject usually disliked by most students. The instructor had never taught the course before, and had but a casual knowledge of the subject. He managed, however, to keep several pages ahead of the class in text, so they did not discover the shallowness of his knowledge. He had an engaging personality, and managed so to arouse the interest of his students in the subject and in himself that every student acquired a deep interest in the course, and a thorough knowledge of this normally dry and uninteresting, although valuable, subject. He also recalls a course in hydraulic machinery given by an instructor who was an outstanding authority, a man who had done much pioneering work in the design and development of pumping machinery and turbines. This instructor, however, droned through his lecture and filled the blackboard with formulas and mathematical developments of theory. Few in this class acquired much knowledge of the subject by the end of the course.

Were it possible to develop a faculty that had as one fourth to one third of its members engineers who had from fifteen to twenty years of experience in the practice of engineering, at least one half of which time was in a fairly responsible capacity, and if freshman and sophomore students were provided the opportunity to sit for three or four hours per week under some of these instructors, and under some of the older instructors who had achieved eminence in engineering—men to whom they could look with respect and even awe—

this lack of interest would not exist. Furthermore, if freshman and sophomore students were told—and they should be told repeatedly—just how their course had been developed, why each subject had been included therein, the interrelationship between all subjects in the course, and the relationship of the structure and pattern of the course to their future practice, they might within this two-year period gain a picture of what they were doing, why they were doing it, and the direction in which they were headed. In addition, it is believed that when the deans or department heads commenced to justify to themselves the courses given, prior to presenting such justification to the younger students, the conclusion might be reached that some of these courses could not be justified and they might be eliminated. It might also be discovered that other courses could only be justified because Professor Soandso, who has been giving a certain course for years, objected to teaching any other subject.

Mr. Perry states that the need of salesmanship by every man is but a partly true statement. A number of discussers decry the need of salesmanship by engineers, and question whether it can be taught. Salesmanship involves a knowledge of the product sold, of people, and the faculty of clear and logical expression, verbal and written. One never completes his education in these subjects any more than he does in engineering subjects, but the student is entitled to be told, with emphasis, of the importance to him of salesmanship. He should, at a minimum, be able to make out an effective application for employment, although only one graduate out of ten can do this. He should be able to write a clear and effective one-page letter, and to meet a prospective employer without stammering and stuttering. Few graduates can do these things. Criticism, by instructors, of the manner in which students recite and of the quality of expression found in their papers and reports will often prove as effective as formal courses in literature and expression.

The writer believes it requires salesmanship for a graduate to secure his first engagement, and that salesmanship is required in order for him to advance to higher positions. He has known and worked with many so-called financially successful and eminent engineers, and with many engineers who were far less successful financially and far less eminent, but who possessed equal ability and capacity. Too often it has been found that the only difference between men in the two groups is the amount of salesmanship practiced by the more financially succesful and more eminent engineers.

It is not so readily possible to determine the quality of engineering service as it is the quality of other professional service. A doctor loses or cures a patient. A lawyer wins or loses a case. Their successes or failures are soon known, and as these accumulate their professional reputation or standing is enhanced or disappears. Floods that wash out dams, earthquakes that destroy buildings, loadings that cause bridges to fail may not occur until years after the death of the engineer responsible for their design or construction. One engineer may develop an ore deposit in one manner and another develop it differently. In either case, the success or failure of the mine is usually credited to the character of the deposit, or to the market, not to the engineer. One engineer may plan, design, and construct a water supply project costing millions of dollars. This aids in making him successful and eminent, even though twenty

years after the project has been completed, it is found to have been greatly overdeveloped, and cannot meet interest on its bonds. Another engineer may develop a much smaller project costing thousands instead of millions, but which is very successful and able to pay off its bonds before maturity. This latter engineer is seldon heard of unless he possesses the quality of salesmanship.

It is admitted that engineers cannot advertise the value and worth of their services in the same manner as do makers of soap and laxatives, but the "better mouse trap" theory became obsolete with the advent of newspaper and periodical advertising. Some engineers have jobs seek them; but the writer is familiar with too many engineers and engineering organizations, public and private, large and small, who have their salesmen or contactmen, sometimes dignified by the title of public relations counsel, to believe that work well done is the only requirement for securing more and better work to perform. Engineers write papers and publish books for the purpose of giving the profession the benefit of their experience and knowledge, or for the purpose of advertising themselves, but usually for both purposes. They join organizations, civic, religious, fraternal, social, and professional, to make contacts. The better an engineer is known the greater the opportunity he will have to perform more interesting and more important work; and he can become known more quickly and effectively if he tells the world about himself in a dignified and ethical manner. Too many instructors fail to realize that it is a wasted effort to teach a student how to design a structure, a piece of machinery or equipment, if the student cannot convince a prospective employer or client that he is capable of doing such a job and, therefore, should be given an opportunity to do it.

There appears to be, on the part of most of the engineering educators who discussed this paper, a mistaken view of the writer's proposal that the current four-year engineering course be limited to fundamentals in the broad field of engineering, and broadened to include subjects in business, the humanities, and expression. This proposal constituted only a part of the program suggested, and was urged in view of prevailing opinion and experience over several decades that few engineering students will, for economic and other reasons, spend more than four years in residence at any engineering school to secure an engineering education. Attention is called to the statements under the heading, "Postwar Engineering Education," that the postwar market for engineering services can be expected to require:

> "(1) A longer period of study and training for engineers * * *; (2) the broader knowledge of a wide range of both technical and nontechnical subjects necessary to provide maturity of judgment; and (3) the greater demand * * * for executives and administrators who have had background and experience in engineering."

The requirements set forth under items (2) and (3) are pointedly emphasized in a survey released[47] in May 1947, which covered personnel placement bureaus in one hundred and one colleges and universities in the United States. This study finds a notable trend on the part of employers toward seeking

[47] "Twelfth Annual Survey of College Placement Experience," Northwestern National Life Insurance Co., Minneapolis, Minn., 1947.

college graduates with engineering degrees for positions in sales work, office administration, and other nontechnical lines. It also finds that employers, except when seeking men for research and design activities, lay far more stress upon a graduate's ability to "get on" with people, his personality, and his extracurricular activities while in school, than they do upon his scholastic record. It also states that most large concerns that have instituted training programs for entering employees who are graduates give little weight to any specialization which a graduate may have undertaken while in college, preferring to train these graduates themselves in specialized subjects. These findings confirm those in the study made by the American Association of Engineers in 1940, which were quoted by the writer.

To meet these requirements, a four-year course as outlined was suggested, to be followed by an opportunity for postgraduate study, either in residence or through extension or correspondence, in specialties or in nontechnical subjects, with the goal of a professional degree when a suitable amount of such study had been satisfactorily completed. These two items are the keys to the program suggested in the paper, and must be considered as being inseparable.

To effect such a program would not call for anything revolutionary. It would mean harder work for instructors during the time necessary to adapt existing courses to new conditions, and to develop new courses. It would also call for some education of, and cooperation and understanding by, personnel directors and employers in the realization that the graduates of the four-year courses they would then be employing would not be so highly trained in certain specialties, but would have a broader training in fundamentals, could be more flexibly used, and could be expected to secure their specialized training during the early years of their employment. A few years of experience with graduates of this type should soon convince employers of their increased value. The greatest burden would fall upon the graduate working for his professional degree at the same time he was earning a living, and also upon his wife and family if he acquires these during such period. These should realize, however, that the sacrifices called for will be compensated later.

The discussion by Mr. Wagner of the composition and structure of the engineering profession, and the information which he presents pertaining to German technical education prior to World War I are of extreme interest. Fig. 1 was presented to illustrate the statement that the practice of engineering is a mass activity—not mass production as mistakenly inferred by Mr. Balog—and that any product created by engineers is the work of many persons with varying kinds and degrees of technical training, experience, and ability. Mr. Wagner states that if engineering is considered as being pyramidal in shape, as against the cubical shape of other professions and trades, a large number of engineering graduates, regardless of their qualities or abilities, are doomed to remain in positions in the middle or lower sections of the pyramid, because there is not room for them at the top. This fact is true of any group activity of this character. An army is composed of a few generals, a substantially larger number of captains and lieutenants, a still larger number of noncommissioned officers, and a great number of privates. A private may advance to be a general, but only if he has outstanding ability and undergoes arduous training. All privates and

sergeants receive specialized training, but not that of a character provided for potential generals.

It may be, as Mr. Wagner states, that other forces and organizations will solve the problem of fitting engineering education to conditions prevailing in practice, without the aid of engineering schools or societies, and that this may develop a pattern somewhat similar to that existing in Germany before World War I, adapted, however, to American conditions. This does not remove from engineering schools or societies the responsibility of endeavoring to solve the problem. Naturally, however, until far more information is available as to the number of engineers required to perform technical duties in the various branches of engineering in each functional bracket (from nonprofessional up through subprofessional, and preprofessional, to professional), and the number of so-called "routineers" among the professional workers, and until more information is available upon which opinion can be developed as to future trends in these numbers, little more than an approach can be made to fitting the general pattern of engineering education to the requirements of the future market.

Adoption of the educational program suggested in this paper would undoubtedly decrease the number of those who embark upon an engineering course with the expectation of achieving full professional status, and would increase the number of those who will be satisfied to seek an education which will qualify them to perform subprofessional or routine professional duties. Mr. Perry questions the wisdom of decreasing the number of professional engineers without, at the same time, making plans to supply adequately the demands for a subprofessional group. He is correct in this view.

In medicine, as Mr. Wagner states, those who do not expect to achieve professional status are only required to follow an educational program equipping them for subprofessional work. Laboratory technicians, nurses, and pharmacists are not secured from the ranks of those who embark upon, but for some reason fail to complete, the entire medical course, or who, having secured their medical degree, lack certain qualities or characteristics necessary for professional practice, and fall back upon subprofessional work. Every person seeking to practice medicine knows at the outset of his premedical course, and usually in high school, that he must complete this course, his medical course, and a year or more of internship, and then submit to an extremely difficult and searching state examination before he can be admitted to practice. He also knows that he can specialize only after further and prolonged study. One does not hear of the need of providing undergraduate courses in medicine in order to maintain the interest of the student.

Professor Grinter agrees that courses in public speaking, English literature, technical writing, history, and public affairs, when given under cultured and stimulating teachers, would produce results in broadening students; but he questions how a student can study such courses in sufficient quantity and still "end up as a trained engineer." This raises the question, "Can a trained engineer be produced by having a student successfully pursue four years of study at an engineering school, even at a school of the highest caliber?" The answer is manifestly in the negative. The author believes, however, that if a student should follow a four-year course, such as is outlined in this paper,

supplemented by graduate study of sufficient scope, character, and time to attain the professional degree, he will more closely approach the status of a trained engineer than will the graduate of current four-year courses after he has practiced from six to ten years.

Mr. Williams states that engineering education consists of cultivating the "growth" of the faculties to observe, to think, and to express thoughts, and of the personal attributes essential to citizenship and a full life. This is a most worthy objective, but if the writer's contacts—with thousands of graduates of all ages, in all branches of the profession, and holding all degrees of responsibility—are a guide, such objectives have been attained in an extremely small percentage of engineering graduates.

Professor Dougherty suggests that the writer has overlooked many changes that have taken place in engineering education. A review of courses offered in a considerable number of engineering schools; discussions with younger graduates; and a reading of news items in technical journals pertaining to engineering faculty additions, to improvements in school facilities and equipment, and to changes in courses, give the impression that there has not been any material change in engineering education since the period prior to World War I, beyond that of keeping well abreast of technical developments and practices.

Professor Dougherty also suggests that Fig. 1 be presented with professional age as the ordinate. This might be accomplished by use of material presented in Table 3(*b*), but it would serve little purpose. The presentation of Fig. 1, which was not drawn to scale, was, as heretofore stated, to indicate the fact that, as the relative amount of engineering work performed and the degree of responsibility attached to such work increased, the number of engineers required materially decreased. There is, of course, some degree of correlation between age, on the one hand, and the importance of work performed and the responsibility assumed, on the other, but no data are readily available, by which such correlation could be developed. The research suggested by the writer would involve the collection of analysis and interpretation of this type of data; but, until such research has been conducted, a great deal of the activities carried on to improve the profession technically, socially, and economically will be performed in the dark, or at least in the twilight.

Professor Lilly with much truth states that "Education should prepare a man for his leisure hours as well as his working hours." Miss McIver has pointed out that it is not now the fashion to portray engineers as "roughnecks," and that more and more stress is being placed by employers upon appearance. If engineering education is to provide a vocabulary, as mentioned by Professor Lilly, and also to provide for leisure hours, it would seem that the students should have an opportunity to acquire a nontechnical vocabulary; for many of his contacts following graduation will include those who do not possess or understand a technical vocabulary and with whom communication must be had in nontechnical terms. Furthermore, if the student has never been given even a taste or a glimpse of good literature, of music, history, or art, how will he know that these exist, or where to turn to gain an acquaintance with them. Even a course offering contact with these things and involving one hour a week during four years, where no problem sets were assigned and no grades given

except for attendance, would develop an interest on the part of many graduates which would greatly help them in preparing their leisure hours in later years.

Mr. Conwell indicates that the program suggested in this paper shifts the responsibility for developing nontechnical requirements of the engineering profession and for the cultural background of the student from the latter to the school, and shifts some of the responsibility for obtaining technical knowledge from the school to the employer. The writer agrees with this, and believes that it is proper. Under current practice, the student, an immature youth of seventeen or eighteen, presents himself to an engineering school and says "I want to be an engineer. Will you please make one out of me." The school proceeds to work on, and sometimes with, him for four years, filling him with technical theories and practice. At the end of the four years' time it takes those who have survived, the school gives each an engraved piece of parchment with his name on it, and tells him,

> "We have done what we could. We know that you can hold down the first job you are offered, and we hope that you can progress to higher and more responsible positions. We know that some of our graduates have done so, although we have not heard from most of the others."

The employer greets the graduate with open arms, and with high hopes that he has hired a future chief engineer, manager, or even president, and then, in most cases, promptly forgets him, although continuing to grouse about the inadequacies of engineering schools. The world seems very cold to the graduate, except for the opportunity sometimes provided once a month, if he resides in a fair-sized community, of attending the meeting of the local engineers' club or of the local section of a national engineering society. At such meetings he may sit next to his boss at dinner, talk about something else besides his daily tasks, and may also listen to some older fellow engineer "advertise" himself by telling about work he has recently performed. The graduate is on his own. His alma mater and his employer, as well as his profession, do have a responsibility toward him which they have, to a great degree, left unfulfilled.

Mr. Amirikian emphasizes the "practical problem of placement of the finished product," referring to graduates in law, medicine, and engineering. The engineering graduate is accepted by most employers as an apprentice in a trade, with the difference that he has acquired a familiarity with some of the tools which he will use in his trade, and is able to use a few of the simpler ones in minor operations. A further difference in this analogy, however, is that a plumber's apprentice can only become a plumber and not a bricklayer, whereas a civil engineering graduate may, and frequently is, apprenticed in mechanical, electrical, mining, or chemical engineering, and is then required to learn the use of a new type of tools.

The discussion by Mr. Thomas illustrates what can be developed concerning engineers and the practice of engineering from existing basic data. Table 7, showing that since 1929 the Society has taken three new members to gain one, serves to raise some extremely pertinent questions, among which is included, "How many engineers must be graduated from the current engineering courses to produce one professional engineer successfully practicing at the

age of, say, forty seven?" An analysis of the relation of losses in Society membership to total membership, other than from death, shows that the high loss ratio was not due to the depression of the 1930's. Table 8 indicates that one half the number of engineers who were practicing their profession at the age of thirty had left their profession fifteen years later, or by the time they were forty five, and that such loss ratio increases for engineers over thirty years old. Although the basic data upon which these conclusions rest may not be a strictly representative sample, nevertheless, the conclusions are interesting and significant, and involve a sample of sufficient size to warrant their being given serious consideration. Mr. Thomas' study resulting in Fig. 2, which shows the increase of employment of younger civil engineers in public service, might well be extended to other branches of engineering. Fig. 2 shows the deterioration of private practice in civil engineering as a field of opportunity for younger engineers. Such trend commenced long before the depression, so it cannot be charged to conditions created thereby.

Miss McIver's discussion of the methods and source of material used by the American Association of Engineers in its studies of engineers and of the practice of engineering indicates what can be accomplished through the use of existing data. The writer is in thorough agreement with her conclusions as to the disadvantages of the use of questionnaires to which voluntary replies are requested, and as to the unreliability of results secured where basic data are collected by this method. The unreliability of these results was established in a dramatic manner by a poll of the 1936 presidential election. Such polls had been taken a number of times previously and showed a high degree of accuracy with election results, but the sampling in the 1936 poll was apparently very unrepresentative.

Past studies of engineering compensation, based on voluntary replies to questionnaires, bear no resemblance to the material presented in Table 3(*c*), which is based on the salaries of every technical engineer in the country. Those receiving income disproportionately low compared to their age, length of experience, or position held, either do not reply or give higher than actual figures, while recipients of high income always proudly answer. The results, therefore, are heavily weighted by replies from those receiving higher incomes. Even if sampling by this method is truly representative, there is no way of determining this fact. Polls compiled by George Gallup and by the magazine *Fortune*, based samples carefully selected so as to be representative, although extremely small in number compared to the universe sampled, are statistically sound and have been proved to be highly accurate. Furthermore, sampling, whether it be on the basis of voluntary replies to questionnaires, or through personally interrogating the respective sample, only gives current views, attitudes, and facts. Trends can best be determined by utilization of existing basic data.

Experience records of engineers as a source of basic data in the study of the composition and structure of the engineering profession were suggested as the best available information. These records include the type and kind of work performed by engineers, the extent of responsibility which was placed upon them, the relative importance of the work which they were performing at various stages of their careers, the general rate of progress which they have made

in their profession, the changes from one branch or specialty to another, and the changes from one location or employer to another. They were not suggested, as Mr. Balog has assumed, as a source of information as to the accomplishments or capabilities of engineers. The writer agrees with Mr. Balog in his statement that:

> "Anywhere and everywhere, structures can be observed which disclose the need for engineering services—instances where the use of expert engineering talent would have resulted in savings, improved appearance, and increased engineering employment."

If engineers were better salesmen this statement would not be true. Agreement is not possible, however, with Mr. Balog's statement that:

> "The management of engineering organizations, at present, may be a purely parasitic activity in which mere trader's cunning, or certain dexterity in the use of technical terms, is sufficient qualification for a position above the engineer."

The writer's statement—

> "The current practice of engineering, except by those who serve in advisory or consulting capacities, is a mass-production activity, in which each individual serves as a cog in a machine and performs his specific duties as a unit of a much larger group"

—was not quoted in its entirety by Mr. Balog. The creations of engineers, particularly when they involve structures or facilities, are not those of mass production, but are those of an organized group of technical men working with others. Before a large engineering structure can be designed and built, its location, site, income possibilities, and many other features must be studied. Alternate preliminary plans and designs must be prepared, the economics of each plan and design investigated. Final plans are then prepared and a cost estimate made. The structure then must be built, and working plans prepared, foundation conditions explored, and changes made in the design during construction.

Tracers, draftsmen, designers—structural, mechanical, electrical, and others—rodmen, instrumentmen, chiefs of party, laboratory technicians, concrete technologists, resident engineers, inspectors, and a multitude of other personnel are required, some performing perfunctory duties, some routine work, and others technical work requiring extremely high skill, knowledge, and judgment. The designer is but one member of the team—an important one it is true, but no more important than many others. At the top is the manager. He coordinates, plans, drives, and pushes, makes the important decisions, or confirms those made by others. If he devoted his time primarily to the engineering phases of the operation which he was managing and, as Mr. Balog suggests, "assigns subordinates to the nontechnical duties of the managerial position," one wonders if the job would ever be completed, and if so at what cost.

Again, the questions posed by Mr. Wagner arise. Was the chief engineer of the Golden Gate Bridge (San Francisco, Calif.) practicing engineering only during the times when he stopped in the drafting room to discuss with his chief

designer the arrangement, size, and thickness of plates to be used in the design of the towers, and the unit stresses of steel to be allowed in the cables of this, the longest suspension bridge in the world? Or was he practicing engineering during the entire period from the time such bridge was conceived and determined in his mind to be feasible until the time it was completed and opened to traffic?

Mr. Golzé well expresses the viewpoint of the engineering employer when he speaks of the need of the U. S. Bureau of Reclamation, a large public engineering organization, which is passing into the stage of operating the engineering projects it has built, and clearly foresees the need for engineers who can administer and operate these projects, as well as design and build them.

Mr. Dykstra gives an excellent description of the type of engineer whose services will be required in coming years when he states,

> "In modern technological civilization there is great need for the technicians called engineers, but, to master this mechanical civilization, technical men must be aware of the implications of modern life and they must feel at home in a changing world. A structural engineer should know more than how to build a bridge. He should be able to state whether a bridge is necessary, what will happen if it is built, and, perhaps, be able to suggest an alternate solution for what is traditionally thought of as a bridge problem."

During the fall of 1946, the writer encountered an article[48] from which the following excerpts are quoted:

> "Because the engineering schools have always made it their chief aim to impart the technical information needed in industrial production, and because both scientific knowledge and industrial practice have grown so rapidly, the attention of technical schools has been focused chiefly upon keeping up to date in science and practice. The university emphasis on research in natural science has also tended to magnify the importance of technique and to minimize the importance of personality * * *. Therefor it seems necessary to consider the question whether this emphasis on technique is producing a new and higher type of engineer, or whether the engineering profession still stakes its faith on the fundamental thesis that personal character is, after all, the real foundation for achievement * * *."
>
> "Progress is being made towards the conception that there is really but one profession of engineering, in spite of its apparent division into the several well known branches. War conditions have not only hastened public recognition of the engineer as an expert in applied science and fostered solidarity in the profession, they have also opened to him new fields of activity * * *."
>
> "Hence there has arisen a pressing demand for men who can deal with labor and with business administration in the engineering spirit * * *."
>
> "These new opportunities for the engineer have been gradually developing for a number of years, but the profession as a whole has been slow to discern them. The war has focused attention on them and precipitated a general recognition of them. It is also evident that the mastery of these new activities depends in greater measure than does mastery of the traditional types of engineering on the personality of the man * * *. Therefor as engineering expands into new fields now opening before it, the conception that character, judgment, efficiency, and the understanding of men are no less necessary than technical knowledge and skill will become more and more impelling, and it will become more and more essential that schools of engineering pay greater attention to the effect of their work on the personal development of students * * *."

[48] *Engineering News-Record*, October 24, 1918, p. 742.

"The net result is that curricula and methods of instruction that were devised to supply the intellectual element in production by imparting knowledge of natural science must be reorganized to meet the new industrial demand for engineering administrators and the larger professional demand for men of strong personality * * *."

It should be noted that these excerpts appeared in an article published in October, 1918, and are quotations from the report on the three-year study of engineering education made by Charles R. Mann, under the joint auspices of the Carnegie Foundation for the Advancement of Teaching and the Joint Committee on Engineering Education of the national engineering societies.

The writer feels that Mr. Mann's statements, made less than a month prior to Armistice Day of World War I, are even more pertinent today.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

MOMENT-STIFFNESS RELATIONS IN CONTINUOUS FRAMES WITH PRISMATIC MEMBERS

Discussion

BY GEORGE H. DELL

GEORGE H. DELL,[7] ASSOC. M. ASCE.—Contrary to Mr. Oesterblom's inference, the moment-stiffness relations were not intended to provide a new method of analysis. They are but a means of calculating changes in the moments as the design is progressively modified and, therefore, are merely an auxiliary tool for use in connection with the moments obtained by one of the standard methods of analysis. In describing the use of moment-stiffness relations, the writer, far from trying to eclipse the Hardy Cross method by offering a better one, recommended and used the Cross method exclusively in the calculation of the moment-stiffness relations. A method cannot possibly be a "convenient substitute" for the Cross method when it relies on that method for its results.

The use of moment-stiffness relations places no restrictions on the designer's choice of methods for analyzing the structure or for designing the members, but it does offer a means of calculating the moment changes with rapidity and with a degree of accuracy such that the resulting design is unlikely to contain errors of any consequence.

One of the advantages of using moment-stiffness relations, not previously mentioned, is that it is not necessary to calculate the revised moments in all parts of the structure for a given arrangement of loads, but only at points where the moments control the design of members. Another important advantage worthy of re-emphasis is that, in redesigning a given member, the effects of a proposed change in stiffness are immediately apparent from the moment-stiffness relation, and one is thus able to anticipate and obviate the danger that after the member is revised it will be more seriously stressed than before.

NOTE.—This paper by George H. Dell was published in May, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1946, by I. Oesterblom, and April, 1947, by William A. Conwell.

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The true moment change that occurs as a result of revisions in the stiffnesses of any or all the members can be expressed in the form of a Taylor series. The writer doubts whether such an expression is what Mr. Oesterblom had in mind in stating that "It is doubtful if any one will ever be able to give definite mathematical expression to the accumulation of moments along their devious paths of approach * * *." It is more likely that he was referring to the problem of determining the maximum positive and negative moments in a structure, such as the three-story building described by him. Inasmuch as the paper (except for Appendix 2) was not concerned primarily with methods of analysis, the introduction of this problem is beyond the scope of the paper. However, it seems evident that if separate analyses are made to determine the moments throughout the structure, as a result of loads applied to one member at a time, the signs will indicate which members should be loaded in order that the maximum moment may be obtained at a given point. A more difficult and equally pertinent problem is that of finding the particular combination of axial forces and moments that will produce the greatest stresses in the columns.

Mr. Oesterblom's difficulty in following the derivation of Eq. 7 may be clarified by reviewing the statement (in the paragraph containing Eq. 1) that ψ is a linear function of the elastic deformations, which, in turn, are functions of the stiffnesses of the various members. The moments can be determined uniquely by a system of linearly independent "slope-deflection" equations, regardless of the values of the stiffnesses, and hence the moment changes may be legitimately represented by a Taylor series development,[8] provided the series converges. Furthermore, since the independent variables (the K-values) occur linearly in the slope-deflection equations, it is entirely proper to apply the principle of superposition in the determination of the first-order moment changes, resulting from a series of successive changes in stiffness in the same member, or in different members. In addition, since the slope-deflection equations are linear, the moment changes of second, and higher, orders may similarly be obtained by superposition; that is, the terms of a higher order may be found by applying, to the basic structure, certain fixed-end moments obtained in solving for the terms of the preceding order. Thus, Eq. 2 can be rewritten as follows:

$$dM_{AB} = M'_{AB}\, i_{AB} + \delta M'_{AB} \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Eq. 25 expresses the fact that the first-order moment changes are found by solving for the balancing moments, $\delta M'$, resulting from the fixed-end moments, $M'\, i$. It can be shown[9] that the second-order changes are given by:

$$\delta^2 M_{AB} = \delta M'_{AB}\, i_{AB} + \delta^2 M'_{AB} \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Eq. 26 expresses the fact that the second-order moment changes $\delta^2 M$ may be found by using the quantities $\delta M'\, i$ as fixed-end moments.

It does not appear feasible to establish a general criterion for the convergence of the series, and even if such a proof were available, it is necessary that the

[8] "The Effects of Errors or Variations in the Arbitrary Constants of Simultaneous Equations," by G. H. Dell, *Bulletin No. 309*, Univ. of Illinois Eng. Experiment Station, Urbana, Dec. 6, 1938, Sections 6, 7, and 18, and Appendix A.

[9] "Adjustments in the Design of Statically Indeterminate Structures," by G. H. Dell, thesis presented to the University of Illinois at Urbana, Ill., in May, 1943, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

series converge rapidly in order that the moment change, given by Eq. 7, be reasonably accurate. For the purpose of indicating the rate of convergence of the series for a particular structure, the writer has computed the changes in the controlling moments of Example 5, up to and including the third-order increments, as shown in Table 5. It should be noted, in comparing these results with those of the final analysis as given originally, that there is a small uncertainty in the third or fourth significant figure in both instances, as a result of slide rule inaccuracies, differences in procedure, and small discrepancies in the values of the stiffnesses.

TABLE 5.—MOMENT CHANGES OF HIGHER ORDER, EXAMPLE 5

Symbol	Original moment	Moment Changes			Total moment
		First term	Second term	Third term	
M_{BA} ..	+822	+47.6	−2.59	−0.60	+866.4
M_{BC} ..	−755	−31.3	−7.90	−1.68	−795.9
$M_{AA'}$ ..	+238	−50.9	−2.76	−0.49	+183.8
$M_{BB'}$ ..	−133	−46.5	+4.06	−0.60	−176.1
$M_{CC'}$ ..	−132	+16.4	+0.76	+0.12	−114.7

Mr. Oesterblom asks: "Can a moment at any point be found independently by taking advantage of the stiffness elements of the entire frame?" It has been shown that the process of determining the moment changes by Eq. 7 is equivalent to performing an analysis of the basic structure to determine the moments resulting from the use of the $(M'\,i)$-quantities as fixed-end moments. From this it follows that all joints of the structure are balanced; in addition, all sidesway conditions are satisfied, so that, theoretically, all the moments are solved, although, as previously stated, it is necessary to compute only the ones that control the design of given members.

The next question—"Can moment corrections be applied at all points by the section adjustment at a few points?"—seems to indicate that Mr. Oesterblom has in mind a method of finding the changes in all the end moments due to a single stiffness change, rather than a method of finding the change in a particular moment because of changes in all the stiffnesses. The writer has treated such a method elsewhere;[9] it consists in applying to the basic structure the M'-moments of the member whose stiffness is changed, and finding the resulting moments throughout the structure. The latter values give the coefficients of the partial moment changes in the moment-stiffness relation.

Time and space do not permit a detailed description of the method in this closure; however, as it does not involve the use of m-moments, the procedure might be advantageous in the design of the three-span, seven-story structure which Mr. Oesterblom mentions in connection with the question, "Can the author write the moment-stiffness relations for all points of such a frame?" In further reply to the latter question, it should be noted: First, that there is seldom any need for the moment change at more than one point in a particular member; and, second, that Eq. 7, being perfectly general, will enable one to write the moment-stiffness relation for any of the end moments if the required moments, resulting from unit couples and unit horizontal forces, are known for the basic structure. Furthermore, if moment-stiffness relations are required for some intermediate point on the span, they may be obtained by a suitable combination of the moment-stiffness relations for the ends of the member.

It is regrettable that Mr. Oesterblom finds Eq. 7 complex, but it is not possible to formulate a general statement of the first-order moment changes without including all the factors involved. As a matter of fact, the complexity is a function, not of the equation, but of the structure itself. Doubtless, in an extensive structure, the magnitudes of a great many of the m-moments would be so small as to permit their omission from a particular moment-stiffness relation.

There is probably no advantage in using moment-stiffness relations in the design of continuous beams; the principal field for their application is in frames in which the columns are subjected to considerable moments and, consequently, where sidesway plays an important part. Therefore, the procedures recommended by Mr. Oesterblom as being superior (the application of one cycle of moment distribution, or the use of the equation of three moments) would be extremely inappropriate and inaccurate. In Example 5, the final moments (Table 3, Col. 14), in foot-kips, as given by one cycle of distribution are as follows: $M_{BA} = 817$; $M_{BC} = 803$; $M_{AA'} = 205$; $M_{BB'} = 46$; and $M_{CC'} = 25$. The errors are 5.7%, 0.9%, 11.4%, 74%, and 78%, respectively. By applying three cycles of distribution, which would be closely equivalent to using the equation of three moments, the corresponding moments are 821, 794, 231, 57, and 31, with errors of 5.3%, 0.2%, 25%, 67%, and 72%, respectively.

It is true that in cases where only minor revisions are involved in the design of a frame there is no need for moment-stiffness relations; however, it is not always possible to foretell whether a particular problem will involve difficulties. Where difficult situations do occur, moment-stiffness relations will be advantageous.

Mr. Conwell notes that in the frame of Example 5 a given end moment may be appreciably influenced by a change in stiffness of a remote member. This tendency would doubtless diminish in proportion to the number of spans and stories, so that terms with very small m-moments could be omitted from the moment-stiffness relations. He also calls attention to the value of the check afforded by the sum of the coefficients. In this connection, it is desirable that a proof be given of the statement that the sum of the coefficients in a complete moment-stiffness relation should be equal to zero. By making all the stiffness-change ratios equal to a constant value, c, one obtains from Eq. 7:

$$dM_{AB} = c\,(m_{A,AB}\,\Sigma M'_{An} + m_{B,AB}\,\Sigma M'_{Bn} + \cdots + m_{\Delta 1,AB}\,\Sigma H'_1 + m_{\Delta 2,AB}\,\Sigma H'_2 + \cdots + M'_{AB}) \quad (27a)$$

For equilibrium, at any point, J, $\Sigma C_{Jn} + \Sigma M'_{Jn} = 0$ and, for each sidesway condition, $\Sigma Q + \Sigma H' = 0$. The expression for dM_{AB} may then be written as follows:

$$dM_{AB} = -\,c\,(m_{A,AB}\,\Sigma C_{An} + m_{B,AB}\,\Sigma C_{Bn} + \cdots + m_{\Delta 1,AB}\,\Sigma Q_1 + m_{\Delta 2,AB}\,\Sigma Q_2 + \cdots - M'_{AB}) \quad (27b)$$

By taking note of Eq. 5a it is seen that Eq. 27b thus reduces to:

$$dM_{AB} = -\,c\,(M'_{AB} - M'_{AB}) = 0 \quad (27c)$$

The investigation reported by Mr. Conwell in Table 4, dealing with a comparison of designing by successive revisions and direct analyses and by use of moment-stiffness relations, constitutes a sincere and unprejudiced trial, for which the writer is duly appreciative. It tends to confirm the writer's view that, if a final check analysis is necessary, a very considerable length of time is required for frame analysis, whether moment-stiffness relations are used or not. If the final check analysis is omitted, the use of moment-stiffness relations should result in a material saving of time. It is the writer's opinion that moments obtained by means of moment-stiffness relations will seldom be in error by more than 5%. On the average, half the errors would be on the unsafe side and half, on the side of safety.

In the case of a column, whose required area depends on the direct stress as well as on the moment, the resulting percentage of error in cross section would be only a part of the error in the moment. In using rolled sections, excess area is usually provided; whereas, in concrete structures the sizes of the members are such that there may be considerable error involved in the usual practice of using the moment of inertia of a member all the way to the intersection of center lines. In view of these, and other, considerations it would appear that the omission of the final check analysis would have no serious consequences. In this connection, it may not be irrelevant to refer to the common practice of basing the design of members of a truss on the primary stresses, although secondary stresses of considerable magnitude are generally present, which are always additive to the effects of direct stress.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SPACE RESECTION PROBLEMS IN PHOTOGRAMMETRY

Discussion

BY P. H. UNDERWOOD

P. H. UNDERWOOD,[10] M. ASCE.—Since the paper was written with the idea that it might be of interest theoretically and with the hope that it might be of some practical value, it was expected that the methods outlined would be compared with other methods, as has been done in the interesting discussions. Professor Church's method has been used to a considerable extent in practice. As a result, routine procedures have been established to facilitate calculations. It is possible that the writer's method could be simplified by a more careful arrangement of the work and by certain approximations in the calculations.

As stated in the paper, the calculations and the final results were carried out to more decimal places than would be warranted by any practical application. With an exposure interval of 1/100 sec in taking a photograph and with a plane speed of 200 miles per hr, the camera would move some 3 ft during the exposure. Considering this and also the errors from other sources, such as lens errors and the precision required in coordinates for mapping purposes, there would seem to be little value in determining results to fractions of a foot. As Mr. Anderson has stated, a graphical solution to obtain the lengths of the edges of the ground pyramid yields values within a few feet. Following this, a single solution by calculation of the plate pyramids should generally yield results of sufficient accuracy for practical purposes, especially if corrections are made by the cosine multipliers.

Mr. Anderson's solution using scale check lines to determine tilt, swing, and height of camera station is interesting. The writer has been much interested in this method and its modification by Jack Rihn[11] and has published a

NOTE.—This paper by P. H. Underwood was published in September, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1946, by Earl Church; January, 1947, by Ralph O. Anderson; and May, 1947, by George H. Dell.

[10] Prof. of Surveying, School of Civ. Eng., Cornell Univ., Ithaca, N. Y.

[11] "Manual of Photogrammetry," Am. Soc. of Photogrammetry, 1944, p. 274.

paper[12] on this subject, which leads to a fairly exact solution even under such adverse conditions as high relief and considerable tilt. It is believed that this method may be still further extended to give quite accurate solutions for space coordinates of exposure stations.

In his discussion Mr. Anderson raises questions relative to determining tilt when the exposure station coordinates are known, stating that the writer's method of adjustment is not clear to him. To attempt to clarify this point requires considerable explanation.

It should be considered that the position circles involved in the solution are on the surface of a sphere. Since the graphical solution must be made on a plane surface, it is necessary to apply some form of projection. In this case a "zenithal" projection was used, with the center of the projection at the principal point o and with points on this projection located at distances in proportion to their arc distances on the sphere and in directions corresponding to those on the sphere from the point of tangency o. This procedure necessarily introduces some distortion, which has not been investigated, but which is believed to be small in its effect on the use to be made of the projection.

On the sphere it is known that the distances from v (the point where the vertical from the exposure station L cuts the sphere) to a, b, and c are equal to M_A, M_B, and M_C, respectively. On the projection this condition would be true to a high degree of approximation if the tilt is small, since these distances would be measured nearly radially from o.

When the plotting of the projection is done to a large scale (as in Fig. 12), the radii of the "position circles" become so large that it would be impractical to draw them with centers and a beam compass. Thus, M_B is equal to 31° 04′ or to 1,864 min of arc. At the scale of the original drawing for Fig. 12 (1 mm = 1 min), the radius of the circle would be about 75 in.—that is, line no, if drawn to scale, would be more than 6 ft long. It is known that the center of this circle at b is at a distance from o equal to m_b with m_b less than M_B by 19.0 min. Thus, n, at the relatively short distance of 19.0 min to scale from o on the line ob, is the point where the position circle with b as center would cut that line. Consequently the perpendicular nn′ is the tangent to this circle at n. Similarly, perpendiculars kk′ and pp′ are tangents. Compared with the radii of the position circles, these tangents are short and approximate to their respective circles. They should not meet in a point but the "triangle of tangents" formed with their points of intersection as vertices should be small and should aid in closely approximating, as explained in the paper, the point where the circles would intersect, or point v. Computed offsets from the tangents give an even closer approximation to the point sought. Thus, the tangents, giving what was termed first a "triangle of error" and then "the triangle of the tangents," should not meet in a point theoretically, whereas the position circles should: (1) Meet if drawn on the sphere; and (2) fail to meet by only a small amount (which has not been evaluated but which for small tilts should certainly be small) if drawn on the projection. The position of each one of these three

[12] "The Determination of Tilt from Scale Check Lines," by P. H. Underwood, *Photogrammetric Engineering*, March, 1947, pp. 143-155.

circles in the vicinity of the point sought can be found closely by use of the offsets from the tangents as shown in the paper, thus obtaining a close approximation to the vertical point.

Corrrections for *Transactions*: In September, 1946, *Proceedings*, page 940, line 27, after the word "find" insert "the tilt i"; page 948, line 25, change K to k; and page 961, change line 16 to read "Thus, as previously stated, the tilt found by this more exact method * * *."

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DISCUSSIONS

RELIEF WELLS FOR DAMS AND LEVEES

Discussion

BY KENNETH S. LANE

KENNETH S. LANE,[38] Assoc. M. ASCE.—The authors have presented a worthwhile and timely assembly of theoretical considerations and experimental data which form the basis of design of relief wells. Such wells are among the most flexible of the various types of drains used to relieve seepage pressure by tapping pervious foundation strata, in accordance with the principle that control of seepage pressure rather than seepage quantity is the main factor in avoiding underseepage failures—an important principle which the authors have done well to emphasize. It is rare when uncertainties do not exist about a pervious foundation, especially in regard to chance arrangement of minor geologic layers of widely different permeability, which greatly influence seepage behavior. Hence, it is generally necessary to test the effect of several plausible assumptions of foundation conditions. In certain cases the straightforward charts of the paper will quickly furnish results to guide judgment. For several other common foundation cases, reference could well be made to formulas developed by Preston T. Bennett[32] and William H. Jervis, Members, ASCE, and Reginald A. Barron,[32] Jun. ASCE. These formulas give solutions for the head lost in seepage beneath the structure, and thus indicate whether the resulting seepage pressures in the zone of seepage emergence are likely to be sufficient to cause flotation, warranting relief well or other drainage treatment.

For the condition of flotation (when upward seepage pressure approaches, or equals, downward soil weight), two effects are prone to occur: (1) Emerging seepage concentrated in boils of moderate to large size tending to develop into piping which progresses as an underground channel toward the water source; or (2) flow dispersed over the emergence zone of upward seepage, causing a quick condition over a substantial area, which is often accompanied by many

NOTE.—This paper by T. A. Middlebrooks and William H. Jervis was published in June, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1946, by Henry C. Barksdale, Willard J. Turnbull, and Glennon Gilboy; December, 1946, by W. A. Wall and C. A. Stone; January, 1947, by John R. Charles, Horace A. Johnson, P. C. Rutledge, H. H. Roberts and Carter V. Johnson, Frank E. Fahlquist, and Harry R. Cedergren; February, 1947, by John S. McNown; and May, 1947, by Reginald A. Barron, Preston T. Bennett, Charles I. Mansur, and S. J. Johnson.

[38] Chf., Soils and Geology Branch, Garrison Dist., Corps of Engrs., Bismarck, N. Dak.

[32] "The Effect of Blankets on Seepage through Pervious Foundations," by Preston T. Bennett, *Transactions*, Vol. 111, 1946, p. 215.

tiny to small boils. In stressing the fact that boils of all sizes are dangerous, the authors give two examples, of the first effect (piping), and also mention the second effect—a general quickening of the foundation. In the latter state the soil behaves as a heavy liquid and has only minor shear strength; a mat of vegetation or root-bound topsoil will generally weave under foot, and seepage is likely to escape from many small boils.

The writer recalls one instance near West Springfield, Mass., where more than 1,000 small boils were counted in a reach of less than 1,000 ft behind a levee, which was then subjected to about two thirds of the design head. Fortunately, the flood was of short duration and seepage reduced the effective head by building up tailwater so rapidly that the levee escaped failure. Subsequently, a relief well system was designed to meet this condition.

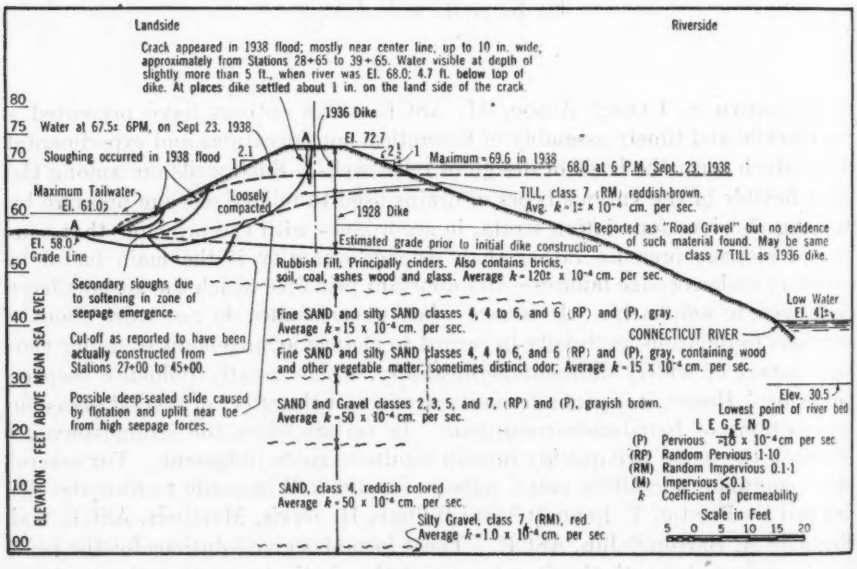
Landside
Riverside
Crack appeared in 1938 flood; mostly near center line, up to 10 in. wide, approximately from Stations 28+65 to 39+65. Water visible at depth of slightly more than 5 ft., when river was El. 68.0; 4.7 ft. below top of dike. At places dike settled about 1 in. on the land side of the crack.
1936 Dike
El. 72.7
Water at 67.5± 6PM, on Sept 23. 1938
Sloughing occurred in 1938 flood
Maximum = 69.6 in 1938
68.0 at 6 P.M. Sept. 23. 1938
Maximum Tailwater El. 61.0±
Loosely compacted
1928 Dike
TILL, class 7 (RM) reddish-brown. Avg. k = 1± x 10⁻⁴ cm. per sec.
Reported as "Road Gravel" but no evidence of such material found. May be same class 7 TILL as 1936 dike.
El. 58.0± Grade Line
Estimated grade prior to initial dike construction
Rubbish Fill. Principally cinders. Also contains bricks, soil, coal, ashes wood and glass. Average k = 120± x 10⁻⁴ cm. per sec.
Secondary sloughs due to softening in zone of seepage emergence.
Low Water El. 41±
Fine SAND and silty SAND classes 4, 4 to 6, and 6 (RP) and (P), gray. Average k = 15 x 10⁻⁴ cm. per sec.
CONNECTICUT RIVER
Cut-off as reported to have been actually constructed from Stations 27+00 to 45+00.
Fine SAND and silty SAND classes 4, 4 to 6, and 6 (RP) and (P), gray, containing wood and other vegetable matter; sometimes distinct odor; Average k = 15 x 10⁻⁴ cm. per sec.
Elev. 30.5
Lowest point of river bed
Possible deep-seated slide caused by flotation and uplift near toe from high seepage forces.
SAND and Gravel classes 2, 3, 5, and 7, (RP) and (P), grayish brown. Average k = 50 x 10⁻⁴ cm. per sec.
LEGEND
(P) Pervious >10 x 10⁻⁴ cm per sec
(RP) Random Pervious 1-10
(RM) Random Impervious 0.1-1
(M) Impervious <0.1
k Coefficient of permeability
SAND, class 4, reddish colored Average k = 50 x 10⁻⁴ cm. per sec.
Silty Gravel, class 7, (RM), red Average k = 1.0 x 10⁻⁴ cm. per sec.
Scale in Feet
5 0 5 10 15 20
ELEVATION - FEET ABOVE MEAN SEA LEVEL
80
75
70
60
50
40
30
20
10
00

FIG. 31

With such a general quick condition there is real danger that slides may develop on the downstream or land-side slope of the water barrier embankment, from loss of toe support, as the quick condition results in a drastic loss of shear strength of the foundation soil adjacent to the embankment toe. If the slide, or succession of slides, extends back to intersect the water surface, it is followed immediately by overtopping, and a crevasse results.

A partial example of dangerous loss of toe support is the near failure of Springdale Dike during the September, 1938, flood. This levee was built by local interests along the Connecticut River near Holyoke, Mass. Its section is homogeneous, not specially compacted, and contains a shallow cutoff, strangely located near the land side. Fig. 31 shows foundation conditions and behavior observed, except in the land-side area where tailwater prevented observations. During the flood, electric failure occurred within the pumping

station and tailwater was built up rapidly, partly from seepage, but largely from unintentional diversion through an unclosed cross connection from a sewer communicating with the river. As an explanation of surface evidence observed, line AA in Fig. 31 has been drawn as a possible surface of a slide of the land-side slope which may have been in progress until accidental creation of a tailwater pond lowered the effective head, and very likely was a major factor in averting complete failure. To no small degree the unfortunate location of a partial cutoff at the land side, instead of at the river side, would contribute toward such a slide by increasing seepage pressure in the land-side part of the levee to nearly full headwater pressure—shown as having occurred from the water level observed in the center-line crack. Although the Springdale Dike foundation might be treated with relief wells, the reconstruction design in-

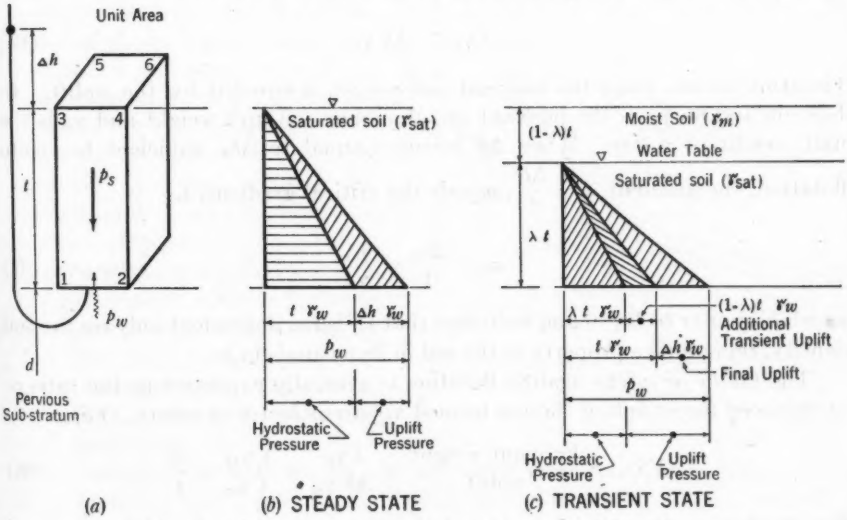

FIG. 32

cluded a continuous pipe drain (extending from 4 ft to 10 ft into the pervious upper layer of fill), which the writer considers as more positive than intermittent relief wells for foundations not too highly stratified, and where the most pervious important foundation layers lie at such depth that they can readily be tapped by a drain.

As indicated by the authors' examples, most of the present relief well installations have been placed behind existing structures as repair measures. From successes in such usage, relief wells are now being designed increasingly as part of initial construction. To aid judgment in determining when pressure-relief treatment is initially needed, it is of interest to investigate the equations of flotation in more detail than is permitted by Eqs. 3 to 6. Figs. 32(*a*) and 32(*b*) show the steady state case usually considered for investigating flotation of a top stratum which is saturated, with the water table at the surface. Seepage is flowing upward and Δh represents the piezometric pressure or uplift head acting at the base of the top stratum, as estimated from computing head lost

in seepage beneath the structure from solutions given by the authors and by others.[32] For the state of flotation, the total upward pressure—

$$p_w = t\,\gamma_w + \Delta h\,\gamma_w \quad \ldots\ldots (65a)$$

—equals the total downward pressure—

$$p_s = t\,\gamma_{bs} + t\,\gamma_w \quad \ldots\ldots (65b)$$

—assuming no resistance on the sides of the parallelepiped 1–2–3–4–5–6, Fig. 32(*a*). This assumption is substantially correct when flotation occurs, because the soil then acts as a heavy liquid with no shear strength. Accordingly:

$$t\,\gamma_{bs} + t\,\gamma_w = t\,\gamma_w + \Delta h\,\gamma_w \quad \ldots\ldots (66a)$$

or

$$t\,\gamma_{bs} = \Delta h\,\gamma_w \quad \ldots\ldots (66b)$$

Flotation occurs when the buoyant soil weight is equaled by the uplift. In Eqs. 65 and 66 γ_{bs} is the buoyant or submerged soil unit weight and γ_w is the unit weight of water. When Δh becomes equal to Δh_c, sufficient to create flotation, the gradient, $i = \frac{\Delta h}{t}$, equals the critical gradient, i_c:

$$i_c = \frac{\Delta h_c}{t} = \frac{\gamma_{bs}}{\gamma_w} \quad \ldots\ldots (67)$$

which is similar to Eq. 6 and indicates that i_c, being dependent only on the soil density, represents a property of the soil in its natural state.

The factor of safety against flotation is generally expressed as the ratio of unbalanced forces acting (herein termed apparent factor of safety, $(\mathrm{FS})_a$):

$$(\mathrm{FS})_a = \frac{\text{buoyant weight}}{\text{uplift}} = \frac{t\,\gamma_{bs}}{\Delta h\,\gamma_w} = \frac{1}{i}\frac{\gamma_{bs}}{\gamma_w} = \frac{i_c}{i} \quad \ldots\ldots (68)$$

For strict correctness it is considered the factor of safety should be expressed as a ratio of total forces acting (herein termed true factor of safety ($\overline{\mathrm{FS}}$)):

$$(\overline{\mathrm{FS}}) = \frac{t\,\gamma_{bs} + t\,\gamma_w}{t\,\gamma_w + \Delta h\,\gamma_w} = \frac{\frac{\gamma_{bs}}{\gamma_w} + 1}{1 + \frac{\Delta h}{t}} = \frac{1 + i_c}{1 + i} \quad \ldots\ldots (69)$$

Eqs. 68 and 69 are compared in Fig. 33, which shows that the function $\frac{1 + i_c}{1 + i}$ is not a sensitive one. It has a definite maximum in the vicinity of $(\overline{\mathrm{FS}}) = 2$ and has the further disadvantage of not varying directly with Δh which is related directly to total head acting. Since it eliminates these disadvantages, the function $\frac{i_c}{i} = (\mathrm{FS})_a$ is considered the flotation factor of safety in usual practice, with which the writer concurs.

The authors suggest (in the text following Eq. 6) that "it may be that design for a safety factor of 1.5 against developing the critical gradient is satis-

factory." If the authors intend this to be computed as $(\overline{FS}) = 1.5$, it becomes of the order of $(FS)_a = 3.0$ when converted in Fig. 33. However, if the authors intended this to be obtained from the usual equation, $(FS)_a = \frac{i_c}{i} = 1.5$, then the writer is inclined to feel that the suggested value is low for dams, although probably reasonable for most levees except those protecting property of particular importance or value. To a considerable extent the choice of a numerical value for a design objective should depend on the severity of the assumptions used because, in the analysis of flotation, a factor of safety often represents a factor of ignorance to allow for the great influence on seepage behavior caused by discontinuous stratifications of widely different permeability. Fortunately, for decidedly stratified foundations a lower factor of ignorance is applicable to a relief well than to a usual drain treatment because, except for a drain of comparable depth, the relief well treatment is the more positive. Once it has been decided to use relief wells in a levee design, it will often be found that the additional cost of increasing safety to the desired objective by a moderate decrease in spacing is not great.

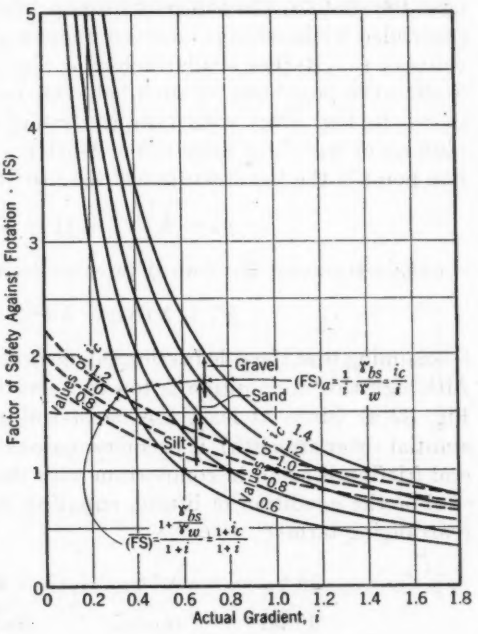

FIG. 33

The most difficult problem is to decide if wells are needed. In studying relief well applications to several levees around built-up areas of moderate property value, the writer has found the following a convenient approach which is advanced here more to stimulate thought than as a proposal for general application. Two flood heights are considered—(1) design flood and (2) top-of-levee flood. The latter equals design flood plus freeboard. If analysis reveals conditions that are considered moderately dangerous for the design flood, and decidedly so for the top-of-levee flood, then pressure-relief treatment is recommended. For less severity, consideration is given to taking a calculated risk by initially omitting extensive treatment and providing for later installation if needed. As the authors mention, the relief well is a particularly flexible tool for such later installation. Once pressure-relief treatment is adopted, an attempt is made to secure definite safety for the design flood and at least a slight margin of safety for the top-of-levee flood.

Greater conservatism is felt warranted in the case of dams, including consideration of the possibility that partial quickening of the downstream

foundation with attendant reduction in shear strength may begin at a gradient below that computed as the critical from Eq. 67. The writer is inclined to the belief that this is a definite possibility from occasional qualitative observations of a quicksand device. In several performances the soil column parted near the middle; the top part lifted a matter of several inches and remained suspended while seepage emerged under a gradient somewhat below a probable value of i_c. Rather similar behavior also has been reported by others.[39] To explore the possibility of such behavior in nature, consider the transient case shown in Fig. 32(*c*) with tailwater rising toward the surface from an initial position of the water table below depth t. When tailwater is rising from point 1 to point 3, the top stratum will tend to lift when the total upward pressure—

$$p_w = \lambda\, t\, \gamma_w + (1 - \lambda)\, t\, \gamma_w + \Delta h\, \gamma_w \ldots\ldots\ldots\ldots (70a)$$

—equals the total downward pressure exerted by the partly saturated soil—

$$p'_s = \lambda\, t\, \gamma_{bs} + \lambda\, t\, \gamma_w + (1 - \lambda)\, t\, \gamma_m \ldots\ldots\ldots\ldots (70b)$$

—assuming that the side friction can be neglected around the periphery 3–4–5–6. Although this assumption is too conservative for a small area as indicated in Fig. 32(*a*) (since at least the moist soil above the water table retains substantial shear strength), it becomes reasonable for large areas where the peripheral friction is small in comparison with the weight, p'_s.

For the condition of lifting, equating $p_w = p'_s$ from Eqs. 70*a* and 70*b* and rearranging terms:

$$\underbrace{\Delta h\, \gamma_w}_{\substack{\text{Final}\\ \text{uplift}}} = \underbrace{\lambda\, t\, \gamma_{bs}}_{\substack{\text{buoyant}\\ \text{soil}}} + \underbrace{(1 - \lambda)\, t\, \gamma_m}_{\substack{\text{moist}\\ \text{soil}}} - \underbrace{(1 - \lambda)\, t\, \gamma_w}_{\substack{\text{additional}\\ \text{transient uplift}}} \ldots\ldots (71)$$

$$\underbrace{\text{Final uplift}}_{\text{net downward pressure}} =$$

Defining $(i_c)_m$ as the lifting gradient at the time that lifting of the partly moist top stratum begins:

$$(i_c)_m = \frac{\Delta h_{cm}}{t} = \lambda \frac{\gamma_{bs}}{\gamma_w} + (1 - \lambda) \frac{\gamma_m}{\gamma_w} - (1 - \lambda) \ldots\ldots\ldots (72a)$$

Let

$$\beta = \frac{\gamma_m}{\gamma_w} = \frac{S\,(\gamma_{sat} - \gamma_d) + \gamma_d}{\gamma_w} \ldots\ldots\ldots\ldots\ldots\ldots (72b)$$

in which S denotes percentage saturation. Making necessary substitutions of β from Eq. 72*b* and i_c from Eq. 67:

$$(i_c)_m = \lambda\, i_c + (1 - \lambda)\, \beta - (1 - \lambda) \ldots\ldots\ldots\ldots\ldots (72c)$$

or

$$(i_c)_m = \lambda\, [i_c - (\beta - 1)] + (\beta - 1) \ldots\ldots\ldots\ldots\ldots (72d)$$

Expressing the apparent factor of safety against lifting, $(FS)'_a$, as the ratio of net downward pressure to final uplift (see Eq. 71) and using the same sub-

[39] "Cofferdams," by Lazarus White and Edmond A. Prentis, Columbia Univ. Press, New York, N. Y., 1940, p. 13.

stitutions as in Eq. 72*c*:

$$(\mathrm{FS})'_a = \frac{1}{i}[\lambda\, i_c + (1-\lambda)\,\beta - (1-\lambda)] = \frac{(i_c)_m}{i} \ldots\ldots\ldots (73)$$

in which $i = \dfrac{\Delta h}{t}$ is for the final condition when the water table has reached the surface as in Fig. 32(*b*).

An expression can also be derived for true factor of safety against lifting which (omitting the algebra) results in:

$$(\overline{\mathrm{FS}})' = \frac{\text{total downward}}{\text{total upward}} = \frac{p'_s}{p_w} = \frac{1' + (i_c)_m}{1+i} \ldots\ldots\ldots (74)$$

Eqs. 73 and 74 are similar in form to Eqs. 68 and 69; and, using $(i_c)_m$ for i_c, then are solved easily from the curves in Fig. 33.

Eq. 72*c* shows that $(i_c)_m$ varies directly with λ for a given soil and is a minimum for $\lambda = 0$. It is always less than i_c for less than full saturation until $\lambda = 1$, whereupon $(i_c)_m = i_c$. With other variables held constant, $(i_c)_m$ varies directly with β and is a minimum for dry soil. Fig. 34 shows this characteristic for a typical loose sand ($i_c = 1.00$) where the value of $(i_c)_m$ is reduced nearly one half for a dry soil and for $\lambda = 0$. Fig. 35 shows the effect of changes in density with constant saturation, drawn for a common specific gravity of $G = 2.65$ and saturation of $S = 70\%$.

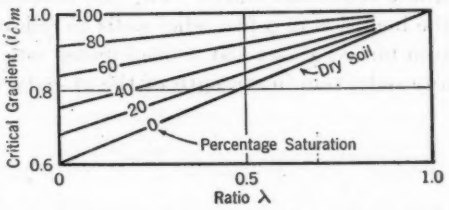

FIG. 34.—EFFECT OF SATURATION, WITH CONSTANT DENSITY ($i_c = 1.00$; $e = 0.65$; $\gamma_d = 100$; AND $G = 2.65$)

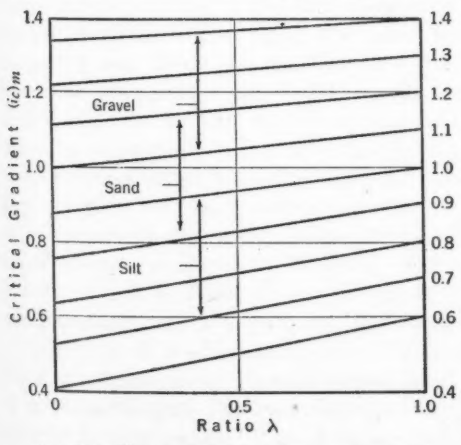

FIG. 35.—EFFECT OF DENSITY, WITH CONSTANT SATURATION ($S = 70\%$ AND $G = 2.65$)

The computation of the lifting gradient, $(i_c)_m$, as appreciably less than i_c, is believed to explain the parting and lifting in the quicksand device, which was generally observed at the first beginning of upward flow when the soil was dry to moist. After seepage emerged for some time and was then shut off, successive resumptions of upward seepage did not again cause lifting, presumably as the soil was then saturated and $(i_c)_m$ equaled i_c for all values of λ according to Fig. 34. The quicksand device used was not adapted to close measurement of actual gradients; hence verification of Eq. 72*d* is as yet qualitative rather than quantitative.

In nature, the case of moist soil with a rising water table is a transient state. Since piping is not likely to start working backward from the seepage outlet

until the steady state has been attained and has established that outlet, most flotation problems should be analyzed for the gradient i_c in the steady state. However, a few cases are visualized where analysis on the basis of the lower gradient $(i_c)_m$ may be of considerable importance. Consider a top stratum of clay, tight and impervious, underlain by a material of such greater permeability that the seepage from it can accumulate rapidly to fill such gaps as might be formed by lifting of the top stratum. With the common case of a sheet pile cofferdam, braced at the top and toed into above top stratum, it is easy to visualize loss of toe support by lifting of the top stratum sufficiently for failure by rotation around the top brace. Another example is that of a dam or high levee, with such a critical condition of downstream slope stability that even a small loss of toe support would be undesirable. Pending the assembly of quantitative data on the possible reduction in shear strength at gradients below i_c, the writer has attempted to allow for this by considering $(i_c)_m$ as the point where reduction starts. For one important dam so analyzed, the result was the inclusion of a few relief wells to relieve seepage pressure in a pervious stratum buried about 130 ft deep under soft varved silt in order to insure against any reduction in strength of the already weak silt foundation.

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DISCUSSIONS

RIGID-FRAME STRUCTURES SUBJECT TO NONUNIFORM THERMAL ACTION

Discussion

BY WILLIAM A. CONWELL

WILLIAM A. CONWELL,[11] M. ASCE.—In analyzing rigid frames subject to thermal action, the author has fully demonstrated the severity of the stresses involved. It is unfortunate, however, that the demonstration is largely a qualitative one rather than a quantitative one. With few exceptions, as stated in the paragraphs following Eq. 7, the stresses are "beyond the limit within which the equations of this paper apply." The stresses encountered are beyond the elastic range, but the methods of analysis are based on elastic behavior and real solution lies only in drastic revision of the type of structure.

The author's method of computing deformations due to thermal action, as demonstrated in Fig. 2 and the subsequent detailed applications to frames, is unique in that it sets aside the usual approximations made in small-angle geometry and treats the problem as a precise surveying layout. This action has forced the author out of the usually accepted method of slide-rule computation and has led him to the statement (in Section 5) that "Displacements must be computed by logarithms, whereas the slide rule offers abundant accuracy for all other numerical values." With a view to determining whether the comparatively large deflections involved in this study required such means of computation, the writer computed the deflections by 10-in. slide rule in the usual manner, as shown in Table 3. The comparison with the author's results (shown in Col. 8, Table 3) would seem to indicate that properly read slide-rule computations can be expected to give reasonably accurate results.

Although it is often desirable and sometimes necessary to know the deflections of a frame under various hypothetical conditions of cutting, support, etc., it is believed that the importance of such deflections has been emphasized unduly in this paper. The designer is first interested in bending moments, shear and axial forces, and then stresses. If these can be obtained without consideration of hypothetical deflection conditions he is usually happy to obtain

NOTE.—This paper by Carl C. H. Tommerup was published in June, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1946, by I. Oesterblom, and Frank R. Higley; January, 1947, by A. L. Miller; and March, 1947, by Charles O. Boynton.

[11] Gen. Engr., Structural Eng. and Design Dept., Duquesne Light Co., Pittsburgh, Pa.

them in that manner. Having obtained these elements of design he may later elect, or be required, to compute the actual deflections, and these furnish real information as to the ultimate action of the structure.

TABLE 3.—COMPARISON OF COMPUTATIONS FOR DEFLECTIONS, SLIDE RULE VERSUS LOGARITHMS, IN THE FRAME OF FIG. 4

Deflections	Point D in Relation to Point E		Point C from Point D		Point B from Point C		Total, Point B from Point E	
	Computations	Inches or radians	Computations	Inches or radians	Computations	Inches or radians	Slide rule	Logarithms
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Δx	0.00316×80	0.2528	$0.000158 \times 120^2 \times \frac{1}{2}$ 0.01264×120	1.137 1.518 2.655	$0.00316 \times (-80)$	−0.2528	2.655	2.720
Δy	$-0.000158 \times 80^2 \times \frac{1}{2}$	−0.506	0.00316×120	0.379	$0.000158 \times 80^2 \times \frac{1}{2}$ 0.0316×80	0.506 2.528 3.034	2.907	2.883
θ	0.000158×80	0.01264	0.000158×120	0.01896	0.000158×80	0.01264	0.04424	0.0442

Section 2 furnishes an example of the emphasis on deflections. In the sixth paragraph of Section 2, prior to the application of the elastic energy equations, the author logically concludes, without reference to deflections, that "Any point of the frame is affected only by a moment that is of uniform magnitude all around." At this point the entire problem may be solved by considering the 1-in. element of the frame shown in Fig. 26. Under thermal action this element tends to assume a wedge shape with its inside fiber $0.0000079 \times 400 \times 1 = 0.00316$ in. longer than the unheated outside fiber. The bending moment induced by this tendency is sufficient to maintain the faces of the element parallel, and is equal to that necessary to turn the face of the element through an angle $d\phi = 0.00316 \times 1/20 = 0.000158$. Such bending moment is quickly determined by substituting known quantities in the expression $d\phi = \frac{M\,ds}{E\,I}$, whence $M = 0.000158 \times 2{,}000{,}000 \times 14{,}230 = 4{,}500{,}000$ in.-lb.

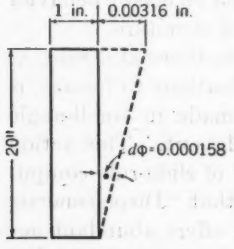

FIG. 26.—DEFORMATION OF THE FRAME ELEMENT

If there is justification for an elaborate computation of thermal deformations, such nicety should be extended to other phases of the calculations. Experience shows that the best results in the solution of statically indeterminate structures are obtained when the same basis is used for each part of the computations. For instance, the use of precise methods in certain steps of a calculation, and not in others, does not necessarily improve the ultimate result over a balanced calculation, on a lower level of accuracy, in which the same factors have been disregarded throughout. The author uses precise methods of calcu-

lation for thermal deformations; but he uses them in connection with the less accurate equations of elastic energy, the derivation of which disregards all quantities resulting from the multiplication of two or more differentials. It is this lack of balance which contributes to the discrepancy between the bending moments of 4,500,000 in.-lb and 4,600,000 in.-lb in Section 2 rather than "the slight approximations made in computing displacements," the reason assigned by the author. In support of this contention is the writer's calculation of the preceding paragraph which checks that made by the author on the basis of angular rotation. In further corroboration of this point of view is the fact that, if the writer's value for Δx (taken from Col. 7, Table 3, as $2 \times 2.655 = 5.310$ in.) is substituted with other known quantities in Eq. 7, a value of 4,500,000 in.-lb, consistent with other balanced calculations, is obtained for M. It is not claimed that these checks, which are the product of consistency, indicate a more accurate value of M; but they certainly enhance the confidence of the designer in his calculations. A more accurate value may be obtained only by conducting a completely balanced calculation on a higher plane, not by improving only one step in the process.

In Section 1 the author states: "Because of the greater ease, member lengths, as a rule, are taken along their inside surfaces." No difficulty is attached to the use of inside surfaces as a basis for calculating thermal deformations except the fact that the comparatively small effect of the corners is thus neglected. Some allowance for corners could have been made by taking member lengths along a plane midway between their outside and inside surfaces, although this in itself is a matter of small moment. Since the bending moment in the frame of Section 2 is dependent only on angular deformations of the frame and not on change in length of the members, the place of measurement of member lengths is immaterial. The same, however, may not be stated of the structures studied in Section 3 and Section 4.

An examination of Fig. 12 reveals that not only have the lengths of the members been taken along their inside faces, but the neutral planes have likewise been taken in that position. The neutral plane of member CD is actually some 90 in. from the line of action of T_{BE} if it is assumed to be about midway between the inner and outer surfaces of the member. This is not a bad assumption. The bending moment actually developed in member CD because $T_{BE} = 36{,}300$ lb would be $36{,}300 \times 90$ or 3,267,000 in.-lb. Although specifications would permit the use of a face-of-support bending moment of 2,900,000 in.-lb in the design of member BC, they would normally require the higher value for member CD; but, more deeply, T_{BE} has been determined on the basis of a bending moment of 2,900,000 in.-lb in member CD and its value would change were the moment other than this value. There is thus some question as to the conformance of the frame analyzed by the author and that which it was intended to investigate. The properties of, and the conditions affecting, the frame analyzed by the author are, upon study and inspection, as follows:

1. Tie member BE is treated substantially as required by the actual design data;
2. Members BC and DE have lengths of 80 in. each and have their neutral planes initially at a distance of 120 in. from each other;

3. Member CD has a length of 120 in. and has its neutral plane 80 in. from the line of action of member BE;
4. Members BC, CD, and DE are subject to the angular deformation resulting from a temperature differential of 400° between their outside and inside faces; and
5. Members BC, CD, and DE are subject to linear expansion resulting from a uniform temperature increase of a full 400° throughout their length.

There is some question as to whether this frame sufficiently conforms to the one being investigated. In the latter, members BC and DE have effective lengths of about 90 in. and member CD, a length of about 140 in. The linear expansion to which each of these members is subject is that resulting from an average increase in temperature of only 200° throughout its length.

A similar situation obtains as regards the frame of Section 4. A study reveals the following with respect to the frame analyzed by the author:

(1) Members AF and GM have lengths of 557 in. each and have their neutral planes initially at a distance of 340 in. from each other;
(2) Members AG and FM have lengths of 340 in. each and have their neutral planes initially at a distance of 557 in. from each other;
(3) Members BH and EL have lengths of 340 in. each and have their neutral planes initially at a distance of 98 in. from each other;
(4) Members AF, GM, AG, and FM are subject to the angular deformations resulting from a temperature differential of 400° between their inside and outside faces; and
(5) All members, not only BH and EL, are subject to linear expansion resulting from a uniform temperature increase of a full 400° throughout their length.

Again there is question as to whether this frame meets the design conditions outlined in Fig. 13; particularly with regard to the fact that spans BH and EL are the only members of the given frame that undergo the full 400° rise in temperature, whereas all other members are subjected to an average increase of only 200°. That this condition is a fact is further borne out by Fig. 15 in which member FM is shown as elongating an amount about equal to that of EL.

With a view to checking the statement of the foregoing paragraph (and, incidentally, demonstrating how well the method of moment distribution lends itself to these problems) a solution was developed for a frame having the foregoing characteristics and represented as the frame actually solved by the author in Section 4. This solution is shown in Fig. 27. The fixed-end moments for each member were determined by the method outlined by the writer in Fig. 26 and the fourth paragraph of this discussion. Moments were distributed in the usual manner. The values are seen to differ somewhat from those of the author but this difference is again the result of the unbalance of elaborate deformation calculations used in conjunction with the ordinary equations of elastic energy. As a means of further checking this point, the writer, in the manner previously demonstrated in Table 3, calculated Δx at point N, Δy at point F, and θ at point F, obtaining 3.19 in., 5.04 in., and 0.08375 radians, re-

spectively. Substitution of these values in Eqs. 16, 18, and 20, and simultaneous solution, yielded the following data which check those obtained by moment distribution: M_D, 4,595,000 in.-lb; M_E, 4,645,000 in.-lb; and M_F, 2,836,000 in.-lb. By methods similar to those used in the treatment of sidesway, any difference in linear expansion between exterior and interior members of this frame may also be easily handled by moment distribution.

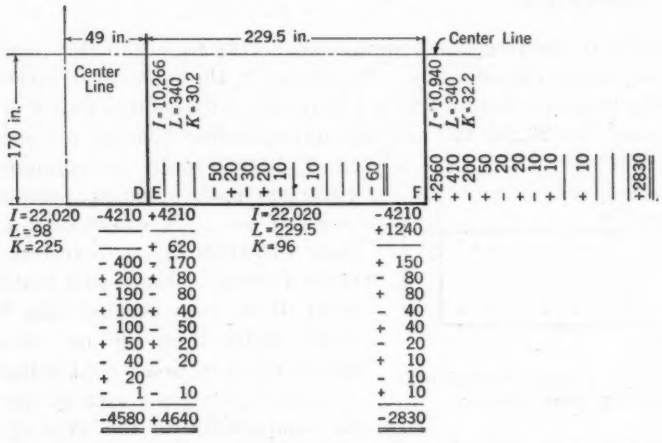

FIG. 27.—SOLUTION, BY MOMENT DISTRIBUTION, OF THE FRAME ANALYZED IN SECTION 4

In the solution of reinforced-concrete structures subject to elastic deformation only, it is not important that the absolute moment of inertia of the section of each member be determined. It is usually sufficient to establish relative values with respect to some base, the most important item being that the same procedure be used in determining the moment of inertia of each member. An example of such procedure is the often-used assumption that each member is adequately reinforced, and, accordingly, that the moments of inertia are proportional to the cubes of the depths of the members.

When other than elastic deformations are involved, as in the present study, determination of the absolute moment of inertia for the purpose of computing deflections is essential. This paper uses the same moment of inertia for calculating deformations as for determining stresses, a procedure which is open to question.[12] After delineating the differences in action between reinforced concrete and homogeneous beams but arriving at the conclusion that it would be desirable to employ the same methods for calculating deflections, F. E. Turneaure, Hon. M. ASCE, and E. R. Maurer specify the following procedure to bring calculated deflections within the range of observed test deflections:[12]

> "For the reasons stated above, the deflection formulas for homogeneous beams will be used for reinforced-concrete beams, but modified in accordance with the following assumptions:
>
> "1. That the representative or mean section has a depth equal to the distance from the top of the beam to the center of the steel;

[12] "Principles of Reinforced Concrete Construction," by F. E. Turneaure and E. R. Maurer, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., 1919, Chapter VI.

"2. That it sustains tension as well as compression, both following the linear law;
"3. That the proper mean modulus of elasticity of the concrete equals the average or secant modulus up to the working compressive stress; and
"4. That the allowance for steel in computing the moment of inertia of the mean section should be based on the amount of steel in the mid-sections."

Application of the foregoing assumptions to the frames of this paper would influence the results considerably. For example, the moment of inertia of the section of the frame of Section 2 would be based on the section shown in Fig. 28. Its value would be 26,200 in.^{4} and the corresponding bending moment would be 8,250,000 in.-lb as compared with 4,500,000 in.-lb previously computed. In physical terms, it will take much more than 4,500,000 in.-lb to restore the deformed frame of Fig. 4 and reduce Δx at point B to zero because this frame is much stiffer than the net moment of inertia of its section would indicate.

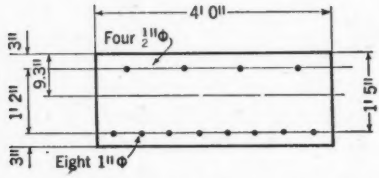

FIG. 28.—EFFECTIVE FRAME SECTION FOR DEFLECTION COMPUTATIONS

Use of the gross area of the section for computation of the axial elastic deformations as employed in this paper is scarcely consistent with the use of net moment of inertia but would fit well into the use of the total section outlined by Messrs. Turneaure and Maurer. (The upper 17 in. of concrete in Fig. 28 is considered effective in both tension and compression, in the computation of deflections.)

As to notation, x and y are defined as horizontal and vertical ordinate distances, while actually all values of x and y lie in a horizontal plane. The symbol m is defined as the moment of a unit force, whereas throughout the paper it is used as the bending moment caused by the various auxiliary forces and moments. Letter symbols $\bar{x}$ and $\bar{y}$ are not defined, although investigation of their use usually discovers their meaning. It is suggested that some definite and compact form of notation be adopted for the undefined quantity "area" which has not the same value for each member of a frame although it seems to have, as it appears in the various equations and formulas, devoid of subscripts or other means of identification.

In paragraph 15 of Section 2 the statement that "when the members of a simple closed frame, of any shape, are subjected to a nonuniform temperature change through their 'depth' (h) they undergo no deformation" should be modified by the words "except axial elongation."

The author states Eq. 14 as four terms, only two of which contain the quantity T'. He then follows with the statement that "The auxiliary force T' might have been assumed as unity at the outset since it cancels * * *." It is not clear how T' can be canceled from an equation when it appears in only two of the four terms. Since the value of the auxiliary force is at the choice of the computer there is no reason why it should not have originally been taken as

unity. In any event it should have been assumed as unity or disposed of by cancellation before it had an opportunity to appear in Eq. 14, which is satisfied only when $T' = 1$.

Another observation which may be made of Eq. 14 is the smallness of the terms expressing axial deformations as compared with that concerned with angular deformations. Although the author normally disregards axial deformations, he has made a particular point of including them in this problem by stating that "For a slender tie rod, stored-up elastic energy of both axial forces and moments must be included in the elastic energy equation * * *." With a view to ascertaining this necessity the writer solved the frame by moment distribution as shown in Fig. 29, disregarding all axial deformation including that of the tie rod. The results are not far from the author's results.

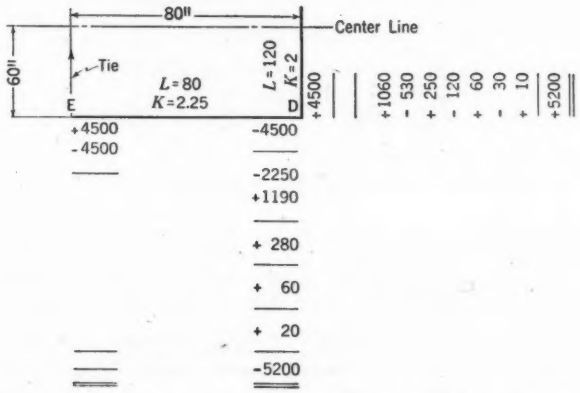

FIG. 29.—SOLUTION BY MOMENT DISTRIBUTION, NEGLECTING ALL AXIAL DEFORMATION; FRAME ANALYZED IN SECTION 3 (I = 14,230 THROUGHOUT; MOMENTS ARE IN INCH-KIPS)

A study of this paper would have been made more facile by inclusion of such basic data as the location of reinforcing rods, the positions of which had to be ascertained in each instance by working back from dimensions which the author substituted in formulas; by including a statement that the values of A, and I given in Table 2 are per-foot height of wall (or that the walls are only 12 in. high); and by avoidance of generalities in the statement of problems such as that in the first paragraph of Section 4: "A cross section through the exterior walls is substantially as shown in Fig. 1(*b*)."

The computations of Section 4 would have been much simplified by making use of both axes of symmetry, taking the midpoint of member EL as a base, cutting the frame at point N and at the midpoint of member FM, and so confining all treatment to the upper right-hand quadrant.

Much of the "Summary" is devoted to a discussion of construction details which were not previously treated and which, accordingly, could well have been placed in a section of their own.

Corrections for *Transactions:* In June, 1946, *Proceedings,* on page 769, change line 17 to read "* * * elastic curve is illustrated in Fig. 11(*d*) as compiled from Fig. 11(*c*)"; in Fig. 10(*a*), "80 *T*" should be "80 *T*'"; on page 770, line 8, computation opposite section 3, change "0.200" to "0.020"; in Fig. 15, the *y*-component of distance between E′ and F′ shown as "1.405" should be changed to "1.450"; and, in Fig. 17(*a*), the hatching in the two M_D-areas should be completed. Other corrections were published in November, 1946. *Proceedings,* page 1324.

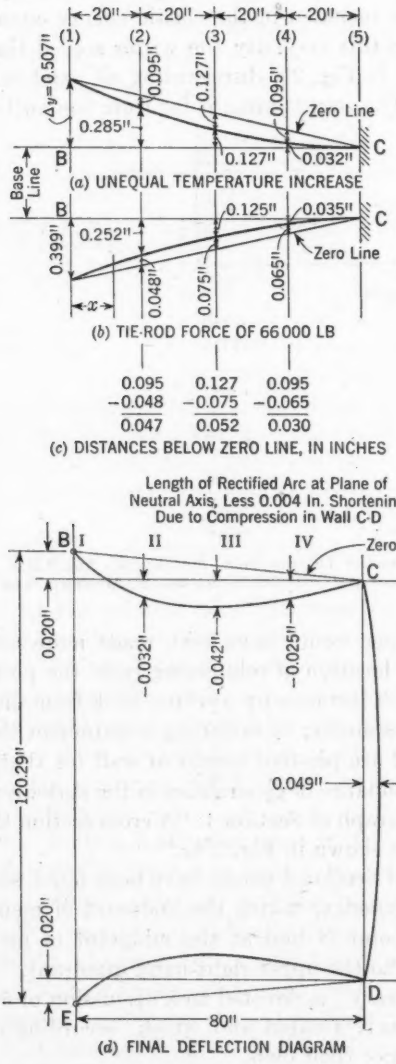

FIG. 11.—CONSTRUCTION OF DEFLECTION DIAGRAM IN RELATION TO THE ZERO LINE

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

CLEANING AND GROUTING OF LIMESTONE FOUNDATIONS, TENNESSEE VALLEY AUTHORITY A SYMPOSIUM

Discussion

BY JAMES B. WARD, J. C. NOWELL, JOHN F. PARTRIDGE, AND PORTLAND P. FOX

JAMES B. WARD,[10] JUN. ASCE.—To those concerned with the investigation, preparation, and treatment of foundations, the experiences set forth in this Symposium are invaluable. Since dams are built to serve a variety of purposes, it is no longer practicable or possible to locate the sites only in areas where good foundations are available. It is often necessary to locate these structures where they will best serve the purpose for which they were intended, and to improvise methods and procedures to meet the foundation conditions as they are. Such was the case in the selection of the Fort Loudoun Dam and Douglas Dam sites, the cleaning and foundation grouting of which are so ably presented by Mr. Pauls and Mr. Taylor. Moreover, the authors of this Symposium have demonstrated the utility of adequate geologic information in the planning and again in the construction of dams.

To appreciate the magnitude of exploration and foundation treatment required for the successful completion of the Fort Loudoun Dam, one has merely to examine the drilling, mining, and grouting quantities. The fact that more than 800,000 lin ft of drilling (including 584,000 lin ft of diamond core drilling, 16,000 lin ft of large-diameter calyx drilling, about 10,000 cu yd of concrete to backfill cavities and mined-out seams, and approximately 2,000,000 cu ft of cement and cement-admixture grouting) was necessary, testifies to the enormity of the undertaking. Seldom, if ever, have the ingenuity and combined efforts of engineers, geologists, and construction specialists been so challenged.

NOTE.—This Symposium was published in December, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1947, by Berlen C. Moneymaker, and C. E. Blee; and May, 1947, by Robert A. Laurence.

[10] Dist. Geologist, Soils and Geology Section, U. S. Engineer Office, Mobile, Ala.; formerly Resident Geologist, Fort Loudoun Dam, TVA, Lenoir City, Tenn.

Undoubtedly, the most striking feature of the Fort Loudoun foundation is the fact that subwater table cavities and weaknesses were explored and treated at what are thought to be unprecedented depths, below any previous water table level. The method in which this was accomplished was well covered in the Symposium. The formation of such deep-lying cavities has been the subject of much discussion. Observations made almost daily during the construction of Fort Loudoun Dam support the explanation postulated in Mr. Pauls' paper. It may be emphasized that the clogging or damming up of subterranean channels by the insoluble residue from the solution of the impure carbonate rock appears to play an important role in the formation and rerouting of cavities and channels to deeper levels as solution continues. This may well explain the scarcity of cavities at great depths in the purer or more soluble carbonate rocks, such as those encountered at Douglas Dam. Moreover, the statement that the formation of cavities may be more dependent on structural deformation than on the relative solubility of the rock also deserves emphasis.

Of considerable interest and importance to those engaged in the selection and investigation of dam and quarry sites in limestone and related rock formations is the condition encountered in the north abutment ridge. What appeared as a steep and apparently sound linestone bluff developed into nothing more than a false front with extensive weathering and solution just behind it, causing a series of deep troughs separated by intervening pinnacles. This condition may be found where steep bluffs have been formed by encroaching meanders and may be due to the fact that overburden is not able to form on a steep bluff that has been or is being cut back or undercut and, hence, rain and surface water drain away immediately. In the case of ridges behind bluffs, it is possible for the overburden to accumulate, forming a catchment for the rain water. The water, in turn, seeps downward to, and finds access along, joints or other areas of structural weakness in the rock. The ground water is thus recharged in a manner that permits continuous solution. In the exploration and treatment of a formation of this nature, horizontal and low-angle core drill holes may be used most advantageously.

It was apparent at the outset of foundation exploration that detailed and reliable geologic information, properly portrayed and prepared well in advance of construction, was mandatory for the effective and economical preparation and treatment of this highly defective dam foundation. Thus, a staff of geologists consisting of a resident geologist, with from two to three assistants (an organization larger than is usually required on a project of this nature), was in residence during most of the construction period; but, to re-emphasize the magnitude of the geologic and foundation work involved—at the peak of construction as many as sixteen diamond core drills and six calyx shot drills were in continuous operation—three shifts, seven days each week. During the course of the project several hundred geologic drawings, including sections, sketches, plans, and models were prepared which portrayed the foundation condition and aided in the determination of the required excavation, as well as the type and probable extent of foundation treatment. The exploratory drill pattern was such that geologic sections could be prepared not only along ranges parallel and normal to the axis of the dam structures, but also, by interpolation,

along any other direction desired. With a myriad of such sections, they were difficult to use unless they were studied in their relationship with one another. For this reason, what are thought to be novel methods of displaying subsurface conditions were devised. After considering peg, glass, and cardboard models, it was evident that some type of transparent or sectional model would be best suited for the visualization or interpretation between the various related sections. Hence, after experimentation, it was found that a transparent model could be constructed most economically by using rigid celluloid film positives, developed from photographic negatives of the geologic sections. These film positives were assembled in an upright position on a rectangular wood base, on which an outline of the particular dam structure had been superimposed. The transparent sections were properly oriented by dovetailing, and were correctly arranged on, and attached to, the wood base by fastening them through metal eyelets to piano wire that was suspended along the top and bottom of four upright steel rods securely anchored at each corner of the rectangular base. The completed model enabled at a glance the visualization of the foundation conditions in an accurate three-dimensional manner.

Another type of model used to advantage may best be described as a hollow, sectional model of plywood construction. It consists simply of mounting a front and reverse print of a geologic section on the front and back of a piece of thin plywood of similar dimensions, and arranging a series of related sections so mounted on a wooden base as to correspond with their location to each other and to the dam structure being represented. The sections are arranged and held securely on the base in an upright position by the slit grooves provided in the sides of a 3-in. upright board wall built around the base. Both longitudinal and transverse sections were dovetailed together in the manner of the transparent model described herein; and the sections were cut with a band saw along the line of the proposed excavation indicated thereon, thus enabling one, simply by removing the upper part of a section, to study the foundation conditions that existed before, and those that would exist after, excavation. This model was invaluable both in the planning for the removal of a maximum of the required excavation with mechanical equipment and in the determination of the method and probable extent of foundation treatment required.

That the difficult foundation conditions at Fort Loudoun were successfully overcome is attested by the fact that seepage through the foundation at full reservoir is inconsequential. Thus, it is indeed gratifying that the contributions of the geologists on this project have been so generously acknowledged in Mr. Pauls' paper. Assisting the writer at intervals during the construction of this project were the following geologists: Earl C. Van Horn, William V. Conn, Coy L. Watson, and Chilton E. Prouty.

J. C. NOWELL,[11] M. ASCE.—In considering Mr. Taylor's able paper on the extensive cleaning and grouting operations required at Douglas Dam, it may be of interest to connect it with the study of the original, preliminary drilling records and with the selection of the position of the axis of the dam. Early

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drilling had disclosed numerous cavities, but restrictions on the authorized scope of the preliminary program had prevented drilling sufficient intermediate holes to determine the extent or the interrelation of the numerous major cavern areas which were encountered later. The Douglas project was suddenly authorized for immediate construction about two months after the Japanese attack on Pearl Harbor, Hawaii, with the unprecedented schedule of closure set at fourteen months, to provide additional power for expanding aluminum production.

It was essential to locate the dam axis immediately and to begin construction without awaiting the results of additional exploratory drilling. In the interests of speed it was perhaps fortunate that the topography closely limited the location of the axis, especially at the bluff on the left abutment. Brief study was given to swinging the right end of the axis slightly to the west in an effort to place the north abutment farther downstream from the creek just above the dam. This creek had cut somewhat of a gash into the sloping north bank, indicating a possible area of fractured rock with increased subsurface weathering.

On the other hand, such a shift in the north abutment would have resulted in obvious disadvantages in the layout of the structures. It was decided, therefore, to proceed full speed on the preferred basis of a straight alinement at right angles to the stream bed. Subsequent drilling indicated such lack of improvement in the condition of the rock downstream as to have resulted in no major reduction in the cleaning and grouting operations for foundation treatment. However, in limestone and dolomite areas especially, it seems advisable to emphasize that a preliminary drilling pattern on 100-ft squares (as mentioned in Mr. Taylor's paper) is insufficient to assure any definite opinions as to the expense or time required for the construction of a major dam. It was fortunate that the foundation conditions actually encountered did not materially delay the completion of the project as a whole. However, this should be credited to the resourceful methods of the construction personnel, rather than to any adequate exploratory program, which had been impossible due to the lack of authorization for extensive investigations.

Mr. Taylor has well described the rather wide variety of methods used in the foundation treatment for the Douglas project. It is interesting to review the principal factors considered in determining the specific treatment to be applied to any particular part of the foundation. In addition to the usual requirements, such as adequacy, permanency, and cost of treatment, special emphasis was placed on speed to meet the unusually fast schedule for closure of the dam in the spring of 1943. With wartime demands on the aluminum company's operations and on the atom bomb project at near-by Oak Ridge, Tenn., the question of speed, rather than that of cost, was given primary consideration. In general, therefore, mass excavation and other mechanical methods were selected, rather than alternatives requiring more handwork and extended periods of time for the foundation treatment. For example, when a difficult cavernous condition was disclosed under the north half of the spillway apron, a redesign was determined on dropping the apron grade in this area. Thus, a heavy general shovel cut, excavated to an elevation below the weathered rock,

was decided upon rather than the filling of the cavities below the cap rock. The latter measure would have been especially slow because of cofferdam leakage conditions described in Mr. Taylor's paper.

Similar need for the speed of mass excavation and concreting was one of the prime considerations in deciding to carry the left abutment blocks in the vicinity of Station 19 down to the level of a bedding cavity which was followed down the dip to about El. 890 (see Fig. 15). Brief consideration was given to an alternative scheme based on treating and leaving in place the partly fractured and somewhat weathered rock above the bedding cavity, with the load on the foundation then reduced by the adoption of a concrete core wall embedded in a rolled fill section, rather than the standard concrete gravity dam section. Such a scheme indicated a possible saving in expense, but would have affected completion dates adversely, especially as it would have been necessary to place the rolled fill in winter. It was again obvious, therefore, that the removal of the questionable rock not only would result in a more conventional and conservative design, but would be essential to meet required completion dates since mass excavation and concreting would eliminate much tedious drilling, grouting, and other "dental" work in treating cavities, which would have delayed the project.

As Mr. Taylor has stated, the heights of the caves on both the right and left abutments were increased by drift mining to permit better access for men and equipment in placing the concrete plugs. This also assisted in the bonding and keying of the concrete to the sound rock. It was particularly required for the left abutment where heavy sound rock, both above and below the cave, permitted the use of a narrow concrete plug designed to take the upstream headwater load by shear into the roof and floor of the cave.

It would be of interest to have available even an approximate estimate of the total extra cost of mass methods used in the interests of speed to reduce cleaning and grouting operations. In considering such problems, the large quantities of extra excavation and concrete involved are likely to appear inordinately expensive as compared with the estimate usually prepared for cavern filling, grouting, and other similar measures. On the other hand, the large quantities in themselves tend to reduce unit cost. Also, such mass measures eliminate the uncertainty of difficult "dental" work and questionable cavern grouting; and they provide a conservative design with little chance for the development of trouble from future leakage. Especially with wartime and postwar shortages of labor, there are obvious advantages in substituting machine operations for handwork. It seems probable, therefore, that little if any actual extra expense was encountered on the Douglas project, in lowering mass excavation grades, as compared with the expense of the extra and especially difficult foundation treatment that would have been necessary if the original tentative excavation grades had been held.

In spite of continuous wartime pressure to keep the job on schedule, regardless of the foundation difficulties encountered, it would appear that the project manager (Lee G. Warren, M. ASCE), Mr. Taylor, and others not only maintained a conservative point of view, which assured a thoroughly safe foundation, but also simultaneously used a wide variety of resourceful methods in the cleaning and grouting of the foundations for the Douglas project.

JOHN F. PARTRIDGE,[12] M. ASCE.—The profession is indebted to Mr. Pauls for his fine exposition on the treatment of the foundation for the Fort Loudoun Dam on the Tennessee River. This paper, together with previous papers convering other Tennessee Valley Authority (TVA) projects,[13,14,15,16] provides a strong background for a technique for handling foundation problems, which problems a few years ago would have been the cause for rejection of otherwise highly advantageous dam sites. This is of great import to the national economy, since before the rivers of the United States are completely controlled there will be an increasing scarcity of ideal dam sites.

Great strides have been made by the construction industry, in the development of methods and machinery for applying principles of modern foundation treatment, but much remains to be done in this field. The continued cooperation of geolgists, design engineers, construction personnel, and the supervisory field engineers is essential to progress in this work.

Standard TVA practice dictated that geologic data of an exploratory nature be furnished continuously to the design section, and preliminary designs be referred back to the construction forces for criticism on their suitability to current construction procedure. Both final designs and construction schedules incorporated the suggestions and recommendations of the field forces. In certain instances the field geologists were granted the supervision of construction activities, under the general supervision of the field engineers. This procedure was facilitated by the organizational setup which placed all construction activities pertaining to grouting and foundation treatment under an associate engineer, reporting directly to the construction superintendent, and nominally under the technical direction of the construction engineer.

The human equation, involving the vigilance of the staff of geologists, the core drillers and core drill inspectors, and the supervisory engineering forces, is the largest single factor in the successful treatment of foundation rock which has been weathered to the degree encountered at Fort Loudoun Dam. Each linear foot of the cutoff curtain is a potential hazard. The repeated thinning and swelling of the area of decomposition, as well as the weathering along the bedding seams and along the intersecting planes of fractures, require an alert and constantly repeated inspection, to insure the continuity of the plane of the cutoff curtain. Seams and intersecting joints in the plane of the curtain must be mined out, cleaned, and washed, until a "live-rock" contact, a "concrete-to-live-rock" contact, or a "grout-to-live-rock" contact is secured. Every step in the work (including both drilling and grouting) must be described accurately and conscientiously. Each step must be recorded and evaluated—including the logging of the drill holes, the plotting of drill holes to show all joints and fractures in the rock formation, and the recording, by stages, of all grout quantities. Effectiveness is often a matter of relative values. When there is

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[13] "Unusual Cutoff Problems—Dams of the Tennessee Valley Authority: A Symposium," *Transactions*, ASCE, Vol. 110, 1945, p. 947.

[14] "Foundation Experiences, Tennessee Valley Authority: A Symposium," *ibid.*, Vol. 106, 1941, p. 685.

[15] "Masonry Dams: A Symposium," *ibid.*, p. 1154.

[16] "Large Core Drills Aid Construction at Chickamauga Dam," by James S. Lewis, Jr., *ibid.*, Vol. 105, 1940, p. 850.

doubt, intermediate check holes should be drilled. A liberal drilling program in the exploratory stages of the work, will result in economies in the entire program.

In all phases of the work at Fort Loudoun Dam, the resident geologist and and the grouting superintendent had "their hands full." The great footage of cores from all core drilling operations meant many hours of examination and logging. Drilling and grouting were continuous operations. It was from the records of these operations that the picture of the weaknesses in the foundation rock was built up. Actual core losses, where present, painted a clear picture. In other instances it was only possible to piece together a clear view of the situation, by observing the "relative density" of lines of fracture (dip, strike, and oblique joints), revealed by plotting the logs of drill holes on cross-section paper, as profiles of each respective range. Except for this method of detection, much time would have been lost and much grout wasted, because to stop persistent leakage it would have been necessary to make a general coverage of an area, instead of proceeding first with a selective coverage of suspicious areas, revealed by the multiplicity of minor fractures detected in the recovered cores.

The drilling and grouting program at Fort Loudoun Dam involved the following total quantities (see Table 1):

Description	Quantity
Diamond drilling, in linear feet	445,839
Wagon drilling, in linear feet	197,226
Calyx drilling, in linear feet	15,007
Grout, in cubic feet	1,067,387
Concrete, in cubic yards	9,816

Construction was started in July, 1940, and power was delivered over the transmission lines in the autumn of 1943. It is obvious that the drilling and grouting program had to be pressed to completion throughout all stages of the construction work.

The general scheme of foundation treatment was formulated during the exploratory stages of the work with a pattern of holes on 50-ft, 60-ft, and 100-ft centers. The detail was developed and expanded as intermediate and supplementary drill holes supplied additional information. The logging of all holes drilled required the presence of a large force of geologists and inspectors. All cores were preserved until the information could be plotted and coordinated, formations were proved, and the data and findings were checked by the local staff and the board of consultants. Exploratory holes were subsequently augmented by intermediate holes, reducing the spacing at each drilling by one half. Each set of holes was grouted before redrilling and grouting, unless washing between holes was done to increase the efficacy of the grouting or to trace any interconnection of cavities and seams. Packers in various holes, and a liberal use of dye introduced into the wash water, aided in the latter operation.

Drilling and grouting were continued through all stages of evacuation. When concreting was started, pipe casing was carried up through the concrete, to facilitate subsequent drilling operations. There were some exceptions to

this procedure when drilling through the concrete proved necessary. In the case of the grout curtain for the spillway, powerhouse intake, and the service-bay sections, the heel of the dam at the upstream face, at ground level, was poured as an integral part of the dam structure, in a projection upstream. This provided a base for setting up the diamond and the calyx drills, and permitted the drills to be in continuous operation without interference with concreting operations as the various blocks of the dam were raised above. All grout holes and calyx drill shafts in these areas were drilled through this projecting lip, and thus afforded a positive connection between the grout curtain and the monolithic structure of the dam proper. The concrete backfill, in the mined-out sections and the large cavities along the line of the cutoff, was placed under pressure, induced by pouring upgrade, through successive vertical shafts (36-in. calyx drill holes). These areas were drained of trapped air and subsequently grouted through grout pipes placed before the concrete was poured. Further consolidation was insured by the regular pattern of curtain drilling and grouting, done after the completion of mining and concreting operations.

Reference is made by Mr. Pauls to the formation and the character of the cavities, particularly in the zone of saturation. This deep weathering and the formation of cavities below the water table caused no small annoyance, before the significance of the process of their formation was recognized. Water was fed into the subwater table strata, over a wide area outside the cofferdam, wherever bedrock (with or without an overburden of gravel) was in contact with the waters of the river channel. Percolating to great depth, by direct or indirect paths, the waters emerged within the cofferdam area, either as seepage over a wide area, or as flow in quantity at points of weakness in the rock formation. Emergence usually occurred along dip joints or weathered bedding seams, particularly at the trace of their intersection. Leakage along the strike joints usually was more direct and in greater quantity, but at lesser depth.

The resultant characteristics of the area inconvenienced the work, through the difficulty of tracing the source of the leakage, (*a*) in the control of the water for construction purposes, and (*b*) in the determination of the foundation treatment, and the probable extent of such treatment. Once the principle of deep percolation was recognized, it was evident that control of the leakage was possible only by interception, since control of the rock surface outside the cofferdam was obviously impractical.

Interception was possible by stage drilling and grouting, (1) on the river-bank, immediately outside the cofferdam, (2) through the base of the cofferdam, (3) at any point between the cofferdam and the area of excavation for the dam, (4) along the upstream or downstream face of the foundation excavation, or (5) within the area of the excavation proper. Grouting with asphalt, a mixture of cement and clay, and neat cement grout were all used successfully. Asphalt proved to be much better than coal-tar pitch. The latter is slower to melt, more likely to chill and plug the hole, and it emits fumes of brownish hue which create dangerous as well as disagreeable working conditions. An asphalt with melting point of 165° F to 175° F, proved to give the best results. Experiments with clay-cement mixes showed best results with 9 cu ft of clay (absolute volume) to 3 ft of water. Lesser amounts of cement were unsatis-

factory and showed considerable shrinkage. In holes showing very large "takes," a 9:2 mix was used, completing with a 9:3 mix at refusal. In the 9:3 mix actually 12 cu ft of clay were used instead of 9 cu ft, because of the loose measure. Although the water content never exceeded 7.5 cu ft per batch, it was varied below this amount, in accordance with the desired fluidity. The clay was obtained from borrow pits in the vicinity of the dam. It was red in color, nonplastic, free of sand, but given to crumbling on drying. It was screened through half-inch hardware cloth, to remove large lumps, vegetation, or other large objects that might cause damage to the pumps or clog the lines. Central station grout mixing was used successfully throughout all grouting operations. Mixes were pumped with facility, and at times the varying distances exceeded 1,000 ft.

In general, asphalt grouting was used in the more open formations, particularly along the outcropping of badly weathered seams, or where the joints being treated carried water in quantity sufficient to carry away the cement and cement-clay mixes as fast as pumped in. Sandbags were placed over surface leaks as they developed, thus allowing the asphalt to cool and harden sufficiently to prevent further loss of asphalt. Cement-clay mixes were used when large takes were experienced, particularly in cofferdam grouting, in the treatment of the north reservoir rim, and in the grouting of the saddle dam. For the cutoff and curtain grouting under the lock and the concrete sections of the dam, neat cement grout was used. Cement grout was used similarly in the treatment of the south embankment and the north core wall (including the north abutment rim immediately upstream from the core wall).

Seepage from deep penetration was stopped by consolidation grouting in a pattern over the entire area of excavation. Interception of heavy leakage required a concentration of effort to provide grout in quantity at a point a sufficient distance down the dip of the rock to permit hardening and so to offset excessive leakage. A range of grout holes, parallel to the strike of the rock, gave a curtain at right angles to the intersecting dip or oblique joints and tapped the weathered bedding seams. Such a condition in block 10 of the spillway section is described by Mr. Pauls. There the asphalt grouting proved most effective in the open weathered formation along the outcrop of the bedding seams, and was secured by subsequent grouting with cement below (down the dip) and in back of the asphalt curtain.

The drilling and grouting program for the south embankment started in the early stages of construction, and was pushed constantly in order to complete the work before beginning to place the earth fill embankment. The magnitude of the task was not understimated and the job was completed with a minimum of interference with construction operations. Interference resulted chiefly from a lack of proper facilities for the disposal of wash water from washing operations preceding the grouting. In preparing for the cutoff trench (which was excavated southward from the lock to a point well beyond the old river channels), the first step was the excavation of the overburden of silt, clay, sand, and gravel. This was done for a sufficient width upstream and downstream from the line of the proposed concrete core wall, to give space for the use of power scrapers and rollers during the placing and compaction of the earth fill in the embankment.

The removal of the overburden probably increased the lateral leakage during the grouting, but an even greater advantage resulted from then being able to observe directly the results of the stage drilling and grouting.

In describing the treatment of the foundation rock where the core wall of the south embankment crossed the old river channels, it probably would have been more informative to have stated that sufficient water to require control thereof by grouting entered the area from upstream along the strike of the rock formation. The leakage was traced by dye, placed in seepage ponds which formed in borrow pits at the inner toe of the upstream cofferdam embankment, several hundred feet distant. A line of diamond drill holes, on 5-ft centers, drilled parallel to, and some 20 ft upstream from, the cutoff trench and a similar line with fewer holes 20 ft downstream virtually encircled the area. The grouting consolidated the enclosed area, intercepting from upstream the direct seepage or flow into the area, and from downstream, the backflow of subsurface water from within the cofferdam enclosure.

During the planning stage preceding construction, it had been suggested in anticipation of leakage under the cofferdam, that a line of grout holes be drilled and grouted, at intervals, along the entire length of the cofferdam. The alternative was to begin stripping and excavation, for the dam, and, as leakage developed within the area of excavation, to drill and grout to the extent necessary. The first would have greatly increased the scope of what was estimated to be a large and expensive drilling and grouting program. The second presented the disadvantage that if grouting of the cofferdam later proved necessary, in order to reduce the leakage within the area of the excavation, it might be difficult to prevent excessive loss of grout where the overburden had been removed by stripping and excavation. The decision was to proceed with the excavation and to follow with the grouting to control the leakage as it developed. No question was ever raised as to the soundness of this decision. Leakage under the cofferdam was serious in a few areas only. Some of these areas required intensive treatment; others required no treatment.

Permeability and porosity tests, in the area of the south abutment of the dam, indicated that the overburden to the south of the zone in which a core wall was considered necessary was essentially impervious, and that a satisfactory cutoff could be obtained through stage grouting without resorting to an open trench and concrete core wall. However, the provision in the scope of the project for a future canal and flood channel from the Little Tennessee River resulted in the necessity of reinforcing the overburden on the upstream face of the south embankment at the extreme south end. At that point the excavation for the proposed channel would be flooded by the impounded waters. The cut exposed certain lenses of gravel extending toward the axis of the dam and well under the body of the embankment. The reinforcement of the overburden consisted of removing the exposed gravel, to a depth depending upon its texture and its position, and its replacement with embankment material rolled in place.

With respect to the lock, the spillway, and the powerhouse sections, Mr. Pauls' paper, complete with illustrations, appropriate sections, and photographs, gives an excellent picture of the problem, and little can be added without needless repetition.

At the north abutment it should be visualized that the vertical oblique joints, transverse of the line of the north core wall, were continuous for an unknown distance upstream and downstream from the line of the cutoff curtain; and that they existed as open cavities from the rock surface to depths exceeding 200 ft, although no evidence of their presence appeared on the surface of the hillside. They were irregular in outline, with branching arms funneling into the main trunk; at times they exceeded 5 ft in width; for the most part they were filled with a residuum of arenaceous material saturated with water below the water table, to form a quicksand.

The presence of some such cavities was partly expected, because large sinks and open cavities were observed immediately following the first rains, and subsequent to stripping the overburden along the ridge above the north abutment for the construction of access roads and a storage area for concrete aggregate. At a later date, when work was started on the north core wall, the significance of these earlier sinks was recognized; and further disturbance of the natural ground surface along the north rim in the vicinity of the north abutment was avoided.

In the construction of a watertight cutoff in the area, it was appreciated that any core wall, or curtain, constructed by backfilling with concrete after all weathered material and residuum had been removed by mining operations, must be functional in design to withstand the development of a pressure head against the wall with reservoir full. To prevent rupture of the wall, and any movement of the ruptured section in a downstream direction, the concrete backfill was wedged against such movement, by cutting into the side walls of the joint cavity. Steel reinforcing bars were also placed to insure a monolithic structure with adequate support by the rock formation in place. This reinforcement was kept clear of known locations of intended calyx drill holes; although, if an occasional bar became displaced, there was no complication because the calyx drill would cut both steel and concrete with equal facility. In the treatment through the deep holes at Stations 0+86.8 and 0+96.8 (Fig. 12), the 7-ft stages mentioned could not always be maintained because of the varying degree of consolidation of the residual material filling the cavity. The safety of the men was the primary consideration fixing the depth of material removed at any one time. Stages of excavation actually varied from 4 ft to 10 ft.

Because of the nature of the material filling the cavity along these vertical joints, consolidation by grouting at the prevailing depths was variable in its effectiveness. Apparently the external pressure balanced the grouting pressure (external, relative to the grouting system, and internal, relative to the quicksand or filling medium), and the displacement, by grout, of the ground-table water in the quicksand was extremely local in its effect. The writer suggests that the grouting of such a medium, be selected as a problem for research in the field of soil mechanics. The displacement of the water of saturation, in a quicksand or in a sandy clay residuum under pressure, with slow or quick setting chemical grouts or neat cement grout, has not been reported, to the writer's knowledge, in current engineering literature.

A mechanical difficulty encountered was the difficulty in maintaining the intended spacing of the grout holes for their full depth. The drill holes were

deflected in the drilling, or were divergent because of inaccuracies in the leveling up of the drills themselves. This resulted in both unequal spacing at depth and in divergence from the plane of the drill curtain. Often the holes failed to strike the cavity, resulting in a loss of the hole and of the time consumed in drilling the hole. In drilling to depths of 200 ft this time loss was serious.

The time consumed in stage drilling and grouting, compared with the time of step procedure in drilling, mining, concreting, and redrilling with calyx drill holes, was a continually disputed point. In the writer's opinion, a more efficient operation would have resulted from a more extended grouting program.

Reference to Fig. 12 reveals a badly weathered area just beyond the end (block 9) of the north core wall. This region was thoroughly explored, and was deemed to be a critical area. Because of its depth, laterally, from the impounded waters in the reservoir, it was believed to be safe against the action of percolating waters. To insure against such lateral percolation, the rim treatment upstream from the north abutment grouting was definitely tied into the north core wall.

During the entire treatment of the north core wall area, and the north rim as far as the saddle dam, a check was maintained of the general effectiveness of the grouting program. This was done through a pattern of drilled wellholes, covering the area of the hillside extending from the future water line of the reservoir, up over the crest of the ridge and down the opposite slope away from the reservoir, to the base of the hill at Muddy Creek. Water-level readings were made twice daily and plotted as water-level contours. Changes in water table elevations were thus detected and, with due allowance for the effects of wash water, were used in the grouting operations, as a means of detecting the efficiency of the grouting operations.

PORTLAND P. FOX,[17] ASSOC. M. ASCE.—Few, if any, dams in the world have received such thorough and extensive foundation exploration and treatment as those described by Messrs. Pauls and Taylor. The authors are to be commended for an excellent presentation of much data of interest to engineering geologists.

Of the nine dams constructed on the main stem of the Tennessee River, only one was constructed on noncarbonate rocks. Watts Bar Dam is constructed on interbedded, fissile shale and sandstones of variable hardness. At the other eight main dams founded on carbonate rocks, open and filled cavities and channels, of various sizes and shapes, were found along bedding planes, faults, unconformities, and joints. The conditions at each site are quite different from the others because of the geology of the area. When confronted with numerous and different problems at each site, the geologist found the particular site at a given time a little worse than any others.

It is difficult to state that any one site is worse than another, but the foundation at Fort Loudoun Dam offered about the most varied types of cavities. The persistent steep dips and the variable physical character of the limestones and shales were chiefly responsible. Kentucky Dam was noted for both small and large persistent cavities parallel to bedding planes from El. 300 above sea

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level to El. 30 below sea level. Large, and nearly vertical, solution channels developed along joints to a depth of 200 ft below the top of rock. The foundations of Chickamauga and Guntersville dams ably demonstrated the effectiveness and the influence on solution in limestone by thin interbedded layers of shale or bentonite. Hales Bar Dam foundation demonstrated concentrated honeycomb cavities developed along, and adjacent to, two minor thrust faults in a thick-bedded, relatively pure limestone. Fort Loudoun Dam foundation demonstrated the great depth to which solution takes place in steeply inclined limestones in a warm humid climate and the control of the geologic structures on solution of the limestones.

The papers form valuable records for other engineers, but they would be even more valuable if the authors would give the unit cost of the excavation, the average unit cost of the diamond, wagon, and large-diameter calyx drill holes, and the unit cost of the two types of groutings used.

The success in the rapid and efficient treatment of these most difficult foundation problems during World War II was due to the availability of highly trained construction personnel.

The writer has had the opportunity to explore a number of dam sites on the upper Missouri River and its tributaries. Several of these sites are located on limestones as soluble as any in the Tennessee Valley Authority (TVA) region. There is a marked contrast in the degree of cavitation in limestones in this cold dry climate as compared with conditions in the TVA region. Relatively pure limestones comprise the foundation rock at the Yellowtail Dam in south central Montana on the Big Horn River. Recent core drilling has proved that cavitation is limited.

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DISCUSSIONS

EXPERIMENTAL OBSERVATIONS ON GROUTING SANDS AND GRAVELS

Discussion

BY FREDERICK J. SMITH

FREDERICK J. SMITH,[18] JUN. ASCE.—Petroleum engineers have long been concerned with the problem of leakage of fluids behind the casing of oil wells. At times their problem has been the same as that described at Baltimore, Md., in Mr. Machis' paper—namely, downward percolation of water between the casing and the well bore. At other times the problem has been the leakage of high-pressure gas or oil from the producing formation, upward between the casing and the borehole to shallower sands or to the surface. When such a leak reaches the surface around the casing there is danger of a "blowout," which results in the development of a crater around the well as the soil and subsoil is blown away by the escaping oil or gas. There is also the possibility of of the ultimate loss of the well by rupturing of the surface connections as their support is removed, as well as the danger that the escaping oil or gas may catch fire, thus adding to the general chaos.

Various methods have been developed by petroleum engineers to combat this leakage between the casing and the wall of the borehole. Some of these methods are applicable in the water works field, and others may be adapted for such use. The writer would like to submit the following suggestions, based on current petroleum engineering practices, as a solution of the leakage problem outlined in Mr. Machis' paper. It is assumed that drilling will be done by the rotary process, although these suggestions can be adapted for use in cable tool holes.

New Wells.—When a water well is drilled in an area where contaminated waters are found at shallower depths than the potable water to be exploited, the following program is suggested:

The well should be drilled to the water sand, but should not penetrate more than 3 ft or 4 ft into the sand before the protective casing is set. As shales and

NOTE.—This paper by Alfred Machis was published in November, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1947, by James B. Hays; and March, 1947, by William S. Foster.

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clays are prone to slough or "cave," a caliper survey[18a] of the borehole can be made before placing the casing in the borehole; the number and size of the "caves" can be established; and an estimate of the amount of cement needed to fill the annular space between the casing and the wall of the borehole can be prepared. Also, in light of the "caving" nature of most shales, it would be wise to have the bottom of the casing at least 1 ft below the top of the sand to prevent subsequent caving, and the development of leakage around the casing shoe, after completion of the well. The casing should be run to bottom and cemented using the "casing method" quoted by Mr. Machis (see heading, "Grouting New Casing: Casing Method of Cementing"). Enough cement grout can be pumped, by normal oil-field cementing practices,[18a] to establish circulation of the cement and to get "returns" of the cement at the surface outside the casing if so desired. During placement of the cement, the casing should be setting on the bottom of the hole. As soon as all cement has been pumped into the hole, the casing should be picked up and suspended with the bottom of the casing 1 ft below the top of the water sand.

After sufficient time has elapsed for the grout to set (from 24 hours to 72 hours, depending on local conditions), the cement remaining in the casing is drilled out, and the hole is opened 1 ft below the casing shoe (which still leaves from 1 ft to 2 ft of cement in the hole). The next step is to determine whether or not any leakage exists around the casing shoe, due to a faulty cementing job. This determination can be made by using any one of several available formation testers[19] with the packer set in the bottom joint of the casing. The testers consist of a packer and a series of valves which are run into the hole on the bottom of the drill pipe or tubing. This is the standard "drill stem test" familiar to petroleum engineers. A valve, below the packer, excludes fluid from the drill pipe, and maintains atmospheric pressure inside the pipe as the testing tool is lowered into the hole. This valve is opened after the packer is set, and the resulting pressure differential between the formation pressure of the zone below the packer and the atmospheric pressure in the drill pipe allows the formation fluid (if any is present) to flow into the drill pipe above the packer. The valve automatically closes when the packer is unseated, trapping the fluid in the drill pipe; the pipe is pulled from the hole, and the amount and nature of the fluid recovered is checked by the engineer supervising the test. A pressure recording device can be used with this testing tool to record the shut-in rock pressure, and the flowing pressure of the zone can be tested.[18a]

If the drill stem test does not recover water, the cementing job can be considered a success, and the well can then be completed using the standard practices for the area. If water is recovered on the test, however, it should be analyzed to determine if it came from the fresh-water sand below the casing shoe, or if it is contaminated water that is leaking between the casing and the wall of the borehole—indicating a poor cement job. in the latter case, the bottom 10 ft to 20 ft of the casing should be gun perforated[18a] (or bullet perforated) and cement grout should be squeezed[18a] through these perforations

[18a] For names and addresses of commercial concerns engaged in the various types of special services address an inquiry to the Editor, Technical Publications.

[19] "Formation Testing and Testing Equipment," by J. C. Padgett, *The Oil Weekly*, March 24, 1947, pp. 18-21.

to seal the leakage behind the pipe. The casing shoe can again be tested following the squeeze cementing. This process of squeezing and drill stem testing should continue until a shutoff is achieved. When no further leakage is apparent the well can be completed.

Old Wells.—Three typical cases applying to old wells can be discussed.

Case 1. Well Completed in a Potable Water Sand and Later Contaminated by Leakage from Behind the Casing.—The first step in the elimination of this type of leakage should be to fill the open hole below the casing with fine sand and gravel in order to protect the face of the producing sand from any possible contamination by the cement grout. To eliminate the leakage, the bottom joint of the casing should be gun perforated, and cement grout squeezed behind the casing through the perforations. When the grout has set, the cement remaining in the casing can be drilled out, and the cement job tested as previously described. This process may have to be repeated one or more times until the leakage is completely sealed off. The sand and gravel can be removed from the hole after the shutoff is obtained, and the well then put back in production.

Case 2. Leak in the Casing.—If a leak in the casing is suspected, it can be located by running a "water survey" or "electric pilot." The writer is not familiar with the details of this type of survey, although the service has been available to the petroleum industry for some time.[18a]

When the leak in the casing is located it can be eliminated in the following manner: A drillable cement retainer or plug should be set in the casing below the leak; the casing is gun perforated in the zone of leakage, and then the perforated section is squeeze cemented. When the cement has set, the cement remaining in the casing should be drilled out to a point below the leak, but above the cement retainer, and the zone of leakage drill stem tested as described herein under the heading, "New Wells." Perforating and cementing should be repeated, if necessary, until no further leakage is noted. After all leakage is sealed off, the cement retainer can be drilled out, the well cleaned out to bottom, and then put back in production.

Case 3. Squeeze Cementing of Shallow Sands Behind the Casing.—At times it may seem advisable to squeeze some cement into the shallow contaminated sands covered by the casing. This would be especially true in plugging an old well where it is impossible to reclaim the casing. These sands can easily be located if a geologist's sample log or an accurate driller's log of the hole is available. Where such logs are not available, or where the records are very poor, the sands behind the casing can be located by a "gamma-ray" survey, a relatively new process that is being used in the petroleum industry. These surveys are based on the natural radioactivity of the sedimentary rocks, the survey instrument recording the intensity of the "gamma-ray" radiation of the formations as the instrument is withdrawn from the borehole. This type of survey is advantageous because it can be run in any type of well fluid, and it will locate the various formations accurately in both cased (one or more strings of cemented casing) or uncased holes. The disadvantage of these surveys lies in the fact that some idea of the normal sedimentary sequence of the area is needed to interpret these logs properly, as sands and limestones give similar reactions.

This disadvantage can be overcome (at extra cost) by making a "neutron" survey in connection with the "gamma-ray" survey, and properly correlating the two surveys (done by the service company[18a]). The "neutron" survey, in contrast to the "gamma-ray" survey, is based on the intensity of radiation from the formation as a result of bombardment by neutrons from a radioactive source suspended below the survey instrument.

It has been found that the best results are obtained, in petroleum geological studies, when the radioactivity logs are used in conjunction with any geologist's or driller's logs that are available from the same area. The aforementioned inability to differentiate between a sand and a limestone should not be too big a disadvantage in the water works field, since a porous limestone is potentially as dangerous as a sandstone as a source of contaminated water.

After the sand bodies (or limestones) have been located, they can be grouted by squeeze cementing following the procedure outlined for case 2.

The Use of Plastics to Seal Contaminated Sands.—Many oil companies have been experimenting with plastics; they have been trying to "shut off" bottom hole water by introducing liquid plastics of various types into the borehole and allowing them to harden. Some good results have been reported from such experiments, and these materials might prove very useful in the water works field. Each contaminated sand encountered in drilling a new well could be impregnated with plastic as soon as it is drilled, thereby eliminating the source of future pollution.

Service Companies.—Communities located in the areas of active petroleum exploitation are familiar with most of the service companies[18a] engaged in the activities described herein. Their characteristically colored cars and trucks are being found throughout a wider and wider area as the search for oil expands.

Locating Cement Squeezed Behind Casing.—By laboratory experiments Mr. Machis has determined the effects of such physical constants as porosity, permeability, pressure, and water-cement ratio on the problem of forcing cement grout into a sand formation. Where the porosity and permeability of a sand have been determined from cores, it should be possible, using Mr. Machis' results, to determine the theoretical quantity of cement squeezed into a subsurface sand body under set conditions of pressure, type of slurry, and time and rate of pressure application. No accurate method has been developed to check, below ground, the theoretical results to be expected from a subsurface squeeze job.

It is possible, however, to determine the vertical extent and the mass distribution of the cement following squeeze cementing. This determination is made possible by adding a radioactive mineral to the dry cement to be used, and by "gamma-ray" surveys of the borehole before and after the squeeze job. The radioactive mineral is usually "carnotite," a potassium-uranium-vanadium oxide, and is added to the cement in the ratio of from $\frac{1}{4}$ lb to 1 lb of carnotite per sack of dry cement. Carnotite, because of its nearly colloidal size and specific gravity, remains in a fairly stable mixture with the cement slurry (usually 15.5 lb per gal), and does not tend to settle out of the slurry.

In practice a "gamma-ray" survey of the borehole is made prior to the squeeze job, and is used as the reference line for all future surveys. The per-

forated zone is squeeze cemented, using the radioactive cement, and a second "gamma-ray" survey is run after the hole has been cleaned out. By superimposing the second survey upon the original, the vertical distribution of the radioactive cement behind the pipe can be determined by the sharp changes in the "gamma-ray" intensities between the two surveys at the upper and lower limits of cement travel.[20] Some idea of the mass distribution of the cement can be obtained from the comparison of the two surveys when it is understood that the greater the mass of treated cement at any point, the greater will be the "gamma-ray" intensity. In all such comparisons of "gamma-ray" surveys, in connection with radioactive squeeze cementing of a sand section through perforations, that the writer has seen, the "gamma-ray" intensities were highest opposite the sand section, and tapered off to the original values a short distance up or down the hole. The concentration of the "gamma-ray" intensities is due to (*a*) a filter cake mass being formed in the annular area between the casing and the wall of the borehole; or (*b*) to penetration of some of the solid particles of cement and carnotite into the sand, or (*c*) to a combination of both.

In some extreme cases two or more "radioactive" squeeze jobs have been performed opposite the same zone. A comparison of the "gamma-ray" surveys made after each squeeze job showed a progressive increase in "gamma-ray" intensities for each succeding survey, indicating the probability that at least a part of the radioactive cement slurry had penetrated the sand body during each squeeze job.

This combination of radioactive cement and "gamma-ray" surveys does not give definite values to the amount of cement forced into a formation; nor does it supply the depth of penetration of the cement into the formation. The method does give an answer to the problem of the vertical travel of the cement behind the casing, and an approximate location of the main mass of the cement forced into the annular area between the casing and the wall of the hole.

Summary.—Mr. Machis' paper is an excellent addition to the literature on grouting of sands and gravels. His experiments have supplied data of much value for the solution of grouting problems. Mr. Machis has noted (see heading, "Repair of Leaking Wells"):

> "The highly sucessful utilization of this process in the oil well industry warrants close study for possible adaptation to the water well industry."

Therefore, the writer has tried to outline these oil-field practices in the hope that they will aid in the practical application of the results of Mr. Machis' experiments. A positive method of locating the cement squeezed into a formation through perforations is also included.

[20] "Some Practical Aspects of Radioactivity Well Logging," by W. J. Jackson and J. L. P. Campbell, *Technical Publication No. 1923*, A.I.M.M.E., pp. 14–16.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

REFLECTIONS ON STANDARD SPECIFICATIONS FOR STRUCTURAL DESIGN

Discussion

BY I. OESTERBLOM, AND JOHN M. HAYES

I. OESTERBLOM,[11] M. ASCE.—It is most gratifying for an old practitioner in structural engineering to see a paper such as that by Professor Freudenthal on "design specifications," for it shows how far the profession has gone since the beginning of the century. At that time there were very few controls and these were in the larger cities only. Worse yet—the few that had been set up were heavily tainted with politics. It may be amusing now to remember that concrete "had no business aboveground," and when it was most graciously allowed it had to be heavy—always heavier than brick, for that was good politics.

The writer at that time was designing a big stable in Brooklyn, N. Y. The stalls "had to be" separated by 8-in. brick walls—or 12-in. concrete; the 4-in. reinforced concrete partitions the writer had proposed were laughed out of existence. Nothing was mentioned about how the concrete was to be made, however, so that into the walls ultimately went all kinds of refuse salvaged from the lot, including some stable manure.

That time is gone now, seemingly, because the authorities are willing to listen to such excellent rationalization as is presented by the author; and this again raises the very important question: Who were the men that made the most recent codes and to what extent were they governed by the law of probabilities? This is a very pertinent question. Even today one may examine official schedules of permissible live loads, and find that there are great variations even for cities in which they should be the same. Any recent assembly of loads will show variations to 100% where there should have been no variations.

In the light of the present paper, therefore, it is proper to ask: How, and by whom, were the codes made? Were they made by politicians or by practical builders? Were professional engineers also consulted? If engineers were consulted, to what extent were their opinions respected and followed? If they

NOTE.—This paper by Alfred M. Freudenthal was published in February, 1947, *Proceedings*.

[11] Engr., Carbide and Carbon Chemicals Corp., South Charleston, W. Va.

were not consulted, why not? Also what was done to tell the world that engineering judgment should have been included—for the sake of equity and safety for those who "foot the bills."

Bridge builders, as a rule, have been quite free from political interference. Therefore, Example 1 is well chosen; but there should have been another similar example for the common building loads in regard to which more rationalization is sorely needed. Usually there is traffic load on all floors, with people temporarily at rest also included. This load varies for different kinds of service; but it is usually very light. It is the incidental loads of equipment that run the specified loads so high—and keep the code makers in such deep doubt that their guesses seem like a chapter from "Alice in Wonderland." Can it be stated now that the Freudenthal method has given designers the means to combine the light traffic loads with the heavy equipment loads into a rational average—one for each type of building? If this be so, it would be interesting to review the principal classes of building service and study how the rational live loads compare with their political and regional predecessors.

By the way, that problem is also a challenge to the author—of both interesting and practical significance.

Example 2 is not of equal significance in that it deals with permissible stresses for which the variability largely has been eliminated by more definite and better controlled processes of manufacture. This might not yet be true for a butt weld; it certainly will be true soon. If it were not so, one may well ask if the probability method is in order. If a load were assumed too light, it would not be serious, but if a weld strength were too low at a critical point the result might be disastrous—and to include such a weak point in the setup for probability will not save a structure when it is cursed by such an element of possible breakdown.

JOHN M. HAYES,[12] Assoc. M. ASCE.—Very pointedly, this paper emphasizes the importance of standard specifications for structural design and clearly advocates the adoption "of the scientific method in their preparation." Too often design practice, as reflected in standard specifications, has lagged behind research, theory, and the study of the action of existing structures. There seems always to be somewhat of a constant conflict between theory and practice. This is futile, since only an integral fusion of both will produce successful results. The great English statesman, Benjamin Disraeli, has stated, "A practical man is a man who practices the errors of his forefathers."

Normally, theory precedes practice. However, this is not to belittle practice. Hardy Cross, M. ASCE, has written:[13]

> "* * * a designer of engineering structures upon which the integrity of property values and the safety of human life depend has no right to accept theoretical results unless he finds for them some support in experience or in common sense."

This seems paradoxical, but it only emphasizes the complexity of the problem.

[12] Dist. Bridge Engr., Public Roads Administration, Little Rock, Ark.

[13] "Limitations and Application of Structural Analysis," by Hardy Cross, *Engineering News-Record*, October 17, 1935, p. 535.

Specifications for structural design should reflect practices based on sound thinking, thoroughly blended with theory and experience. This experience should be tempered with information derived from a complete study of structures in service. Every engineer should recognize his obligation to gather data on the actions of structures in service at every opportunity and disseminate this information to the profession. In common with practitioners in every field, the structural designer must rely on authentic information from other engineers and related workers. This necessitates complete cooperation and coordination of all fields of engineering.

A structural designer must not only be able to interpret standard specifications and apply them to routine design problems, but must also have a working understanding of the theory and practice on which they are based. He must be thoroughly conversant with their limitations, and, therefore, should be able to cope with special problems that arise in design. Too often, specifications reflect personal prejudice and blind admiration for past practice.

The writer can do no more than to lend emphasis to the author's closing statement:

> "Nothing must be taken for granted and every relation assumed must be carefully examined by the most appropriate method, be it logical induction or statistical inference. Only thus will the impact of modern scientific development make itself felt in the engineer's everyday work."

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DISCUSSIONS

THEORY OF INELASTIC BENDING WITH REFERENCE TO LIMIT DESIGN

Discussion

BY GEORGE WINTER, AND D. C. DRUCKER

GEORGE WINTER,[6] M. ASCE.—In presenting this comprehensive and realistic solution of the problem of inelastic bending, the author has performed an unusually valuable service to the profession.

In his discussion[7] of the pioneering paper on "Theory of Limit Design" by Prof. J. A. Van den Broek, M. ASCE, the writer stated:

> "Any design theory that does not provide the means of determining the deformations corresponding to any given load up to the ultimate one is incomplete and questionable."

It is precisely this problem which is ably solved in this paper. Other attempts in this direction have been made, including the writer's sketchy treatment in the quoted discussion and some work in Great Britain. However, the paper is the first to be, at the same time, complete and practical. The extension to the strain-hardening regions and the discussion of shear stresses are particularly valuable.

From a practical point of view, the method is necessarily rather complex for use in design. The confirmatory evidence it presents for the original, simpler method of limit design is therefore doubly welcome. Although limited in design application, the author's paper should prove particularly valuable in research work in the interpretation of results of tests beyond the elastic range.

The main point of this discussion is to register a sincere request that the complete form of Table 1 be published with the closing discussion.[5a] The practical use of these methods is all but impossible without the complete tables

NOTE.—This paper by Alexander Hrennikoff was published in March, 1947, *Proceedings*.

[6] Associate Prof., Civ. Eng., Cornell Univ., Ithaca, N. Y.

[7] *Transactions*, ASCE, Vol. 105, 1940, p. 673.

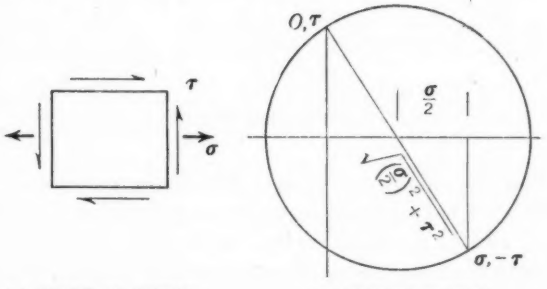
[5a] Tables 1(*a*) and 1(*b*) are greatly condensed forms of the complete tables, and they are confined to values of functions m, n, u, and $\frac{dq}{dm}$ for mild steel I-beams and channel beams with $\kappa = 1$ and 1,000 ϵ varying between the limits 1.1 and 65, as needed for solving the illustrative examples in the paper. The complete tables contain similar functions for mild steel computed for four values—of $\kappa = 0$, $\kappa = \frac{1}{2}$, $\kappa = 1$, and $\kappa = 1\frac{1}{2}$—and 46 values of 1,000 ϵ varying between the limits 1.1 and 100. The photostats of these tables are available on request at the price of $1.00.

mentioned in footnote.[5a] Indeed, as the author notes correctly, Figs. 15 and 16 are not necessarily accurate enough for numerical work. On the other hand, these tables are computed for values of κ not exceeding 1.5. Cases in which κ is larger, however, are frequent in standard wide flange sections. Thus, for example, for a 14 by 16, 426-lb wide flange section, the value of κ is of the order of 3.0. Similar values may also occur in built-up sections.

It would be of great practical value, indeed, if the author agreed to extend his tables to higher values of κ. For general use, the practical value of the paper would be greatly enhanced if it were possible to reproduce the complete and enlarged tables in the author's closing discussion.

D. C. DRUCKER,[8] JUN. ASCE.—The analysis by Professor Hrennikoff is based not only on plane sections remaining plane in the usual sense (that is, longitudinal strain proportional to distance from neutral axis), but also on the applicability of the stress-strain diagram for simple tension (Fig. 1). Unfortunately, the problem is far more complicated than any for which exact solutions of the idealized theory of plasticity have been found. It is the purpose of this discussion to indicate the necessity for a broader stress-strain relationship when vertical shear stresses are large, and to suggest that the elementary approach may seriously overestimate the load carrying capacity of short WF-beams or I-beams especially when fixed ended.

In the case of rectangular steel beams or any beam in which the shearing stress is small compared with the bending stress, once the unstable plastic yielding of the outer fibers is complete and the strain-hardening region is reached, the author's careful work undoubtedly will give very useful and accurate information. However, WF-beams and other structural shapes are so proportioned that the vertical shearing stresses are appreciable and, if concentrated loads act near the end of a beam (Fig. 24), may control the design. The larger the ratio of depth to span, h/l, the more important the shear stress becomes.



(a) STATE OF STRESS (b) MOHR'S CIRCLE

FIG. 28.—STRESS RELATIONS AT JUNCTION OF WEB AND TOP FLANGE NEAR RIGHT END OF BEAM WITH FIXED-ENDED CONDITIONS

An element at the junction of the web and top flange near the right support of a fixed-ended beam will be subject to tensile stress σ, and shearing stress τ, as shown in Fig. 28. A ratio of 20/12 for σ/τ is somewhat low but reasonable for a fixed-ended WF-beam, although usually too low for I-beams. However, it is interesting to note that if $h/l = \frac{1}{12}$ in Example 2, the nominal shearing stress V/A_w is greater than 12 kips per sq in. when the bending stress is 20 kips per sq in. Substituting $\sigma = 20$ kips per sq in., and $\tau = 12$ kips per sq in., the

[8] Asst. Prof. of Mechanics, Illinois Inst. of Tech., Chicago, Ill.

maximum shear stress at the point is $\tau_{max} = \sqrt{(\sigma/2)^2 + \tau^2} = 15.6$ kips per sq in., ignoring, as is customary, stress concentrations at the fillets and the high stresses actually present near supports or concentrated loads.

Experiments show that the plastic state in structural steel starts when the maximum shear strain energy reaches a limiting value. This criterion is more accurate but not so convenient in nominal stress analysis as the maximum shear stress basis; that is, yielding will occur when τ_{max} equals the yield point in shear—approximately 19 kips per sq in. for a tensile yield point of 33 kips per sq in. As all the elastic stresses are proportional to the load, this means that yielding will occur when the bending stress is $20 \times \frac{19}{15.6} = 24.3$ kips per sq in., or only three fourths of the yield point in tension (33 kips per sq in.). The closer the point to the neutral axis, the smaller the ratio of the bending stress to the tension yield point.

The distribution of shear over the cross section of an I-beam or WF-beam, Fig. 29, will often greatly reduce the load required for initial yielding. Although the effect of such local yielding on the main part of the flange can only be guessed, it would be reasonable to expect a decrease in the yield strength.

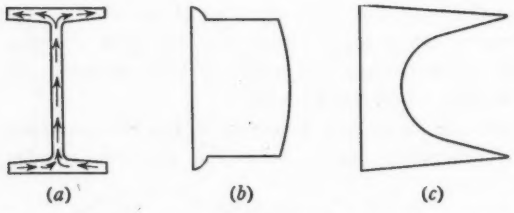


FIG. 29.—DISTRIBUTION OF SHEARING STRESS ON BEAM CROSS SECTION:
(b) NOMINAL, $-\frac{VQ}{Ib}$
(c) ACTUAL, AT FIXED SUPPORTS ALONG CENTER LINE OF WEB

No matter what the ultimate effect on strength, it becomes necessary to think in more general terms than the simple tension test and so Sections 6 and 11 on shear stresses do not seem valid. The resulting shear stress of 24.5 times the average is certainly not a possibliity in an I-beam, because yield must occur long before so high a stress is reached (at 19 kips per sq in. or less for the steel of the illustrative examples). As the integral of the shear stresses over the area of the cross section must give the total shearing force, the vertical shearing stresses in the zone of yielding cannot disappear; nor, correspondingly, can the bending stresses at two neighboring sections become equal in the manner shown in Fig. 18. Certainly, the distribution of the bending stresses is not given simply by the stress-strain diagrams for tension alone, but is a function of the shearing stress or strain as well as the bending strain.

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DISCUSSIONS

FORECASTING PRODUCTIVITY OF IRRIGABLE LANDS

Discussion

BY CHARLES KIRBY FOX

CHARLES KIRBY FOX,[10] M. ASCE.—The question at the end of the second paragraph of this paper—"What will be the returns" [of an irrigation project]?—is timely and shows that engineers are becoming more "cost conscious."

Two aspects of the paper, however, are open to serious discussion, namely: (1) The use of 32° F as a base and (2) the adoption of a range of available heat units in terms of thousands of degree-days above 32° F. Most plants begin growing when the mean temperatures reach from 42° F to 48° F. Since the rate of growth at temperatures lower than 50° F is slow, a mean monthly temperature of 50° F is a logical base. It is an easy base for computation purposes and is readily correlated with data issued by the United States Weather Bureau. The use of thousands of degrees per day above 32° F would complicate computations considerably. Elsewhere the writer has called attention to the advisability of using the 50° F base.[11]

The late Mr. Muldrow is correct in his observation that some plants cease to grow when the weather becomes too warm.

Closely allied to the question of growing temperatures are the following six factors: (*a*) The soils; (*b*) the quantity of water used: (*c*) the character of the water; (*d*) the transportation facilities; (*e*) the value of fluctuating dollar; and (*f*) special conditions and study required by some, often highly profitable, crops.

(*a*) *The Soils.*—The University of California, at Berkeley, has issued a bulletin[12] in which a soil classification is analyzed from a production point of view. The Soil Survey Bureau, United States Department of Agriculture, had reported on fifty-nine California areas of which six areas (more than 2,500,000

NOTE.—This paper by the late W. C. Muldrow was published in February, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1947, by Arthur F. Johnson, H. W. Van Loo, and Will H. Noble.

[10] Cons. Engr., Los Angeles, Calif.

[11] Discussion by Charles Kirby Fox of "'A National Reclamation Policy': Report of the Committee of the Irrigation Division," *Transactions*, ASCE, Vol. 95, 1931, p. 1357.

[12] "A Method of Rating California Soils," by Walter F. Weir and R. Earl Storie, *Bulletin No. 599*, College of Agriculture, Univ. of California, Berkeley, Calif.

acres) were not complete and nine areas (more than 4,750,000 acres) were not computed because of the alkali content of the soil. The University of California bulletin was concerned with the remaining forty-four areas (15,000,000 acres). Its classification, briefly, is as follows:

Class	Percentage of maximum yield
1	> 80
2	60 to 79
3	40 to 59
4	20 to 39
5	10 to 19
6	< 10

The average total producibility of classes 1, 2, and 3 was 70% or, roughly, two thirds. Classes 4, 5, and 6 are not considered suitable agricultural soils for cultivation and irrigation. The East Mesa of Imperial Valley in California is rated on the same basis. The area of California is given as 158,693 sq miles; and, since the foregoing soil survey is supposed to embrace most of the arable land in California, the forty-four soil-class areas have been reduced to a percentage of California area, as shown in Table 2. All these data show that the areas having the best soils are very limited.

TABLE 2.—Summary of Soil Survey Results (Percentages)

Class of soil	Production Based On:		
	Forty-four areas	California	East Mesa
Acres	(14,858,561)	(101,563,520)	(222,025)
1	13.6	1.95	0.2
2	10	1.43	3.6
3	19.3	2.75	26.2
(Total)	(42.9)	(6.13)	(30.0)
4	....	....	12.5
5	....	....	23.2
6	....	....	34.3

(*b*) *The Quantity of Water Used.*—By plotting more than 2,500 determinations of the duty of water (including many multiple tests), it was determined that, if the quantity of water required to produce the maximum yields were known, it could be assumed that two thirds of that water will produce yields of 90%; and one and one-half times that maximum quantity will only produce yields of 66%, or two thirds.

(*c*) *The Character of the Water.*—Alkali or saline waters, depending on relative concentrations, cause a decline in yields. As proof of this fact one has only to examine the thousands of acres of land in old irrigation districts, which were formerly productive but are now waterlogged and saturated with alkali. This is particularly noticeable in the great Central Valley (California) and the Imperial Valley, as well as in the Yuma Project in Arizona. The acreage yields (not dollar) of the Imperial Valley and the Yuma Project were plotted for the period from 1926 to 1940 and showed a decline of approximately one half, in relation to comparable crop yield data for Los Angeles County in California.

(*d*) *The Transportation Facilities.*—In some cases the transportation facilities will limit the yield of commercial crops.

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DISCUSSIONS

MISSISSIPPI RIVER CUTOFFS

Discussion

BY LYTLE BROWN, AND ANSON MARSTON

LYTLE BROWN,[12] M. ASCE.—The idea of making artificial cutoffs on the Mississippi River is an old one and has been put forward in the past by various people, mostly by those who had but little experience with the river.

In 1859, two officers of the Corps of Engineers, U. S. Army (Capt. A. A. Humphreys, Hon. M. ASCE, and Lt. Henry L. Abbot) made the first comprehensive study of the lower river. These two officers were among the most distinguished that the Corps of Engineers has ever furnished to the United States. Their massive report for many years was regarded as a standard in river hydraulics.[13] It contains an elaborate discussion of cutoffs based on both analysis and on the observation of natural cutoffs. The conclusion reached was unfavorable to cutoffs, but was cautiously stated, and ended with the remark that further discussion was not necessary since it was not practicable to make artificial cutoffs. At that time it was not practicable because the dragline on the hydraulic dredge had not then been developed. Gen. U. S. Grant tested the practicability of this theory opposite Vicksburg, Miss., in 1863.

The fear of injuring navigation by either artificial or natural cutoffs in 1859, when navigation was of supreme importance, made experiments with cutoffs a risky business. Experience with natural cutoffs and the authoritative report of Messrs. Humphreys and Abbot made a policy against cutoffs unshakable until 1933.

The plan for flood control on the Mississippi following the flood of 1927 (sometimes called the Jadwin Plan) contemplated no cutoffs; and, since that plan received the sanction of Congress, radical departures from it were frought with danger.

From 1929 until 1933 there existed among some of the authorities on the river an attitude of hostility toward anything that might in any way alter the Jadwin Plan. This attitude tended toward discouraging further study of new

NOTE.—This paper by Gerard H. Matthes was published in January, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1947, by W. E. Elam, C. L. Hall, H. D. Vogel, and Harry N. Pharr.

[12] Maj. Gen., U. S. Army (Retired), Franklin, Tenn.

[13] "Report on the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, *Professional Paper No. 13*, Corps of Engrs., U. S. Army, 1876.

attacks on the problem of flood control. Col. (later Maj.-Gen.) Harley B. Ferguson, M. ASCE, an officer well known to the writer (then Chief of Engineers, U. S. Army) as an able engineer willing and capable of developing a new attack on the flood control problem or other hydraulic problems, was accordingly appointed President of The Mississippi River Commission in the hope that he would not be crippled by adherence to any hard and fast ideas. General Ferguson was the first and only responsible man who ever brought to the Chief of Engineers the serious proposition to make artificial cutoffs on the Mississippi River. Whatever credit is due for a courageous effort to lower the height of floods on the confined waters of the Mississippi is due to Maj.-Gen. Harley B. Ferguson. There are many who project an idea where danger is involved but there are few with the courage to give it effect and to assume the responsibility. According to the writer's knowledge, Mr. Matthes' paper cannot be questioned in any important particular.

ANSON MARSTON,[14] PAST-PRESIDENT AND HON. M. ASCE.—The writer is keenly interested in this paper on "Mississippi River Cutoffs." Early in 1932, Maj-Gen. Lytle Brown, M. ASCE, then Chief of Engineers, appointed a three-man Mississippi River Engineering Board of Review to restudy and to report upon the "Jadwin Plan" for Mississippi River flood control, which had been adopted by Congress after the great 1927 flood. The Board members were the then Brig-Gen. Harley B. Ferguson, M. ASCE, President; the then Col. George B. Spalding, U. S. Division Engineer at St. Louis, Mo.; and the writer, as a civilian engineer.

It should be emphasized that General Ferguson is entitled to full credit for originating in 1932, and later as President of the Mississippi River Commission for constructing, the extensive and highly successful cutoff program so well described by Mr. Matthes. That the previous Mississippi River Commission policy and much engineering professional opinion had been hostile to cutoffs was demonstrated in the 1929 ASCE Symposium[15] on flood control. Therein Col. Edward H. Schulz, M. ASCE, United States Corps of Engineers, long engaged in Mississippi flood control work, stated: "Cutoffs * * * would be a detriment to the river and are decidedly inadvisable." This had been the long continued policy of the Mississippi River Commission and was supported by prevailing engineering opinion.

John F. Coleman, Past-President and Hon. M. ASCE, stated:

> "For the reasons stated, the writer is of the earnest conviction that levees, with auxiliary works for the protection of banks from caving, will afford the only certain method of controlling the floods of the Mississippi River."

However, in a preceding discussion,[16] Mr. Coleman indicated an open mind on trying out cutoffs "gradually," and "with caution." W. E. Elam, M. ASCE, assistant engineer, Mississippi Levee Board, and a few others ad-

[14] Dean Emeritus of Eng., Iowa State College, Ames, Iowa.
[15] *Transactions*, ASCE, Vol. 93, 1929, pp. 655–969.
[16] *Engineering News-Record*, June 28, 1928.

vocated trying out cutoffs, but it remained for General Ferguson to begin in 1932 to design, and in 1933 to construct, the cutoffs shown in Fig. 1.

One of the early acts of General Brown's Board of Review in 1932 was to ask Brig.-Gen. T. H. Jackson, M. ASCE, then President of the Mississippi River Commission, for suggestions concerning the work. He said nothing to the Board to indicate that he was in favor of making any cutoffs to improve the river. The Yucatan Cutoff (1929) into the Black River Channel has been mentioned (see heading, "Historical") as demonstrating how a properly designed cutoff could improve the river. However, it appears that General Jackson "desisted" from efforts to close the break only after efforts to do so by willow mats failed.

By that time it had become evident that there was danger that the river would break also through the Ashbrook and Leland Greenville Necks (Miss.) in spite of the Commission's costly preventive dikes. In 1933, the river did break the Leland Neck; the writer rode through the break on a government launch down a deep and wide raging torrent within less than 48 hours after it happened.

Construction of the model of the Greenville bends was a wise move, so successful that before the Board of Review concluded its work in 1933 the model was extended to include the whole distance from the Arkansas River to Old River. However, getting ready to quit losing fights to prevent Greenville bend cutoffs was quite different from starting a planned program to improve the river by making many cutoffs. In 1933, it was the writer's privilege during a conference in Washington, D. C., with General Brown, to advise him to approve General Ferguson's plans for the other Greenville bend cutoffs, shown in Fig. 1.

The Board of Review concluded its work in 1933, by which time the cutoff program described in the paper was well under way. Evidently its success had been all that was forecast. The advantage to navigation of shortening the river 170 miles between Memphis (Tenn.) and Baton Rouge (La.) is obvious. It also is obvious that the cutoffs have materially improved the flood-carrying capacity of the river in that reach, although the exact extent is not obvious.

Also, at each cutoff two channels—the cutoff and the still open channel around the old bend—are now available to carry the flow. The advantage of having the bends open to get rid of silt is obvious, but in the past such bends eventually have silted full, or at least have become blocked off from the river by silt. It would be of interest for the author to add some discussion of this matter to his valuable paper. Are there any actual measurements proving the continuity of the river's bed in the reaches north of Old River?

There are some other questions on which the author doubtless can give valuable fact or opinion information. Table 1 shows that the greatest flood to June, 1945, in the Mississippi River reaches from Arkansas City (Ark.) to Vicksburg (Miss.) since the cutoffs therein were made occurred in 1937, and was about 2,000,000 cu ft per sec. It is stated[17] that the 1927 maximum flood flow at Arkansas City was 2,662,000 cu ft per sec. and a still larger future flood

[17] *Transactions*, ASCE, Vol. 93, 1929, p. 707 and p. 874.

is possible. It would be of great interest to readers if the author would add information on the following three points:

1. What, in the author's judgment, is the present maximum flood flow in the Arkansas City-Vicksburg Mississippi River channel reaches which will not dangerously decrease the adjacent levee freeboards?

2. When the Board of Review concluded its work in the fall of 1933 it was of the opinion that, in addition to the completion of the cutoff program, an adequate floodway west of the river from below Arkansas City to Old River would be necessary to insure that part of the Mississippi Valley from flood damage by a future "super flood" of the magnitude then thought possible; it was hoped that the future developments of reservoirs above Arkansas City on the Mississippi River and its tributaries (including Arkansas River) might in time have some measurable value in reducing floods. What is the author's current opinion on this matter?

3. Fig. 1 indicates a well-developed Atchafalaya River channel from Old River to Grand Lake, although in 1932 such channel existed only part way south of Old River. Does this mean that General Ferguson's big dredges have succeeded in extending such channel clear to Grand Lake?

In conclusion, the writer wishes to say that the author is entitled to much credit for his own indispensable services in helping to plan and develop the entire admirable Mississippi River cutoff program described in the paper.

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DISCUSSIONS

STRENGTH OF BEAMS AS DETERMINED BY LATERAL BUCKLING

Discussion

BY EDWIN H. GAYLORD

EDWIN H. GAYLORD,[40] ASSOC. M. ASCE.—In a previous paper on the strength of beams as determined by lateral buckling,[41] it was suggested that the critical stress for a simply supported beam subjected to pure bending could be used as a design criterion for most cases of transverse loading. From the formula for the critical bending moment in pure flexure,[27] it can be shown that the critical stress is expressed by Eq. 4 with the factor k given by:

$$k = \frac{\pi^2}{4}\sqrt{1 + \frac{1}{\pi^2}\left(\frac{l}{a}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (93)$$

Eq. 7 can be obtained from Eq. 4 by substituting approximate values for I_x, I_y, and the torsion constant K, using the value of k given by Eq. 93, with the first term under the radical omitted so that,

$$k = \frac{\pi\, l}{4\, a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (94)$$

Values of k computed from Eqs. 93 and 94 are given in Cols. 3 and 5, respectively, Table 8. In the same table, Col. 2 gives values of k for a uniform load applied at the top flange. These values are taken from Col. 5, Table 1, except as noted. Granting that a design formula should be based on the case of a uniform load applied at the top flange, the errors involved in using the k-values of Cols. 3 and 5 are indicated in Cols. 4 and 6, Table 8. For small values of l/a, both approximate values of k are considerably in error.

NOTE.—This paper by Karl de Vries was published in September, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1946, by George Winter, and David B. Hall; February, 1947, by Theodore R. Higgins; March, 1947, by Neil Van Eenam; April, 1947, by H. N. Hill, and H. D. Hussey; and May, 1947, by H. G. Brameld.

[40] Prof. and Chairman, Dept. of Civ. Eng., Ohio Univ., Athens, Ohio.

[41] "Strength of Slender Beams," by George Winter, *Transactions*, ASCE, Vol. 109, 1944, p. 1339.

[27] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 261, Eq. 159.

TABLE 8.—VALUES OF k

$\frac{l}{a}$	Uniform load at top flange	Pure bending Eq. 93	Col. 3 / Col. 2	Eq. 94	Col. 5 / Col. 2	Eq. 95	Col. 7 / Col. 2	Eq. 97	Col. 9 / Col. 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0.6325..	1.84[a]	2.52	1.370	0.497	0.270	1.81	0.984	1.80	0.978
2.....	2.269	2.925	1.289	1.571	0.692	2.348	1.035	2.258	0.995
3.....	2.788	3.412	1.224	2.356	0.845	2.932	1.052	2.814	1.009
4.....	3.418	3.995	1.169	3.142	0.919	3.594	1.052	3.485	1.020
5.....	4.121	4.638	1.125	3.927	0.953	4.297	1.043	4.202	1.020
6.....	4.872	5.319	1.092	4.712	0.967	5.025	1.031	4.942	1.014
7.....	5.655	6.026	1.066	5.498	0.972	5.768	1.020	5.694	1.007
8.....	6.459	6.750	1.045	6.283	0.973	6.521	1.010	6.455	0.999
9.....	7.280	7.487	1.028	7.069	0.971	7.281	1.000	7.221	0.992
10.....	8.113	8.232	1.015	7.854	0.968	8.045	0.992	7.991	0.985
11.....	8.954	8.985	1.003	8.639	0.965	8.814	0.984	8.764	0.979
12.....	9.802	9.742	0.994	9.425	0.962	9.585	0.978	9.539	0.973
13.....	10.65	10.50	0.986	10.21	0.959	10.36	0.973	10.32	0.969
14.....	11.51	11.27	0.979	11.00	0.956	11.13	0.967	11.09	0.964
20.....	16.7[a]	15.90	0.952	15.71	0.941	15.80	0.946	15.78	0.945

[a] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, pp. 268 and 269, Table 23 and Eq. (*h*).

Since Eq. 93 overestimates values of k for small l/a-ratios, whereas Eq. 94 underestimates them, it is evident that a better value might be obtained by arbitrarily reducing the first term under the radical of Eq. 93. Col. 7, Table 8, gives values of k computed from the equation:

$$k = \frac{\pi^2}{4}\sqrt{0.50 + \frac{1}{\pi^2}\left(\frac{l}{a}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (95)$$

Col. 8 shows that these values check very well with those of Col. 2, even for the extreme values of l/a which were not included in Table 1.

Eq. 95 can be put into more convenient form by using the first two terms of the binomial expansion for $\sqrt{1+x}$. If $x \leqq 1$, the error of this approximation does not exceed +6%. The equation may be written:

$$k = 0.707\,\frac{\pi^2}{4}\sqrt{1 + 0.203\left(\frac{l}{a}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots (96a)$$

and also,

$$k = \frac{\pi}{4}\frac{l}{a}\sqrt{1 + 4.93\left(\frac{a}{l}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (96b)$$

From these equations:

$$k = 1.745\left[1 + 0.075\left(\frac{l}{a}\right)^2\right] \quad \ldots\ldots\ldots\ldots\ldots (97a)$$

$$k = \frac{\pi}{4}\frac{l}{a}\left[1 + 1.75\left(\frac{a}{l}\right)^2\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (97b)$$

The coefficient of the second term in the brackets has been reduced from its binomial expansion value. This was done because the effect of this term is greatest in the region where the k-values of Col. 7, Table 8, are most in excess of the values of Col. 2. The equations give approximately equal values when

$l/a = 2$. Eq. 97*a* is to be used if $l/a < 2$, and Eq. 97*b* if $l/a \geqq 2$. Col. 9, Table 8, gives values of k computed from Eqs. 97; and Col. 10 compares these values with those of Col. 2.

If the values of k from Eqs. 97 are substituted in Eq. 4, with $E = 30 \times 10^6$ lb per sq in.,

$$f = \frac{52.3 \times 10^6}{\frac{I_x}{I_y}\left(\frac{l}{d}\right)^2}\left[1 + 0.075\left(\frac{l}{a}\right)^2\right] \text{ if } l/a < 2 \quad \ldots\ldots\ldots (98a)$$

$$f = \frac{23.6 \times 10^6}{\frac{I_x}{I_y}\left(\frac{l}{d}\right)^2}\left(\frac{l}{a} + 1.75\frac{a}{l}\right) \text{ if } l/a \geqq 2 \quad \ldots\ldots\ldots\ldots (98b)$$

Applying the factor of safety used by the author to Eqs. 98, formulas for design are:

$$f = \frac{31.5 \times 10^6}{\frac{I_x}{I_y}\left(\frac{l}{d}\right)^2}\left[1 + 0.075\left(\frac{l}{a}\right)^2\right] \text{ if } l/a < 2 \quad \ldots\ldots\ldots (99a)$$

$$f = \frac{14 \times 10^6}{\frac{I_x}{I_y}\left(\frac{l}{d}\right)^2}\left(\frac{l}{a} + 1.75\frac{a}{l}\right) \text{ if } l/a \geqq 2 \quad \ldots\ldots\ldots\ldots (99b)$$

Except for the factor a, these equations are in terms of common properties of beams. They are relatively simple to use if tables of the factor a are available.[8] A limit for l/a beyond which the error in the use of Eq. 9 is inconsequential can be established from Table 8. If it be desired that the error not exceed 5%, then the approximate k of Col. 5 should not be used if $l/a < 5$. The error will not exceed 8% if $l/a > 4$. (This is exclusive of errors inherent in approximations to I_x, I_y, and K, through which Eq. 7 is derived from Eq. 4.)

If tables of the factor a are not available, the ratio l/a may be computed from the equation:

$$\frac{l}{a} = 1.24\frac{l}{d}\sqrt{\frac{K}{I_y}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (100)$$

This equation was derived from the relation $C = \frac{E\, I_y\, d^2}{4\, a^2}$, with Poisson's ratio equal to 0.3. The torsion constant K can be computed with sufficient accuracy in most cases by means of the familiar equation $K = \frac{1}{3}\Sigma b\, t^3$, or, if desired, by a more accurate procedure described elsewhere.[10]

Eqs. 99 may be presented in a different form by calculating I_x, I_y, and K terms of the beam flanges only, that is, $I_x = \frac{b\, t\, d^2}{2}$, $I_y = \frac{t\, b^3}{6}$, and $K = \frac{2\, b\, t^3}{3}$. However, the equations are first written in the following form, by factoring

[8] "Bethlehem Manual of Steel Construction," *Catalogue S-47*, Bethlehem Steel Co., Bethlehem, Pa., 1934, pp. 146 to 158 and p. 167.

[10] "Structural Beams in Torsion," by Inge Lyse and Bruce G. Johnston, *Transactions*, ASCE, Vol. 101, 1936, p. 865, Eq. 21.

l/a from the brackets and parentheses, and substituting from Eq. 100:

$$f = \frac{39 \times 10^6}{\frac{I_x}{\sqrt{K\,I_y}}\frac{l}{d}}\left(\frac{a}{l} + 0.075\frac{l}{a}\right) \text{ if } l/a < 2 \ldots\ldots\ldots (101a)$$

$$f = \frac{17.5 \times 10^6}{\frac{I_x}{\sqrt{K\,I_y}}\frac{l}{d}}\left[1 + 1.75\left(\frac{a}{l}\right)^2\right] \text{ if } l/a \geqq 2 \ldots\ldots\ldots (101b)$$

When the substitutions for I_x, I_y, and K are made,

$$f = \frac{26 \times 10^6}{\frac{l\,d}{b\,t}}\left(\frac{a}{l} + 0.075\frac{l}{a}\right) \text{ if } l/a < 2 \ldots\ldots\ldots (102a)$$

$$f = \frac{12 \times 10^6}{\frac{l\,d}{b\,t}}\left[1 + 1.75\left(\frac{a}{l}\right)^2\right] \text{ if } l/a \geqq 2 \ldots\ldots\ldots (102b)$$

The l/a-ratios in the brackets and parentheses of Eqs. 102 are not expressed in terms of l, d, b, and t because of large errors that could result. The reason for the errors is found in Eq. 100. The value of K expressed in terms of the flanges only may be in error by as much as 50%, whereas the value of I_y is practically unaffected by omitting web moment of inertia. In the substitutions outside the brackets, the error in K is compensated for by an error in I_x. For a 12 B 16.5[42] section, $K = 0.052$ in.^{4} and $I_x = 77.5$ in.^{4} if the flanges only are considered; $K = 0.098$ in.^{4} and $I_x = 105$ in.^{4} if the web is included. The ratio $I_x/\sqrt{K}$ for the true values is 336, and for the approximate values, 340. Calculations for several beams chosen at random showed variations of as much as 8%.

If values of the factor a, or K, are not available, their calculation may be avoided by writing Eq. 100 as $\frac{l}{a} = 1.24\frac{l}{d}\frac{I_x}{I_x}\sqrt{\frac{K}{I_y}}$ and substituting the approximate values for K, I_y, and the I_x in the denominator. This procedure introduces the same compensating errors mentioned previously, and results in:

$$\frac{l}{a} = 5\frac{l}{d}\frac{I_x}{b^2\,d^2} \ldots\ldots\ldots (103)$$

This value of l/a may be substituted in Eq. 102b to obtain:

$$f = \frac{12 \times 10^6}{\frac{l\,b}{d\,t}}\left[1 + 0.07\left(\frac{b^2\,d^2/I_x}{l/d}\right)^2\right] \text{ if } \frac{l}{d} \geqq \frac{0.3\,b^2\,d^2}{I_x} \ldots\ldots (104)$$

Eq. 104 should be particularly useful, since it contains only common properties of sections. After selecting a beam according to Eq. 9, the designer need calculate only the second term in the brackets to determine the percentage by

[42] "Steel Construction Manual," A.I.S.C., 5th Ed., 1947, p. 24.

which the allowable stress, as given by Eq. 9, should be increased. Eq. 102*a* is not written with the l/a-ratio expressed in terms of Eq. 103, because the result is cumbersome. However, for $l/a = 1.5$, the value of f given by Eq. 102*b* exceeds by only 5% the value given by Eq. 102*a*, so that Eq. 102*b*, or Eq. 104, probably can be safely used for $l/a \geqq 1.5$. The l/d limitation stated after Eq. 104 is based on this assumption. Thus, Eq. 104 will be applicable to the design of practically any rolled shape or plate girder, and Eq. 99*a* or its equivalent, Eq. 102*a*, may be used for light-gage beams.

A simple criterion to determine whether the second term in the brackets of Eq. 104 is significant can be established. As stated previously, if $l/a > 5$, Eq. 9 will err by about 5%. Therefore, from Eq. 103,

$$\frac{l}{d} \geqq \frac{b^2 d^2}{I_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (105)$$

is an index of the accuracy of Eq. 9.

One other convenient form for Eqs. 98 will be given. The radius of gyration for the axis perpendicular to the web of the I-section and the WF-section varies from about 0.375 d to 0.425 d. If an average value of $r_x = 0.41\ d$ is assumed, formulas for the critical stress for the I-section and the WF-section are:

$$f = \frac{310 \times 10^6}{\left(\frac{l}{r_y}\right)^2}\left[1 + 0.075\left(\frac{l}{a}\right)^2\right] \text{ if } l/a < 2 \quad \ldots\ldots\ldots (106a)$$

$$f = \frac{140 \times 10^6}{\left(\frac{l}{r_y}\right)^2}\left(\frac{l}{a} + 1.75\,\frac{a}{l}\right) \text{ if } l/a \geqq 2 \quad \ldots\ldots\ldots (106b)$$

These equations are particularly simple in form. However, they may overestimate values of f given by Eqs. 99 by as much as 10%, and may underestimate them by as much as 15%.

The following examples illustrate the use of the equations in this discussion. It is assumed that the maximum allowable bending unit stress is 20,000 lb per sq in. Since the American Institute of Steel Construction has adopted Eq. 9 in its 1947 design specifications,[43] diagrams based on that equation[44] can be used for these problems. The data given do not include the moment due to the weight of the beam.

Example 1.—A simply supported beam on a span of 55 ft is to resist a moment of 450 kip-ft. The diagrams referred to indicate that a 30WF190 section is required. For this section, $a = 121.9$ and $l/a = 5.41$. Therefore, the allowable stress is predicted with sufficient accuracy by Eq. 9. Using Eq. 105, $\frac{l}{d} = 22$ and $\frac{b^2 d^2}{I_x} = 23.3$, which indicates that Eq. 9 is in error by somewhat over 5%. Instead of checking with Eq. 105, the second term in the brackets of Eq. 104 may be calculated. It is found to be 0.078, which shows that Eq. 9 underestimates the allowable stress by about 7%.

[43] "Steel Construction Manual," A.I.S.C., 5th Ed., 1947, p. 286.
[44] *Ibid.*, p. 203.

Example 2.—A simply supported beam on a span of 24 ft is to resist a moment of 400 kip-ft. The diagram shows that 24WF110 is the lightest section that can be used. For this section, $l/a = 288/108 = 2.67$, and, since the diagram also shows that the allowable stress was reduced for buckling, the section may be unnecessarily large. Using Eq. 105, $\frac{l}{d} = 12$ and $\frac{b^2 d^2}{I_x} = 25.5$.

The allowable stress for a 24WF100 section is given by Eqs. 99*b*, 102*b*, and 104 as 21,100, 20,800, and 22,200 lb per sq in., respectively. From Eq. 106*b*, with the factor of safety used to obtain Eq. 99*b*, $f = 22{,}300$ lb per sq in. According to Eq. 9, however, $f = 16{,}100$ lb per sq in. Since the more nearly correct values exceed the stated maximum allowable of 20,000 lb per sq in., the allowable moment is $\frac{20 \times 249}{12} = 415$ kip-ft. Because the moment due to its weight is only 7.2 kip-ft, the 24WF100 is satisfactory. Thus, in this example, a few minutes of the designer's time saves 240 lb of steel.

Example 3.—A simply supported beam on a span of 18 ft is to resist a moment of 490 kip-ft. From the diagram, a 30WF124 or a 24WF120 section may be used. The diagram shows that the allowable stress for 24WF120 is greater than 20,000 lb per sq in., and there is no need to investigate the 24 × 12-group further. However, the allowable stress for 30WF124, as given by Eq. 9, is below 20,000 lb per sq in.; and it may be underestimated. The ratio $l/a = 216/105.1 = 2.06$ shows this to be the case; or, using Eq. 105, since $\frac{l}{d} = 7.2$ and $\frac{b^2 d^2}{I_x} = 18.8$, the allowable stress is seen to be underestimated considerably. For a 30WF108 section, the allowable stress as given by Eqs. 99*b*, 102*b*, 104, and 106*b* is 22,500, 22,600, 24,400, and 21,300 lb per sq in., respectively. According to Eq. 9, $f = 14{,}900$ lb per sq in. Then $M = \frac{20 \times 299}{12} = 498$ kip-ft. The moment due to the weight of the beam is 4.4 kip-ft. In this example, the weight saving is 10% compared to section 24WF120, and 13% compared to 30WF124.

The foregoing examples have been chosen to show extreme differences; it is true that there are many cases in which comparable savings could not be made because of the necessarily limited range of standard sections. It has been assumed that Eqs. 98 apply as long as the critical stress is less than the yield point (assumed to be about 33,000 lb per sq in.). Strictly speaking, the equations are not correct if the critical stress exceeds the proportional limit. Transition curves to accompany the equations in this discussion could undoubtedly be developed, but the writer did not attempt them. The author's transition equations are subject to errors comparable to those in Eq. 7. The examples might give the impression that Eq. 7 errs only when the critical stress is in the region of the proportional limit of carbon steel. Two illustrations will show that this is not the case. The critical stress for a 12-in. 14-lb joist[45] on a span of 12 ft is 10,400 lb per sq in. according to Eq. 7, whereas Eq. 98*b* gives a critical stress of 13,900 lb per sq in. For an 8WF17 section on a span of 15 ft, Eqs. 7 and 98*b* give 22,400 and 27,000 lb per sq in., respectively. It is pertinent

[45] "Steel Construction Manual," A.I.S.C., 5th Ed., 1947, p. 24.

to observe that the difference between the two equations is even more significant for beams of high-yield steel.

Since there are some types of loading which subject a beam to pure flexure, or at least approximately so, formulas for the critical stress for this case may be useful. Substitution of the value of k from Eq. 93, as given by the first two terms of the binomial expansion in Eq. 4, leads to the following formulas for the bending critical stress in pure bending:

$$f = \frac{74 \times 10^6}{\frac{I_x}{I_y}\left(\frac{l}{d}\right)^2}\left[1 + 0.04\left(\frac{l}{a}\right)^2\right] \text{ if } \frac{l}{a} < 3 \ldots\ldots\ldots\ldots (107a)$$

$$f = \frac{23.5 \times 10^6}{\frac{I_x}{I_y}\left(\frac{l}{d}\right)^2}\left(\frac{l}{a} + 4\frac{a}{l}\right) \text{ if } \frac{l}{a} \geqq 3 \ldots\ldots\ldots\ldots (107b)$$

The coefficient of the second term in the brackets and parentheses has been reduced from the value obtained through the binomial expansion. For $0 \leqq l/a \to \infty$, the equations give values of f which differ by a maximum of -2% from the values computed with k as given by Eq. 93. The second term in the brackets of Eq. 107a may be omitted if $l/a < 1$; and the corresponding term in Eq. 107b may be omitted if $l/a > 10$. The maximum error resulting from these omissions is about -5%. The value of l/a may be computed from Eq. 100 or Eq. 103 if tables of the factor a are not available.

Mr. de Vries is to be commended for having presented a design formula for beams subject to lateral buckling which, although eminently simple in form, is far superior to previous formulas. In most cases it predicts the critical stress with a good degree of accuracy. The writer has shown how the designer may determine the precision of the formula for any case, and easily obtain a better estimate when necessary. Furthermore, since either Eq. 99b or 102b is simpler to use than is Eq. 104, the writer suggests that the American Institute of Steel Construction list in future editions of its manual values of the factor a not only because it is useful in the problem of lateral buckling of beams, but also because it is necessary in the analysis of torsion.

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DISCUSSIONS

MATRIX ANALYSIS OF CONTINUOUS BEAMS

Discussion

BY A. FLORIS

A. FLORIS,[23] ESQ.—From an academic viewpoint the paper is highly interesting and instructive. To a reader with a theoretical turn of mind, the study of this paper will undoubtedly be a source of pleasure and inspiration. Practicing engineers, however, have at their disposal other more elementary methods of stress analysis, which do not require the mathematical training and equipment necessary for the intelligent use of the method presented.

A brief and unusually clear introduction to the matrix algebra is given by the author for the benefit of those not familiar with the subject. Nevertheless, without a systematic study of matrices it is doubtful whether the author's methods can be used profitably.

The main practical difficulty arising in the stress analysis of hyperstatic structures is the solution of linear nonhomogeneous algebraic equations obtained from considerations of structural deformation. To a certain extent, the best methods are those which reduce the number of these elastic equations to a minimum, or eliminate them entirely. The relaxation methods of stress analysis meet this requirement. However, by their very nature the matrix methods do not.

The solution of linear nonhomogeneous algebraic equations by means of matrices is identical to G. Cramer's rule given in the theory of determinants. Although this rule leaves nothing to be desired theoretically, in practice it is almost useless. By the same token, matrix methods for formal, shorthand solutions have no equals. In numerical problems, however, their usefulness is rather restricted. The greatest success has been achieved in the application of matrix methods to the solution of linear ordinary differential equations with constant coefficients. In this case, use is made of the matric-exponential function.[24,25]

NOTE.—This paper by Stanley U. Benscoter was published in October, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1947, by I. Oesterblom, and Harris Solman; and May, 1947, by Rufus Oldenburger, Horace A. Johnson, Lloyd T. Cheney, and Paul W. Norton.

[23] Asst. to Structural Engr., Pacific Electric Railway Co., Los Angeles, Calif.

[24] "Matrix and Tensor Calculus," by Aristotle D. Michal, John Wiley & Sons, Inc., New York, N. Y., 1947, p. 20.

[25] "Elementary Matrices," by R. A. Frazer, W. J. Duncan, and A. R. Collar, Cambridge Univ. Press, 1938, p. 156.

The statement is made that the stiffness factors are always positive. This is, of course, only true if the deformation due to shears, which are produced by the moments at the joints, is neglected. If this shearing deformation is greater than the bending deformation, the stiffness factors become negative.[26,27]

Inasmuch as the division of two matrices cannot be performed directly, the great importance of the inverse matrix, which transforms the division into multiplication, becomes apparent. Therefore, a mnemonic check on the correctness of the computed inverse matrix can be of some value. Using the matrix (Eq. 38*a*) and its inverse (Eq. 41) the self-explanatory checking scheme is arranged as in Fig. 13.

(3)(11) = 33	(3)(2) = 6	(3)(−5) = −15
(1)(−2) = − 2	(1)(16) = 16	(1)(5) = 5
(2)(7) = 14	(2)(−11) = −22	(2)(5) = 10
45	**0**	**0**
(1)(11) = 11	(1)(2) = 2	(1)(−5) = − 5
(2)(−2) = − 4	(2)(16) = 32	(2)(5) = 10
(−1)(7) = − 7	(−1)(−11) = 11	(−1)(5) = − 5
0	**45**	**0**
(−2)(11) = −22	(−2)(2) = − 4	(−2)(−5) = 10
(3)(−2) = − 6	(3)(16) = 48	(3)(5) = 15
(4)(7) = 28	(4)(−11) = −44	(4)(5) = 20
0	**0**	**45**

FIG. 13.—MNEMONIC CHECK OF INVERSE MATRIX

Division of the bold-face numbers in this scheme, by the determinant of the given matrix (in this case 45), gives the unit matrix or L. Kronecker's delta. Thus, Eq. 30 is satisfied.

The inverse of a two-row square matrix is obtained in a simple manner by changing the signs of the elements in the secondary diagonal and by interchanging those in the principal diagonal. The result is then divided by the determinant of the original matrix, which for a unit matrix is equal to one.

Practically, the application of matrix methods to the analysis of continuous beams by means of the slope-deflection equations and the three-moment equations is not simple, although formally the process is excellent. It requires more numerical work than the corresponding older methods. In addition, the slope-deflection equations are solvable by iteration, and if the shearing deformation predominates, the three-moment equations can also be solved by iteration.[28]

The most valuable section of the paper is the treatment of the moment-distribution method from an advanced mathematical point of view. Here, as

[26] "Shearing Deformation in Continuous Beams and Rigid Frames," by A. Floris, *Journal*, ACI, November–December, 1937.

[27] "Verallgemeinertes Ausgleichverfahren," by A. Floris, *Beton u. Eisen*, May 20, 1939.

[28] "Shear Deformation Included in Three-Moment Equation," by A. Floris, *Civil Engineering*, October, 1937, p. 711.

in the previous parts of the paper, the reader is refreshed by the exact formulation and generality of the statements as contrasted to the oversimplified and, to some extent, dull treatment accorded them in the ordinary textbooks.

The continuous beam is a comparatively simple element in the analysis of hyperstatic structures. It is to be regretted that the author did not include the more complicated continuous and multiple-story frames, inasmuch as the acid test of any new method lies in the analysis of multiple-story frames, for which simple and speedy methods are much needed.

In conclusion, the writer wishes to congratulate the author upon his efforts to bring to the attention of structural engineers the fruitful and elegant methods of matrix algebra through this pioneer work of considerable scientific value.

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DISCUSSIONS

STABILITY OF SOIL SLOPES

Discussion

BY JACOB FELD, AND EARL M. BUCKINGHAM

JACOB FELD,[19] M. ASCE.—The thesis of the paper is to prove—by experiment, by experience, by deduction from the measured stress distribution in gelatin models, and from mathematical analysis of stress distribution in idealized soil masses—that failure within a soil body results from plastic strain. Plastic failure methods applied to cohesive materials are consistent with present thought; but, when the failure of pure granular materials is likewise explained, a new concept has been added.

The basic differences in shear deformations of different states of materials have been explained by E. C. Bingham.[20] When a perfectly elastic solid is subjected to a shearing force, a strain develops and disappears entirely when the stress is removed. In this case of elastic deformation, there is no permanent displacement and, therefore, no flow. If the solid is not perfectly elastic, a part of the deformation remains as a permanent displacement and work has been performed in overcoming the internal resistance of the material. The flow resulting in the displacement may be of three types:

1. Viscous or linear flow in which the amount of the deformation is proportional to the acting force, although such relationship may not become apparent until the force has acted over a measurable time period;
2. Hydraulic or turbulent flow; and
3. Plastic flow in which the slightest shearing force starts a deformation which continues indefinitely.

The phenomenon, plasticity, contains the concepts of internal friction and mobility, acting simultaneously. True plasticity can only be exhibited by a material in the fluid state, for, when the material changes to a solid state, deformation ceases.

NOTE.—This paper by Ek-Khoo Tan was published in January, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1947, by Paul Baumann, and Gregory P. Tschebotarioff.

[19] Cons. Engr., New York, N. Y.

[20] "Fluidity and Plasticity," by E. C. Bingham, McGraw-Hill Book Co., Inc., New York, N. Y., 1922, p. 4 et seq.

The laws of friction in solids and the laws of friction during viscous flow are quite different.[21] For solids, frictional resistance is: (1) Directly proportional to the component of the exterior forces acting normal to the surface of rupture; (2) greater before the beginning of motion than during motion; and (3) independent of the area of contact. For viscous flow, frictional resistance is: (1) Independent of the load; (2) equal to zero if the velocity is zero and is directly proportional to the velocity; and (3) proportional to the area of contact.

A number of these concepts were enunciated by C. A. Coulomb[22,23] as "preliminary propositions" to his studies of the rupture of bodies, pressure of earth, and equilibrium of arches. He postulates that:

a. Frictional shear failure resistance is proportional to the pressure acting normal to the failure surface; and

b. Cohesion is a measure of the resistance which a body presents to a disunion into two parts, and in a homogeneous mass the cohesion is proportional to the area of the section of separation.

The author's statement that failure in earth slopes is a plastic phenomenon requires the assumption that the materials immediately adjacent to the fractures are in a fluid condition; otherwise failure would be localized and not progressive. Examination of slide failures in soil masses indicates that such an assumption is possible. It explains the existence of void spaces along the rupture surfaces. Discrete particles of solid matter in motion are fluid.

To the list of references proving the existence of curved rupture surfaces, the author should add Coulomb's derivation of the same type of surface. It is somewhat involved mathematically, but, nevertheless, it is the earliest derivation of the rupture curve of a soil having both frictional and cohesive resistances.[24]

The apparatus used by the author to induce variable cohesive properties to granular soils is ingenious. For materials with high angles of slope, the fixed bottom surface will affect the properties of the internal mass. The diagram in Fig. 1 does not give the angle of the side walls, but the gelatin models in Fig. 9 show an angle of 29°. The apparatus tests should be repeated with a bottom sloping downward from the toe, so that the angle of soil is at least as large as the largest angle of natural slope to be measured.

The artificially induced "cohesion" by partial evacuation of granular materials brings to mind an interchange of discussions between the late H. de B. Parsons, M. ASCE, and the writer about 1932. In an attempt to explain why readings in lateral pressure tests were lower than expected, Mr. Parsons[25] postulated the existence of a partial vacuum in the soils tested. The writer summarized his arguments by showing how the existence of cohesion within a soil mass gave the necessary explanation of the pressure readings and the

[21] "Fluidity and Plasticity," by E. C. Bingham, McGraw-Hill Co., Inc., New York, N. Y., 1922, Chapter VIII.

[22] "Essai sur une application des règles de maximis et minimis à quelques problèmes des Statique, relatifs a l'Architecture," by C. A. Coulomb, *1773 Mémoires de mathematique présentés à l'Académie Royale des Sciences par divers savants*, Vol. 7, 1776.

[23] "Theorie des Machines Simples," by C. A. Coulomb, Paris, 1821, pp. 318–363.

[24] *Ibid*, p. 345, Propositions 15 and 12.

[25] "Vacuum in Soil Mechanics," by H. de B. Parsons, *Civil Engineering*, December, 1933, p. 690.

internal stress relationship was the same as when a partial vacuum was assumed.[26]

In sections 1 and 2 under the heading, "I. Investigation of Soil Slopes by Models," the author distinguishes between the angle of repose phenomena in "dry cohesionless materials" as purely a surface readjustment and in "cohesive soils" as a deep-seated internal rupture. There must be a zone of gradation in action, and some relationship between the minimum cohesion (probably as a ratio of internal frictional resistance) in a soil to place it into the "cohesion class." All soils that show the superficial phenomenon are not totally lacking in cohesion.

The author explains the existence of cohesion in natural soil deposits (section 2) as the result of capillary forces in the moisture between the finer soil grains. This explanation is not probable. The writer has experimented with pulverized bentonite (a colloidal natural clay) which when perfectly dry shows no cohesive properties and acts exactly the same as a very fine sand. However, a very slight moisture content makes the bentonite a slimy gel. Dry particles cannot be packed close enough (in soils) to permit intermolecular attraction. Powders can be made to stick together without adding a fluid medium, but only under very high pressures. On the other hand, thin moisture films will draw particles closely together; but, even in this case, the force of capillarity of the moisture films is much larger than the gravitational attraction between the molecules of the separated particles.

Cohesion is more likely to be generally explained as the interplay of dissimilar materials in intimate contact. For instance, the introduction of a liquid into a granular mass results in cohesion only if the solid aggregate is wetted by the liquid. If the liquid does not wet (adhere to) the solid grains, the physical resistance of the granular mass is not increased. Actually, for some lubricating types of liquids, the internal friction may be decreased. If a gas and a liquid are added to a granular mass, with conditions such that the gas is occluded on the surface of the grains and the liquid tends to dissolve the gas, a very plastic mix is obtained. Similarly, large cohesive properties result from the addition of colloidal ingredients or the mixing of materials which form colloidal covering of the soil grains. Any of these conditions increases the "resistance to disunion" between grain particles, the increase being many times the natural gravitational attraction of like grains to each other. The author's experimental method permits a study of varying equivalent cohesive strengths in any type of soil to be easily made.

The results given in Fig. 2, at zero vacuum, for loose and dense sand are not consistent with the values in Table 1. Possibly, the dashed line extrapolations in Fig. 2 should be eliminated, and the graphs for loose and dense state extended to meet at a value of 36° for the angle of slope with no vacuum acting.

In section 4 of the test results, the author reports a linear relation between the critical height of an unsupported embankment and the cohesion of the soil. In 1773 Coulomb developed a formula for the height h_f of a bank which will stand vertically unsupported as

$$h_f = c\,\frac{l}{m} \qquad (25)$$

[26] "Vacuum or Cohesion in Soils," by Jacob Feld, *Civil Engineering*, April, 1934, p. 219.

in which c is the coefficient of cohesion; and m and l are coefficients depending on the weight of the soil and value of the internal friction, such that the earth pressure,[24] p, equals:

$$p = m\,h^2 - c\,l\,h \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Coulomb's result also gives a linear relation between the free standing height h_f and the cohesion c.

All the author's methods, as well as the reports by H. G. Moulton[27] and the late J. C. Meem,[28] the late David Molitor,[29] Lazarus White,[30,31] D. P. Krynine,[32] Members, ASCE, and the references listed by the author under the heading, "Introduction," tend toward a solution defining the surfaces of rupture and of stable unsupported slope as circular, conchoidal (Mr. Moulton),hyperbolic (Mr. Molitor), and sometimes spiral. The unanimity of determination is the form of the curve—namely, concave; yet, the problem cannot be declared solved as long as the largest experimental report, from the Panama Canal slide correction work, gives just the opposite type of curve, definitely convex.[33]

EARL M. BUCKINGHAM,[34] ASSOC. M. ASCE.—Thep reparation of this paper has required a considerable amount of painstaking work. To the writer, however, it seems that the planning and interpretation were not commensurate with the effort expended in the laboratory. Tests as simple as working a lump of clay between the fingers will demonstrate the fallacy of trying to simulate the action of impervious cohesive soil by increasing the friction in sand. The forces of capillarity, molecular attraction, and pore pressure are much too complicated to be replaced by a simple external pressure. Part I of the paper must therefore be considered applicable only to clean dry sand.

Although the internal structure of clay has been known for years,[35,36] the profession has been slow to accept the application of this knowledge. Soils that have a high percentage of sand possess a true internal friction, and some of the more permeable silt clays may consolidate rapidly enough to develop an increase in strength when loaded. However, the application of the Coulomb conception of cohesion plus friction to an impermeable clay may be highly misleading. This is true even though values of "cohesion" and "friction" may have been determined in the laboratory.

Consolidation is frequently difficult to prevent in the laboratory; but it might occur very slowly in the natural deposit from which the sample was taken. Instead of developing more frictional resistance, an increase in loading might cause sufficient deflection to develop dangerous pore pressures and ma-

[27] *Transactions*, A.I.M.M.E., 1920, Section 13, pp. 1–20.
[28] *Transactions*, ASCE, Vol. LX, June, 1908, p. 1.
[29] *Journal*, Assn. Eng. Societies, Vol. 13, 1894, p. 12.
[30] *Engineering News-Record*, Vol. 85, 1920, p. 1268.
[31] *Ibid.*, Vol. 86, 1921, p. 439.
[32] *Ibid.*, July 16, 1931, p. 93.
[33] *Proceedings*, ASCE, February, 1947, p. 179 and p. 185, Fig. 2.
[34] Draftsman, City of Oakland, Oakland, Calif.
[35] "The Structure of Clay and Its Importance in Foundation Engineering," by Arthur Casagrande, *Journal*, Boston Soc. of Civ. Engrs., April, 1932.
[36] "Progress Report of the Special Committee on Earths and Foundations," *Proceedings*, ASCE, May, 1933, p. 797.

terially reduce the shearing strength. Figs. 17 and 18 show typ cal results of some shear tests made recently by the writer with equipment of the three-ring type. Fig. 17 represents an unconsolidated estuary mud composed of particles of the silt and clay sizes. Each sample had a constant shearing strength for all normal pressures below a critical value at which the granular structure broke down and the mud began to liquefy and lose its strength.

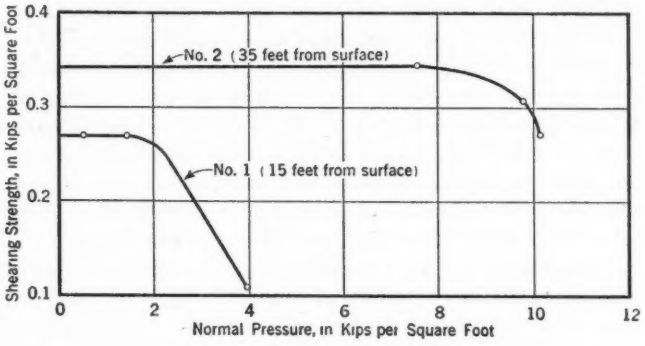

FIG. 17.—SHEAR TESTS, ESTUARY MUD

After the principle had been demonstrated with mud, an attempt was made to obtain the same effect with a stiff and fairly dry marly clay, as shown in Fig. 18. The sample was first tested at zero normal load, and failed at a shearing load of 3,030 lb per sq ft. After shifting the sample to expose new shear faces, a normal load of 12,700 lb per sq ft was applied. When the shearing stress reached 3,030 lb per sq ft, water appeared on the surface of the sample between the rings. Failure did not occur until the shear load reached 5,760 lb per sq ft. In the natural deposit, the escape of even a small quantity of water might not be possible, and slow failure might begin at the stress under which water began to extrude from the test sample. A casually run test would have indicated an "angle of friction" of 12° 10′.

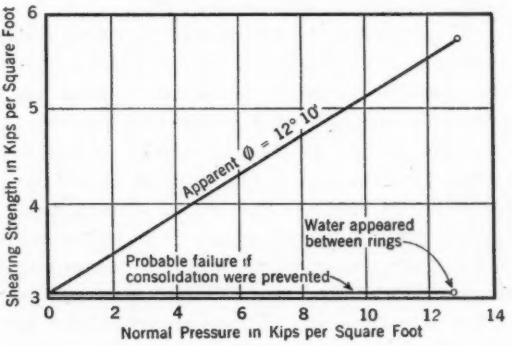

FIG. 18.—SHEAR TESTS, MARLY CLAY

Because a saturated clay ordinarily loses its strength when it is deformed, complete failure may result from a local concentration of stress even if the average stress is below the safe strength of the soil. Progressive failure of this type is common in natural slopes; but it is not considered in any analysis based on "cohesion" plus "friction."

In Part I, section 5, Mr. Tan states: "The significance of a factor of safety of less than unity at failure is not fully understood." Lack of understanding in

this respect is due to the failure to distinguish between the angle of internal friction and the angle of repose. For the arc analyzed, the angle of internal friction, $\phi_i = \tan^{-1} \frac{16.95}{20.95} = 39°$. Failure along either a straight line or a circular arc does not involve internal deformation, and the only resisting forces are those acting on the failure surface. Hence, if the condition that causes failure comes into being slowly, the factor of safety at failure cannot be less than unity. Other failure sections, such as a logarithmic spiral or a broken line, involve internal deformations which are not ordinarily considered in the computation. For this condition, a computed factor of safety of less than unity is possible. Assumed failure sections of all types are arbitrary and probably differ from the section of least resistance along which failure will occur. Therefore, a factor of safety greater than unity at failure is at least theoretically possible for all assumed sections. Uneven distribution of stress should not be of major importance when dealing with a material that can yield without losing its shearing strength. If it were a factor, it would increase rather than decrease the apparent factor of safety at failure.

The author does not state how he computed the distribution of shearing stress by the Swedish arc method. Although the interfacial pressures between slices are neglected when dealing with the arc as a whole, they are quite important when considering an individual slice. The distribution of both stress and frictional resistance depends on these pressures, and a method of solution would be of considerable interest.

The straight-line relationship between critical height of a vertical bank and vacuum is not surprising. Considering unit thickness, volumes and weights vary as the square of the linear dimensions, whereas the areas to which the vacuum is applied vary as the first power. Therefore, it is necessary for the vacuum to vary as the dimensions to maintain similitude. The relation between vacuum and angle of slope is not so simple geometrically, but could probably be reduced to a constant coefficient of friction if the shapes of the failure sections were known. The writer recommends that Mr. Tan include, in his closing discussion, detailed measurements of representative failure sections which he produced. These may well prove to be the most valuable part of the investigation. Unless an actual slip has occurred and has been located, current methods of analysis depend on an assumed section. Although the various methods yield fairly comparable results, experimental data might make possible a more intelligent choice between the several conventional sections.

Corrections for *Transactions:* In January, 1947, *Proceedings*, on page 25, lines 10 and 17, change "top of the slope" to "middle of the sliding curve"; and, on page 33, in the right-hand side of Eq. 6*b*, change the denominator of the first term from "∂r" to "$\partial \theta$"; in the numerator of the last term in Eq. 6*b*, change "$\partial^2\theta$" to "$\partial^2\sigma_\theta$"; in the third term of Eq. 7*a*, change the sign from minus to plus; and change the coefficient "r^2" in the last term of Eq. 7*a* to "r."

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DISCUSSIONS

A RATIONAL EXPLANATION OF COLUMN BEHAVIOR

Discussion

BY ROBERT M. MAINS, AND CHI-TEH WANG

ROBERT M. MAINS,[17] ASSOC. M. ASCE.—In consideration of the voluminous literature on column behavior, a new paper on the subject should perform one of two functions: Set forth solutions for problems hitherto unsolved, or present more convenient methods for applying existing solutions to the practical column. The author's work on theoretical columns is interesting in that it restates, with some changes in notation, problem solutions which are readily available in such books as "Theory of Elastic Stability" by S. Timoshenko.[18] Indeed, many of these solutions date back to the Euler-Bernoulli correspondence concerning problems of the "elastica" about 1740.

In studying this paper, the writer hoped that it would contain forms of solution which would permit more convenient computations of practical columns. However, the methods used in the section dealing with practical columns seem to be much more cumbersome than existing methods. For example, the results of Col. 13, Table 5, which give deflections for Fig. 8, were checked with a brief half page of computation by the use of the numerical procedure of N. M. Newmark, M. ASCE, combined with the relation:[19]

$$w_a = \frac{w_i}{\frac{P_e}{P} - 1} \qquad (54)$$

in which w_a is the additional deflection due to load P; w_i is the sum of the initial deflection and the deflection due to lateral load; P_e is the critical buckling load; and P is the applied end load. The results obtained from this computation agreed with those of the author within 1%, which is certainly adequate for practical purposes. A similar saving in the labor of computation, although not

NOTE.—This paper by Frederick L. Ryder was published in March, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1947, by R. Robinson Rowe, and George Winter.

[17] Asst. Prof., Civ. Eng., Cornell Univ., Ithaca, N. Y.

[18] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936.

[19] *Transactions*, ASCE, Vol. 108, 1943, p. 1181, Eq. 4.

so marked, may be made by the application of the same technique to the problem in Fig. 9.

Although the author's equations for the calculation of deflections are adequately tied in with computations of stress, it is not made clear just how buckling loads are to be determined for the practical column which fails by elastic buckling instead of by yielding. The relationship between deflections and buckling loads is emphasized in Professor Newmark's method, and the buckling load is obtained conveniently in the process of solution.

Since there are practical problems (although not necessarily in bridges and buildings) in which elastic buckling occurs before yielding of the column, the writer fails to see any advantage in clarity of theory or simplicity of application for the author's methods over "step-by-step" processes already available.

CHI-TEH WANG,[20] ESQ.—Although the nonlinear theory of column buckling in the elastic range was first developed by G. Kirchhoff[21] almost ninety years ago, a recapitulation for the average practical engineer still appears to be praiseworthy. Mr. Ryder's paper presents an excellent résumé of the theory, together with an enlightening discussion relating to the stability of the columns. Several points, however, are in need of further clarification.

In the summary of Section 3, it was concluded that "(3) At values of P/A close to the critical, the deflection is indeterminate, and may assume any fairly small value." This was supposedly proved by showing that the curves of $\frac{M_i}{A\,L}$ and $\frac{M_e}{A\,L}$ have the same slope or are tangent to each other at $\frac{\delta}{L} = 0$. It is a simple matter to show that when two curves are tangent to each other they need not necessarily meet at more than one point. Of course, they may To prove that there is actually only one point at which these two curves meet, reference is made to Eq. 13, in which $k^2 = \frac{P}{E\,I}$ and $q^2 = \left(\frac{k\,\delta}{2}\right)^2 = \frac{P\,\delta^2}{4\,E\,I}$. When $P = P_c = \frac{\pi^2\,E\,I}{L^2}$, $k^2 = \frac{\pi^2}{L^2}$, and $q^2 = \frac{\pi^2\,\delta^2}{4\,L^2}$. The left-hand side of Eq. 13 is then equal to $\frac{\pi}{2}$. The elliptical integral on the right-hand side will be equal to $\frac{\pi}{2}$ only if q^2 is zero. Since $q^2 = \frac{\pi^2\,\delta^2}{4\,L^2}$, it is zero only if $\delta = 0$; that is, at the critical load the deflection is zero, and at any other load close to the critical value the deflection, however small, has a definite value and can be calculated from Eq. 13. When this is proved, Eqs. 29*a* and 29*b* are equal only at the point $\delta = 0$ and not at any other point or points close to $\delta = 0$, and the original conclusion (3) is therefore incorrect.

Mr. Ryder mentions in the "Synopsis" that he is merely re-emphasizing the well-known fact that at the critical point the deflection is indeterminate. The so-called "well-known" fact is accepted as such because most writers in strength of materials deal only with the linear theory and, therefore, reach that erroneous

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[21] "Ueber das Gleichgewicht und die Bewegung eines unendlich dünnen elastischen Stabes," by G. Kirchhoff, *Journal für reine und angewandte Math.*, Vol. 56, 1859.

conclusion. Actually, from the nonlinear theory, it is clear that the so-called "buckling load" is the load at which the straight form of the ideal column begins to be unstable. At any load greater than the critical load, the column has two possible forms of equilibrium, the straight form, which is unstable, and the bent form, which is stable. For any load which is less than, or equal to, the critical value, the ideal column has only one form of equilibrium, namely, the straight form. Accepting the result obtained from the linear theory, some writers consider the equilibrium at the critical load as neutral equilibrium since the deflection then seems to be indeterminate. It is evident that such terminology is not correct in the strict sense.

Another objection may be raised with respect to Fig. 5, in which Euler's column formula has been extended to the yield point of the material. It should be noted that Mr. Ryder's theoretical discussion does not hold true if the stress exceeds the elastic limit of the material, namely, the proportional limit. For some alloy steels the proportional limit is approximately 90% of the yield stress. Perhaps in these cases, if a large factor of safety is used, diagrams such as Fig. 5 could be accepted for design purposes, as is the usual practice of civil engineers. For aeronautical engineers, who have to deal with aluminum alloys most of the time, this practice has long been found to be unsatisfactory because the proportional limit in some cases is only about 50% of the yield stress.

Since the author is interested in the "rational" explanation of column behavior, the writer feels that this is a point worth mentioning. The most accurate experiments on columns, up to the present time, probably were those made in the Berlin-Dahlem material testing laboratory by using a very elaborate construction for the end supports of the columns.[22] For structural steel columns with a yield stress of 45,000 lb per sq in., the test results show that there is a marked deviation from the Euler curve above $\frac{P_c}{A} = 39{,}000$ lb per sq in.[23] Theoretically, the use of Euler's column formula for stress beyond the proportional limit cannot be considered to be correct. Between the proportional limit and the yield stress one should use the tangent modulus formula, which recently has been verified theoretically by F. R. Shanley. Fig. 5 should then be modified as in Fig. 10 in which $\frac{P_t}{A} = \frac{\pi^2 E_t}{(L/r)^2}$, and E_t is the tangent modulus of the material and is defined as the slope of the compressive stress-strain curve at the stress corresponding to $\frac{P_t}{A}$. The same objection can be applied to Fig. 6.

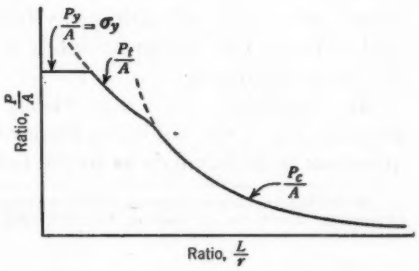

FIG. 10

[22] "Neuere experimentelle Beiträge zur Frage der Knickfestigkeit," by K. Memmler, *Proceedings*, 2d International Cong. for Applied Mechanics, Zürich, 1926, pp. 357–363.

[23] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 174, Fig. 109.

In the "Summary," it is emphasized that in a strict sense there is no critical point for an actual column. This statement is indeed very true. If one uses the linear theory, the load versus deflection curve of an actual column in compression is approximately a rectangular hyperbola with asymptotes $P \rightarrow P_e$ when $\delta \rightarrow \infty$. This condition, originally discussed by R. V. Southwell,[24] is interesting, because from the tests of actual columns the buckling load of a corresponding ideal column can be approximately determined. Thus, a relation between the theoretical discussion of ideal columns and the testing of actual columns can be obtained. It is obvious that when δ is not small the nonlinear equation must be used; the theoretical discussion of actual columns will yield no result concerning the buckling load.

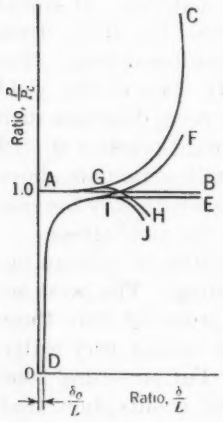

FIG. 11

The behavior of ideal and actual columns as obtained from various rational theories can be summarized in Fig. 11. Suppose that the column is slender and that the material can sustain the compressive stress $\frac{P}{A}$ without failure of elasticity. The loading of an ideal column will follow the curve OAB according to the linear theory and OAC according to the nonlinear theory. At some point G on curve OAC, the deflection δ is so large that the sum of direct compressive stress and bending stress due to the bending moment $P\,\delta$ is sufficient to produce elastic breakdown. Beyond this point the actual curve AGH will begin to drop away from AC, since the material can now sustain less stress than before. For an actual column with an initial deflection of δ_0, the loading of the column will follow the curve DE according to the linear theory, and the curve DF according to the nonlinear theory. At point I, the maximum stress in the column reaches the elastic limit and then the actual curve DIJ will drop away from DF. It should be noted that the upper part of curve DE is approximately a rectangular hyperbola which approaches AB as an asymptote.

In conclusion, the writer wishes to congratulate Mr. Ryder on his clear presentation of the nonlinear theory which will clarify for many engineers the questions in their minds as to the behavior of columns.

[24] "On the Analysis of Experimental Observations in Problems of Elastic Stability," by R. V. Southwell, *Proceedings*, Royal Soc. of London, Vol. 135, 1932, pp. 606–616.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FRICTION COEFFICIENTS IN A LARGE TUNNEL

Discussion

BY WESTON GAVETT

WESTON GAVETT,[13] Assoc. M. ASCE.—Friction coefficients for large tunnels with three types of wall surface such as described are of great interest.

Apparently the piezometers were located with difficulty, for in some instances several types and sizes of tunnel with bends and transitions are included between piezometers, requiring successive approximations to arrive at the coefficient for each part.

TABLE 10.—VALUES OF x IN $H_f = C\,V^x$

Tunnel section	Friction Loss		x
	Table	Column	
(1)	(2)	(3)	(4)
A-B	2	7	2.00
B-C	2	8	2.09
D-E	2	9	2.01
18-ft concrete	8	2	2.18
Unlined rock	8	6	1.89

Figs. 11 and 12, showing the relation of Manning's n to V, indicate a straight line for the 18-ft diameter steel pipe but show curves for the 18-ft diameter concrete tunnel and the unlined rock tunnel. A straight line shows that the exponent x in the equation $H_f = C\,V^x$ is 2.00, whereas a curve indicates that x is greater or less than 2.00. As the friction loss for the 18-ft steel pipe, for which $x = 2.00$, was obtained in a straight run between points A and B with no other types, sizes, or bends to complicate the computations, the question arises as to whether the values of x for the 18-ft concrete tunnel and the unlined rock tunnel are, in fact, more and less, respectively, than 2, or whether the variations are due to the assumptions made in arriving at the coefficients.

TABLE 11.—COMPUTED ROUGHNESS HEIGHT FOR VARIOUS CONDUITS

Type of conduit	Length of sections (ft)	Diameter (in.)	Age (yr)	Roughness height, k (in.)
(1)	(2)	(3)	(4)	(5)
Concrete, lock joint	12	51	6.7	0.008
Steel, riveted	7½	72	17.0	0.55
Steel, riveted	7½	72	23.0	0.77
Steel, riveted	7	48	27.4	0.37
Steel, riveted	7	48	39.2	0.894
Steel, riveted	7	42	27.4	0.23–0.40
Steel, riveted	7	42	43.8	0.85
Steel, lock bar	30	72	4.0	0.18
Steel, lock bar	30	74	0.6–9.6	0.028–0.056

NOTE.—This paper by G. H. Hickox, A. J. Peterka, and R. A. Elder was published in April, 1947, *Proceedings*.

[13] Asst. Engr., Clyde Potts, New York, N. Y.

By computation, the writer obtained the values of x shown in Table 10, using figures for Q, in cubic feet per second, from Table 3, Cols. 4 and 13, and friction losses from other data in the paper as shown in Cols. 2 and 3, Table 10. It will be noted that the values of x vary only slightly from 2 for the observed data in sections A-B, B-C, and D-E.

Of special interest is the finding that Eq. 6, derived from experiments with 2.5-cm, 5-cm, and 10-cm pipes, gives plausible results for the roughness height k for the 18-ft to 22-ft diameter tunnels. Use of the same equation to compute the roughness height in pipes described elsewhere[14,15] gave the results shown in Table 11. The Nikuradse[11,12] sand size appears to be a practical standard for evaluating wall roughness, comparing different types of pipe, and indicating the growth of roughness height with age. Obviously, many types of roughness are not similar to uniform sand; but some standard is desirable, and any form of roughness may be referred to Nikuradse's sand as an equivalent sand size as was done by Von H. Schlichting[16] for various forms of artificial roughness.

[14] "Trends in Coefficients of Large Pressure Pipes," by Charles H. Capen, *Journal*, A.W.W.A., January, 1941, p. 1.

[15] Discussion by Weston Gavett of "Trends in Coefficients of Large Pressure Pipes," by Charles H. Capen, *ibid.*, April, 1942, p. 524.

[11] "Widerstandsgesetz und Geschwindigkeitsverteilung von turbulenten Wasserstromung in glatten und rauhen Rohren," *Proceedings*, III International Cong. on Technical Mechanics, Stockholm, 1930.

[12] "Stromungsgesetze in rauhen Rohren," *Forschungsheft*, Vereines Deutscher Ingenieure, Heft 361, 1933.

[16] "Experimentelle Untersuchungen zum Rauhigkeitsproblem," by Von H. Schlichting, *Ingenieur-Archiv*, Vol. VII, No. I, 1936.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THRUST EXERTED BY EXPANDING ICE SHEET

Discussion

BY EDWIN ROSE

EDWIN ROSE,[6] ESQ.—Temperature conditions in an ice sheet are governed by the fundamental equation for the case of one-dimensional flow of heat:

$$\frac{\partial \theta}{\partial t} = h^2 \frac{\partial^2 \theta}{\partial x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

E. Schmidt (11)[6a] gives an approximate method for determination of temperatures to satisfy Eq. 10 under various boundary conditions. Use of this method is illustrated by the following example. Temperatures are computed in an ice sheet 2 ft thick based on the assumptions given in Section 4 for curve B, Fig. 2(*a*). The 2-ft ice sheet is assumed divided into eight slabs of equal thick-

TABLE 3.—TYPICAL COMPUTATION OF ICE TEMPERATURES, IN DEGREES FAHRENHEIT

ELAPSED TIME		DISTANCE FROM TOP SURFACE								
Intervals (1)	Hours (2)	0 (3)	1 ΔL = 0.25 ft (4)	2 ΔL = 0.50 ft (5)	3 ΔL = 0.75 ft (6)	4 ΔL = 1.00 ft (7)	5 ΔL = 1.25 ft (8)	6 ΔL = 1.50 ft (9)	7 ΔL = 1.75 ft (10)	8 ΔL = 2.00 ft (11)
0....	0	−40	−31	−22	−13	−4	5	14	23	32
1 Δt....	0.72	−32.8	−31	−22	−13	−4	5	14	23	32
2 Δt....	1.44	−25.6	−27.4	−22	−13	−4	5	14	23	32
3 Δt....	2.16	−18.4	−23.8	−20.2	−13	−4	5	14	23	32
4 Δt....	2.88	−11.2	−19.3	−18.4	−12.1	−4	5	14	23	32
5 Δt....	3.60	− 4.0	−14.8	−15.7	−11.2	−3.5	5	14	23	32
6 Δt....	4.32	3.2	− 9.9	−13.0	− 9.6	−3.1	5.2	14.0	23	32
7 Δt....	5.04	10.4	− 4.9	− 9.7	− 8.0	−2.2	5.4	14.1	23.0	32.0

ness, ΔL, equal to 0.25 ft. From Eq. 1, the time interval, Δt, equals 0.72 hour. By Eq. 2, the temperature gradients at time intervals Δt are computed as shown in Table 3.

NOTE.—This paper by Edwin Rose was published in May, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1946, by Ernest Brown, B. R. McGrath, and Robert E. Kennedy.

[6] Engr., Bureau of Reclamation, U. S. Dept of the Interior, Denver, Colo.

[6a] Numerals in parentheses, thus: (11), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

Use of Eq. 2 in a typical computation is shown by selecting three consecutive points $p = 0$, $q = 1\,\Delta L$, and $r = 2\,\Delta L$, at time $4\,\Delta t$. Then the temperature θ'_q at time $5\,\Delta t$ equals $\frac{\theta_p + \theta_r}{2}$ or $\frac{(-11.2) + (-18.4)}{2} = -14.8$, as given in

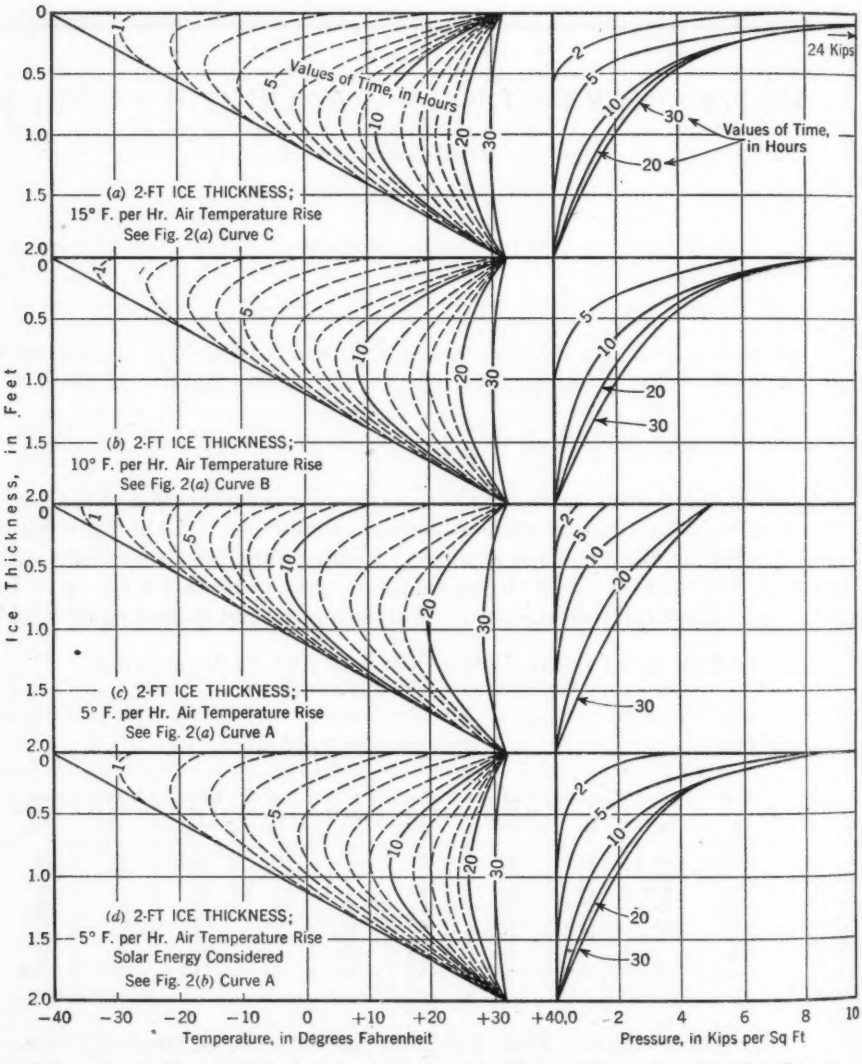
Ice Thickness, in Feet
Values of Time, in Hours
24 Kips
Values of Time, in Hours
(a) 2-FT ICE THICKNESS;
15° F. per Hr. Air Temperature Rise
See Fig. 2(a) Curve C
(b) 2-FT ICE THICKNESS;
10° F. per Hr. Air Temperature Rise
See Fig. 2(a) Curve B
(c) 2-FT ICE THICKNESS;
5° F. per Hr. Air Temperature Rise
See Fig. 2(a) Curve A
(d) 2-FT ICE THICKNESS;
5° F. per Hr. Air Temperature Rise
Solar Energy Considered
See Fig. 2(b) Curve A
Temperature, in Degrees Fahrenheit
Pressure, in Kips per Sq Ft

FIG. 7.—ICE TEMPERATURE CURVES AND RESULTING PRESSURE CURVES FOR 2-FT ICE SHEET

Table 3. The temperatures may be computed in this manner or by the graphical method indicated by Mr. McGrath. Determination of temperature changes in an ice sheet by solution of Eq. 10 for various exposure conditions has been obtained by F. W. Taylor.

Figs. 7 and 8 show estimated temperature gradients and the corresponding pressure curves and total thrusts for several selected examples covering a fairly broad range of conditions. Fig. 7 gives curves and data for a 2-ft ice sheet with air temperature rises of 5° F, 10° F, and 15° F per hr (see curves A, B, and C, Fig. 2(*a*)), and also for the case of 5° F per hr with solar energy effects included as discussed in Section 5 (see curve A, Fig. 2(*b*)). Fig. 8 illustrates the temperature gradients and pressures developed for 1-ft and 4-ft ice sheets with an assumed air temperature rise of 10° F per hr (see curve B, Fig. 2(*a*)).

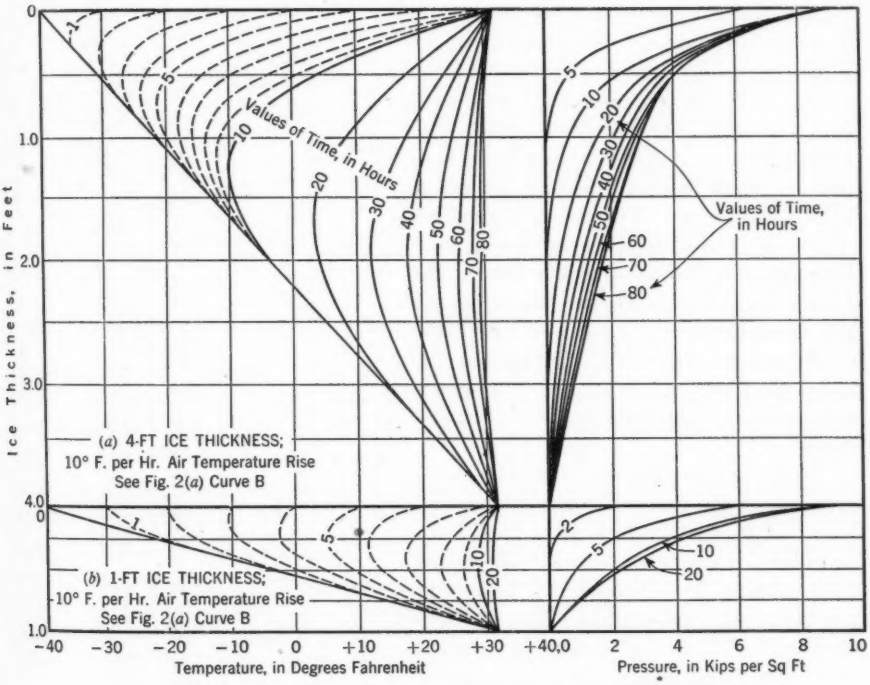
Ice Thickness, in Feet
Values of Time, in Hours
Values of Time, in Hours
(a) 4-FT ICE THICKNESS;
10° F. per Hr. Air Temperature Rise
See Fig. 2(a) Curve B
(b) 1-FT ICE THICKNESS;
10° F. per Hr. Air Temperature Rise
See Fig. 2(a) Curve B
Temperature, in Degrees Fahrenheit
Pressure, in Kips per Sq Ft

FIG. 8.—ICE TEMPERATURE CURVES AND RESULTING PRESSURE CURVES

The temperature gradients shown in Figs. 7 and 8 are estimated for 1-hour time increments. From these several temperature gradients, the pressures at the selected depths in the ice sheet are determined by the data given in Fig. 1, and pressure curves are drawn for several periods of time. These pressure curves are shown in Figs. 7 and 8 for times corresponding to the temperature curves. By summation of the area under these curves, the total thrust developed per linear foot at the various times is given in Table 4. The last value for each condition in Table 4 corresponds to the value of thrust per linear foot plotted on the appropriate curve in Fig. 2.

Results of Mr. McGrath's analysis of ice pressures for an assumed air temperature rise of 10° F per hr, as given in Figs. 4 and 6 and Table 1, show a good comparison with data presented in Figs. 2(*a*), 7, and 8. Mr. McGrath

presents the pertinent thought that pressures likely to develop in thick ice will probably be less than those obtained in the analysis. The writer agrees that some reduction in the pressures as shown in Figs. 2, 6, and 8 for ice sheets thicker than 2 ft would seem justified due to the unlikely occurrence of the assumed constant air temperature of 32° F for a period of from 30 hours to 70 hours following a rapid rise in temperature from −40° F. Inspection of the pressure curves given in Fig. 8 for the 4-ft ice sheet reveals that the maximum pressure ordinate at considerable depth in the ice is not reached until about 80 hours after the beginning of the temperature rise. In addition, the high pressures at and near the top surface of the ice occur within the first 10 hours; therefore, because of the tendency of flow under load, this high pressure would very likely be reduced appreciably before the maximum pressures at the greater depths are reached some 70 hours later. This consideration serves to substantiate the belief that the curves in Fig. 2 are very conservative, particularly for thick ice sheets.

TABLE 4.—TOTAL THRUST OF ICE SHEETS FOR VARIOUS CONDITIONS

Ice thickness (ft) (1)	Air temperature rise (degrees F per hr) (2)	Time (hr) (3)	Total thrust (kips per lin ft) (4)	Fig. No. (5)
1.....	10	2	0.3	8(*b*)
		5	1.6	
		10	2.8	
		20	2.9	
2.....	15	2	1.2	7(*a*)
		5	4.7	
		10	6.3	
		20	7.0	
		30	7.3	
2.....	10	2	0.5	7(*b*)
		5	1.7	
		10	3.6	
		20	4.7	
		30	5.0	
2.....	5	2	0.2	7(*c*)
		5	0.7	
		10	1.5	
		20	3.5	
		30	4.1	
2.....	5[a]	2	0.8	7(*d*)
		5	2.6	
		10	4.0	
		20	4.7	
		30	5.0	
4.....	10	5	1.8	8(*a*)
		10	3.8	
		20	5.3	
		30	6.5	
		40	7.4	
		50	8.1	
		60	8.5	
		70	8.9	
		80	9.1	

[a] Solar energy considered.

By estimating the pressures for lateral restraint with Poisson's ratios equal to 0.25 and 0.50, Mr. McGrath provides a considerable range of values, from which the designer by exercise of judgment may select design values to fit a particular case. The use of Poisson's ratio of 0.50 gives extremely severe restraint conditions. Ice pressures determined on the basis of Poisson's ratio of 0.365 and the assumption of elastic behavior are believed sufficiently conservative. Elastic behavior of ice, as assumed in computing lateral restraint effects, is approached only when the temperature change of the ice is very rapid and the flow strain is small—a condition which rarely occurs, particularly at any considerable depth in the ice sheet. In view of the uncertainty of the value of Poisson's ratio for ice, the use of a range of values may be desirable, but a lower top value is suggested.

The tabulation of ice thicknesses recorded on reservoirs and lakes on various projects as listed by Mr. Kennedy in Table 2 is a worthy contribution of data. Credit is also due those men in the field who performed the difficult task of gathering the data. By a statistical analysis of the data in Table 2,

Mr. Kennedy derives Eq. 9, which expresses the maximum ice thickness as a function of latitude and elevation. This equation, although limited in accuracy due to both the short period of record of data and the omission of pertinent factors cited, can be used to approximate ice thicknesses. It is believed that a more adequate expression might be derived if consideration were given to air temperatures along with the data in Table 2 as fundamental factors influencing ice thickness.

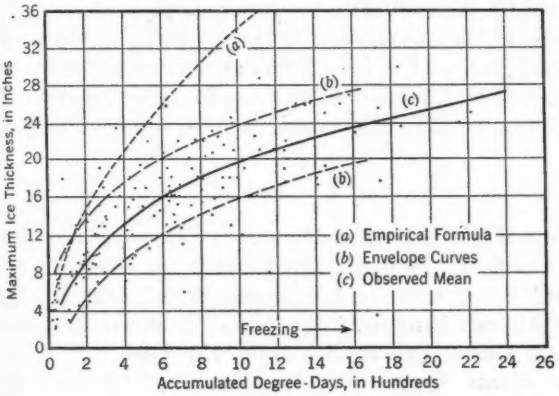

FIG. 9.—MAXIMUM ICE THICKNESS FOR FREEZING DEGREE-DAYS

Mr. Kennedy, in his study of the listed data, plotted the maximum recorded ice thicknesses against accumulated degree-days as shown in Fig. 9. Curve (c) is the mean line drawn by eye, with curves (b) enclosing roughly two thirds of the plotted points. Curve (a) represents the growth of an ice sheet as determined by the empirical formula given by N. E. Dorsey (4*a*) assuming quiet water and no snow. In a recent paper (21), Robert P. Sharp presents a similar curve, which falls approximately midway between curve (a) and curve (c) in Fig. 9. The wide range between the empirical curve and the recorded data in Fig. 9 is probably due to variation of such factors as wind, current, snow, cloudiness, etc. However, Fig. 9 may be used as a guide to predict the expected maximum ice thickness, if air temperatures are known or can be estimated.

In personal correspondence with S. P. Wing, M. ASCE, regarding allowance made for ice pressure in design of dams in Norway, Thor G. Brandzaeg states that, although no official Norwegian regulations are printed as yet:

> "* * * up to 1938 an ice pressure of 10 metric tons per meter (6720 lbs. per ft.) was used for design of dams; since then the ice load has been reduced to 8 tons per meter (5380 lbs. per ft.) for dams with vertical upstream faces and 5 tons per meter (3350 lbs. per ft.) for dams with sloping upstream faces."

In another paper (22), Paul E. Gisiger, M. ASCE, states that:

> "* * * static ice pressure, except in very confined areas or spaces, will, under the most severe climatic conditions, seldom exceed 10,000 lb per lin ft."

The writer agrees with Mr. McGrath that much remains to be accomplished to secure a better understanding of ice pressures. The Bureau of Reclamation under the immediate supervision of Douglas McHenry has undertaken a program to secure both field and laboratory data on ice pressures and related problems. No definite results are as yet available, but some field work has

been in progress. As Professor Brown states, the engineer may not be able to satisfy his desire for precise data and will have to use his best judgment after careful consideration and study of available information. It is believed that, until more precise information is available, the material presented in the paper serves as a conservative basis for the designer and enables him to estimate the expected maximum ice thrust by the rational procedure indicated.

The writer wishes to express his appreciation to those who commented on this paper. It is hoped that interest has been stimulated in this subject to hasten the time when there will be a more definite basis for determination of ice thrusts, despite the large number of variables and the recognized complexity of the problem.

Bibliography.—

(4) **"Properties of Ordinary Water Substance,"** by N. E. Dorsey, Reinhold Pub. Corp., New York, N. Y., 1940, pp. 395–510. (*a*) p. 409.
(11) **"A. Föppl's Festschrift,"** by E. Schmidt, J. Springer, Berlin, 1924, p. 179.
(21) **"Suitability of Ice for Aircraft Landings,"** by Robert P. Sharp, *Transactions*, Am. Geophysical Union, Vol. 28, No. 1, pp. 111–119.
(22) **"Safeguarding Hydro Plants Against the Ice Menace,"** by Paul E. Gisiger, *Civil Engineering*, January, 1947, p. 24.

Correction for *Transactions:* In May, 1946, *Proceedings*, page 581, Figs. 3(*a*) and 3(*b*), the curves farthest to the left should read 5 kips per lin ft not 50 kips per lin ft.